



TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

SIXTIETH ANNUAL MATHEMATICS CONTEST

2016

Statistics

Prepared by:

Department of Mathematical Sciences
University of Memphis
Memphis, Tennessee

Reviewed by:

Mathematics Faculty
Carson-Newman University
Jefferson City, Tennessee

Coordinated by: Alistair Windsor

Scoring formula: $4 \times (\text{Number Right}) - (\text{Number Wrong}) + 40$

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you; if you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly eighty minutes to work.

1. In a statistics class, a student scored 84, 86 and 88 on the first three of four exams. What score must be obtained on the next test for the mean to remain unchanged?
 - A. 84
 - B. 85
 - C. 86
 - D. 90
 - E. 88

2. A polling firm conducting a political poll surveys every fourth shopper leaving a grocery store during the lunch hour. Which of the following best describes the sampling strategy?
 - A. Convenience
 - B. Systematic
 - C. Stratified
 - D. Cluster
 - E. Simple random

3. A forestry company randomly samples the trunk diameter of 200 trees from one of its forests. Which of the following charts could best be used to display this data?
 - A. Bar Diagram
 - B. Scatter Plot
 - C. Pie Chart
 - D. Histogram
 - E. Line Graph

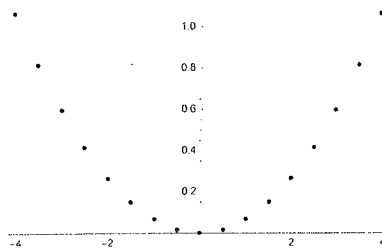
4. A survey was conducted at a local employment agency to determine satisfaction with services rendered by the agency. Every third applicant on a given day is selected and asked to complete a questionnaire. What sampling method was used?
 - A. Convenience
 - B. Systematic
 - C. Cluster
 - D. Stratified
 - E. Simple random

5. Summary statistics for students' test scores are given below.

Summary Statistics	
Mean	71.43
Median	76
Mode	#N/A
Standard Error	2.30465
Standard Deviation	13.63
Sample Variance	185.899
Kurtosis	2.07908
Skewness	-0.386921
Range	52
Minimum	40
Maximum	92

Based on this data, if a student has a z-score of 0.70 to two decimal places, what is her test score?

- A. 70
 - B. 73
 - C. 76
 - D. 81
 - E. 92
6. If it snows each day independently with chance 30% for the next seven days, what is the probability that it will snow on at least two of the days?
- A. 2%
 - B. 32%
 - C. 67%
 - D. 75%
 - E. 92%
7. The scatter plot below shows two datasets.



Which of the following is closest to the Pearson Correlation Coefficient?

- A. -0.5
- B. -0.1
- C. 0
- D. 0.1
- E. 0.5

8. Consider two sets of test scores. On the first test the mean score was 70 with a standard deviation of 8, while on the second test the mean was 80 with standard deviation of 6. A student scored 86 on both tests. The teacher would like to grade both tests on a curve using standardized z-scores for each test. What can be said about the student's performances on the two tests based on her z-scores
- The 86 on the first test is twice as high compared to the mean than the 86 on the second test.
 - The 86 on the first test is half as high compared to the mean than the 86 on the second.
 - The 86 on the first test is equivalent to the 86 on the second test.
 - The scores of 86 will both be considered A's when the scores are curved since they are both two standard deviations above their means.
 - The 86 on the first test is clearly an outlier.

9. To convert from a temperature in Celsius to a temperature in Fahrenheit you use the following formula:

$$T(^{\circ}F) = \frac{9}{5} \times T(^{\circ}C) + 32.$$

Suppose that a set of temperatures recorded in Celsius are converted to Fahrenheit. What can be said about what happens to the mean and standard deviation of the set of temperatures.

- After conversion the mean and standard deviation are higher.
 - After conversion the mean and standard deviation are lower.
 - After conversion the mean may be higher or lower but the standard deviation is lower.
 - After conversion the mean may be higher or lower but the standard deviation is higher.
 - After conversion the mean is higher and the standard deviation is lower.
10. A manufacturing facility creating computer components is comparing two different machines, Machine A and Machine B. A random sample of 250 components is taken from each machine. Machine A has produced 7 defective components but Machine B has produced 10 defective components. Which of the following would be the most appropriate for determining whether there is a statistically significant difference in the machines?
- A one-sample proportion z-test
 - A matched pairs t-test
 - A Chi-square test of association
 - A one-sample z-test
 - A two-sample proportion z-test
11. A game is played with the following rules:

- You roll a fair die.
- If you roll a 1 you win \$12.
- If you roll a 3 or 5 you win \$1.
- If you roll an even number you lose \$5.

What is the expected amount you will win or lose when you play the game?

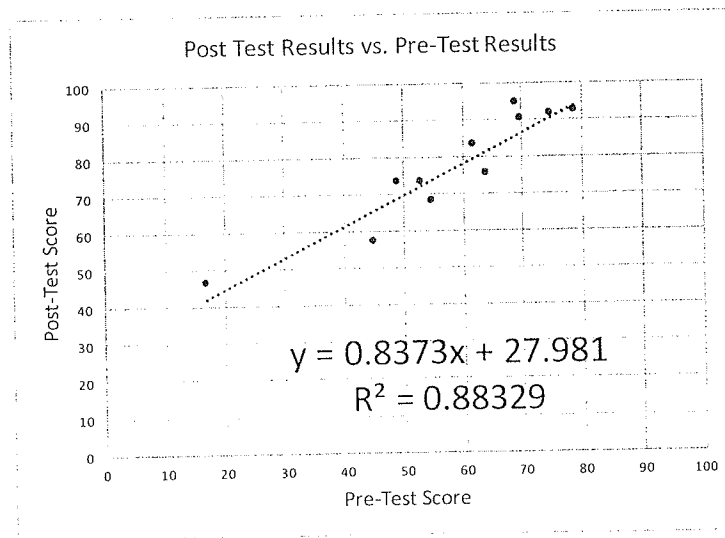
- An expected win of \$1.
- An expected loss of \$1.
- An expected win of 1/6th of a dollar.
- An expected loss of 1/6th of a dollar.
- An expected win of \$2.

12. The mean on a national exam in 2012 was 46.2 with a standard deviation of 10.8. Assuming that the scores are distributed normally, what interval would contain 95% of all scores on the exam?
- 32.1 to 60.3
 - 25.0 to 67.4
 - 40.0 to 52.4
 - 18.6 to 73.7
 - 33.5 to 58.9
13. A survey of 500 students' study habits found that the 190 students who studied in groups had a higher final average than the 310 students who studied alone. The study indicates:
- Studying alone will cause a lower final average.
 - Nothing because the two factors have no relationship.
 - Those students who are prone to lower averages will study alone.
 - There seems to be an association between whether you study alone or in a group and final average.
 - The correlation coefficient between the two variables will be close to 1.
14. A survey was conducted recently of 300 college students to determine which social media outlet was their favorite. The following data table shows the results of the survey.

	Social Media Platform Favored					Total
	WhatsApp	Facebook	Vine	ShapChat	Twitter	Total
Male	76	34	25	10	5	150
Female	70	36	30	7	7	150
Total	146	70	55	17	12	300

- What is the probability that a randomly selected student is Male or favored Vine?
- 0.092
 - 0.167
 - 0.455
 - 0.600
 - 0.683
15. Using the data table from the previous problem, what is the probability that a randomly selected student favored SnapChat, given he was male?
- 0.067
 - 0.028
 - 0.033
 - 0.495
 - 0.800

16. A certification test consists of 12 questions each with four choices, and students must get at least seven correct to pass. If a particular student randomly selects the answer for every question, what is the probability of passing the test?
- A. 0.014
 B. 0.054
 C. 0.250
 D. 0.946
 E. 0.986
17. Which of the following statements about correlation and linear regression is correct?
- A. If the sign of the slope of the linear regression line is negative, the correlation coefficient will also be negative.
 B. The closer the correlation coefficient gets to 1, the higher the slope.
 C. The lower the slope, the lower the correlation coefficient.
 D. A negative slope will result in a correlation coefficient close to 0.
 E. The value of the slope indicates how strongly two variables are related
18. A study looks at a pre-test and a post-test to determine how effective instruction is. Data is gathered on 10 students.



Which of the following is a correct interpretation of the data?

- A. For every 10 point increase of the pre-test scores, the post-test score increases by approximately 8.4
 B. For every 10 point increase of the pre-test scores, the post-test score increases by approximately 8.8
 C. There is no correlation between pre-test score and post-test score
 D. For every 10 point increase on the post-test, the pre-test score increases by approximately 8.4
 E. For every 10 point increase on the post-test, the pre-test score increases by approximately 8.8

19. Based on the data in the previous problem, if you changed the post-test to the x-axis and pre-test to the y-axis, what would be the correct result?
- A. The slope and correlation coefficient values would stay the same but the signs for both would change.
 - B. The new linear regression equation would be $y = 27.981x + 0.8373$.
 - C. The slope would change but the correlation coefficient would stay the same.
 - D. The correlation coefficient would change but the slope would stay the same.
 - E. The slope would decrease.
20. A standard deck of 52 cards has 12 face cards, 4 aces and 36 numbered cards. If two cards are drawn without replacement from the deck, what is the probability of drawing two aces?
- A. 0.45%
 - B. 0.91%
 - C. 1.45%
 - D. 31.02%
 - E. 49.95%
21. A soft drink company uses a filling machine to fill cans. Each 12 ounce can is to contain 355 milliliters of beverage. In fact the amount varies according to a normal distribution with mean $\mu = 355.2\text{ml}$ and standard deviation $\sigma = 0.5\text{ml}$. What is the probability that the mean content of a six-pack of cans is less than 355ml?
- A. 0.8365
 - B. 0.5793
 - C. 0.5000
 - D. 0.1635
 - E. 0
22. A random sample of 1000 Tennessee business owners were surveyed. There were 236 women business owners. The conservative margin of error for this poll is 3.2%.
- A. We can be 95% confident that the percentage of business owners in this survey who are women is between approximately 20.4% and 26.8%.
 - B. We can be 95% confident that the percentage of all Tennessee business owners who are women is between approximately 20.4% and 26.8%.
 - C. We can be 95% confident that the percentage of all US business owners who are women is between approximately 20.4% and 26.8%.
 - D. We can be 95% confident that the number of business owners in the survey who are women is between 204 and 268.
 - E. 95% of all Tennessee residents have fishing licenses with a margin of error of $\pm 3.2\%$

23. A simple random sample of 25 cat owners in the Memphis metropolitan area is selected and asked how much money they spend each month on cat food and litter. The mean of the sample was \$31.25 with a standard deviation of \$6.43. Find the 99% confidence interval for the population mean amount of money spent on cat food and litter each month. Assume the amount of money spent on cat food and litter for Memphis residents follows a normal distribution.
- A. (\$13.30, \$49.26)
 - B. (\$27.65, \$34.85)
 - C. (\$27.97, \$34.59)
 - D. (\$14.72, \$47.84)
 - E. (\$28.08, \$34.48)
24. An SAT coaching program guarantees an average increase on the verbal score of the SAT of 15 points after taking the course. If we wish to test this claim by testing the null hypothesis $H_0 : \mu \geq 15$ versus the alternative $H_1 : \mu < 15$ where μ is the mean increase in verbal test score after taking the course, which of the following would be a Type 2 error?
- A. Concluding that the mean increase is greater than or equal to 15 when it is in fact at least 15.
 - B. Concluding that the mean increase is greater than or equal to 15 when it is in fact less than 15.
 - C. Concluding that the mean increase is less than 15 when it is in fact less than 15.
 - D. Concluding that the mean increase is less than 15 when it is in fact greater than 15.
 - E. Concluding that the mean increase is 20 when it is only 15.
25. In 2013 the average debt of a student with student loans who graduated college with a Bachelors degree was \$24,000. A claim is made that in 2015 the average debt was \$25,000. A sample of 100 graduating students with student loans was selected and surveyed about their debt. The mean of the sample was \$24,778 and the standard deviation was \$3620. Conduct a hypothesis test of the claim with $\alpha = 0.05$ and determine which outcome would be the most appropriate.
- A. There is enough evidence to reject the claim.
 - B. There is enough evidence to support the claim.
 - C. There is not enough evidence to reject the claim.
 - D. There is not enough evidence to support the claim.
 - E. The test could not be conducted because the population standard deviation is unknown.
26. A die is rolled until an even number is obtained. What is the probability that the die will need to be rolled 3 or more times?
- A. 1/36
 - B. 1/16
 - C. 1/8
 - D. 1/4
 - E. 1/2

27. Recruits in the US Navy are given a physical fitness test to assess their overall condition. As part of the test they are required to run 300 yards and the time in seconds that it takes them is recorded. An admiral recalls that the average time for his class of recruits was 49.7 seconds. He wishes to test the hypothesis that the mean running time for current recruits is at least 49, $H_0 : \mu \geq 49$. Based on a 95% confidence interval of (42.14, 48.46) computed from the running times of 10 recruits from the current class, what can be said about the mean running time of current recruits?
- A. The null hypothesis can be rejected at the $\alpha = 0.05$ level of significance.
 - B. The null hypothesis cannot be rejected based on this evidence.
 - C. The p-value for the test is greater than 0.05.
 - D. The mean time for new recruits, $\mu = 45.3$.
 - E. The mean time for new recruits is 49.
28. Suppose that 90% of drivers wear seat-belts. Of drivers involved in serious accidents 60% of those not wearing a seat-belt died while only 10% of those wearing a seat-belt died. What percentage of the drivers that died were not wearing seat-belts?
- A. 10%
 - B. 30%
 - C. 40%
 - D. 50%
 - E. 60%
29. The impurity levels in a chemical used in a manufacturing process are normally distributed with mean μ and standard deviation $\sigma = 2$ ppm. The impurity level must not exceed 50ppm. Ten samples from a batch resulted in a sample mean of $\bar{X} = 48.7$ ppm. Which of the following can be said if the p-value is used to test $H_0 : \mu \geq 50$ versus $H_1 : \mu < 50$?
- A. The null hypothesis can be rejected at the 0.01 level.
 - B. The null hypothesis can be rejected at the 0.05 level but not at the 0.01 level.
 - C. The null hypothesis can be rejected at the 0.10 level but not at the 0.05 level.
 - D. The null hypothesis can be rejected at the 0.30 level but not at the 0.25 level.
 - E. The p-value cannot be used to test H_0 .
30. A random sample of graduating students was surveyed and asked whether they had any student loan debt. The proportion of graduating students from the sample who said they had any student loan debt was 61.16%. The 95% confidence interval based on this data for the population proportion of all graduating students who said they had any student loan debt was (57.615%, 64.705%). What was the sample size?
- A. There is not enough information to determine the sample size.
 - B. $n = 48$
 - C. $n = 181$
 - D. $n = 612$
 - E. $n = 726$

31. A professional basketball player has been a 70% free throw shooter throughout his career. In the current season he made 300 of his 400 attempted free throws. Let p be the player's true free throw percentage. What would be the appropriate null (H_0) and alternative (H_1) hypotheses to test whether the player's free throw percentage has improved?
- A. $H_0 : p = 0.70, H_1 : p \neq 0.70$
 - B. $H_0 : \mu = 300, H_1 : \mu > 300$
 - C. $H_0 : p = 0.70, H_1 : p < 0.70$
 - D. $H_0 : p = 0.70, H_1 : p > 0.70$
 - E. $H_0 : p = 3/4, H_1 : p > 3/4$
32. To study the effectiveness of insulation in saving energy on home utility costs, energy consumption for 100 houses in Ft. Lauderdale Florida was recorded for two years. The first year was before insulation and the second year was after insulation. Which of the following analyses is appropriate for testing the null hypothesis that insulation has no effect on energy consumption?
- A. two sample t-test with equal variances
 - B. two sample t-test with unequal variances
 - C. a matched paired t-test
 - D. a test of equal proportions
 - E. a test of equal variances
33. A study is conducted to determine whether a new method of tutoring changes the average results of standardized test scores. The mean of the standardized test scores without the new tutoring method is known to be 15. The standard deviation of the standardized scores is known to be 40. A sample of size $n = 20$ was used to test whether the new tutoring had a positive or negative effect on the standardized scores. If the sample yielded a mean score of $\bar{X} = 35$, what is the appropriate p-value that should be reported? (Assume the data is normally distributed.)
- A. 0.025
 - B. 0.013
 - C. 0.26
 - D. 2.236
 - E. -2.236
34. If the histogram for a data set is symmetric, which of the following is most likely to be true about the underlying population of interest?
- A. The mean and the median are equal.
 - B. The population is normally distributed.
 - C. There are at least two modes.
 - D. The mean is greater than the median.
 - E. The mean is less than the median.

35. A high fat diet has been associated with increased incidence of breast and colon cancer in several observational studies. Which of the following can we conclude about this finding?
- A. A high fat diet causes either breast or colon cancer but not both.
 - B. A high fat diet causes both breast and colon cancer.
 - C. Having colon or breast cancer causes a person to eat a high fat diet.
 - D. There may be a relationship between eating a high fat diet and breast and colon cancer.
 - E. Drinking wine causes breast and colon cancer and causes a person to eat a high fat diet.
36. A diagnostic test for a disease can be viewed as a hypothesis testing situation with the null hypothesis (H_0) that the person does not have the disease and the alternative hypothesis (H_1) that the person has the disease. Which of the following would be considered a Type II error for this test?
- A. The power of the test.
 - B. The size of the test.
 - C. A false positive.
 - D. A false negative.
 - E. The risk of contracting the disease.
37. A box contains 10 balls. X of them are black and the remaining $10 - X$ are purple. A ball is drawn randomly from the box, its color is noted and it is returned to the box. This process is then repeated. A test of $H_0 : X = 1$ versus $H_1 : X = 9$ rejects H_0 if both balls drawn are black. What is the probability of a Type I error?
- A. 0
 - B. 0.01
 - C. 0.10
 - D. 0.99
 - E. 0.19
38. Twenty-nine measurements of the density of the earth (expressed as multiples of the density of water) yielded a mean $\bar{X} = 5.448$ and a sample standard deviation $s = 0.221$. Assuming that the measurements follow a normal distribution, a 95% confidence interval for the density of the earth is given by:
- A. (5.398, 5.498)
 - B. (5.423, 5.473)
 - C. (5.429, 5.467)
 - D. (5.368, 5.528)
 - E. (5.364, 5.532)

39. Which of the following sets of statistics are substantially influenced by outliers?
- A. mean, median, standard deviation
 - B. variance, range
 - C. median, five number summary
 - D. interquartile range
 - E. median, mean, mode
40. An airline uses sampling to estimate the mean cost of tickets for its passengers. Suppose the airline wants to estimate the mean cost within \$5 using a 99% confidence interval. An estimate of the standard deviation of ticket cost is $\sigma = 50$. How many tickets should be sampled assuming normality?
- A. 664
 - B. There is not enough information to determine the sample size needed.
 - C. 13
 - D. 30
 - E. 1000