

FIFTY-EIGHTH ANNUAL MATHEMATICS CONTEST
sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

Statistics 2014

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Scoring formula: $4R - W + 40$

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft head (No. 2 lead or softer).

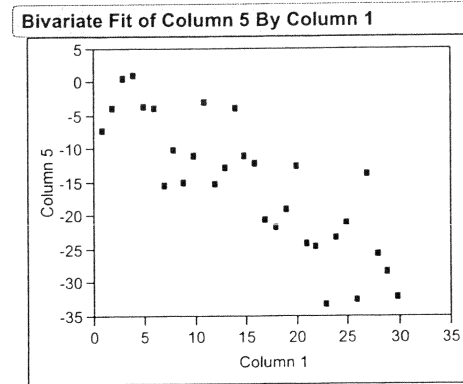
This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly 80 minutes to work.

1) Estimate the value of r for the scatter plot

- (A) $r \approx -.8$
- (B) $r \approx .2$
- (C) $r \approx 0$
- (D) $r \approx -.2$
- (E) $r \approx .8$



2) Which of these animals has the shortest length relative to the length of others in their species if their lengths have the following Z-scores?

- (A) A sardine with a corresponding Z-score of 2.93
- (B) An orca whale with a corresponding Z-score of -1.79
- (C) A great white shark with a corresponding Z-score of -3.56
- (D) A barracuda with a corresponding Z-score of 0
- (E) A dolphin with a corresponding Z-score of 3.14

3) Which of these is true if 29 is the 64th percentile of a data set?

- (A) 64% of the values in the data set are less than or equal to 29
- (B) 100% of the values are between 29 and 64
- (C) 29% of the values in the data set are less than or equal to 64
- (D) 29% of the values in the data set are more than or equal to 64
- (E) 64% of the values in the data set are more than or equal to 29

4) A statistics class has 140 students enrolled. The final exam grades of 5 of the 140 students are randomly selected grades are 93, 72, 83, 98, and 59. Find the sample standard deviation of these 5 final exam grades.

- (A) 250.5
- (B) 15.8
- (C) 14.2
- (D) 200.4
- (E) 12.4

5) Use the stem and leaf to find the second quartile.

- (A) 88
- (B) 93
- (C) 86
- (D) 6
- (E) 98

7		22356
8		114578
9		3458
10		58
11		
12		4
12		4 represents 124

6) A personal trainer plans his client's workouts so carefully that the lengths of his training sessions are uniformly distributed. His sessions always last between 55 and 60 minutes. Find the probability that a session runs more than 58 minutes.

- (A) 0
- (B) 1
- (C) 0.6
- (D) 0.4
- (E) 0.2

7) Which of these CANNOT be an alternative hypothesis?

- (A) $p > .1$
- (B) $p \neq .1$
- (C) $p < .1$
- (D) $p = .1$
- (E) All of them (A – D) could be an alternative hypothesis

8) A confidence interval for the percentage of a population is [38%, 45%]. Find the values of \hat{p} and E.

- (A) $\hat{p} = 0.83$ and $E = 0.07$
- (B) $\hat{p} = 0.415$ and $E = 0.035$
- (C) $\hat{p} = 0.83$ and $E = 0.035$
- (D) $\hat{p} = 0.415$ and $E = 0.07$
- (E) $\hat{p} = 0.45$ and $E = 0.07$

9) When performing a hypothesis test to test that a mean for a population is less than some specified value, it is known that $\sigma = 4.2$, $n = 30$, and the significance level is 0.01. Find the critical value that would determine the critical region of rejecting the null hypothesis.

- (A) $t = 2.462$
- (B) $z = -2.33$
- (C) $t = -2.462$
- (D) $t = -2.756$
- (E) $z = 2.33$

10) An article written by Arash Markazi for the ESPN website blasted USC head coach Lane Kiffin for going for two points after scoring the first touchdown of the game against Minnesota (which they failed on). He cited NCAA data from 2010 to support his argument. The table below shows how many points were scored on a point after touchdown (PAT) attempt and the proportion of times they occurred. Find the expected value of points scored on a PAT attempt.

Points	Probability
0	0.060
1	0.924
2	0.016

- (A) 1.000
- (B) 0.956
- (C) 0.667
- (D) 0.333
- (E) 0.924

11) If the p-value is 0.0723 and α (the significance level) is 0.05 then you should:

- (A) Reject the ALTERNATIVE hypothesis
- (B) Reject the NULL hypothesis
- (C) Fail to reject the NULL hypothesis
- (D) Support the NULL hypothesis
- (E) Reject the test statistic

12) The time a bull rider stays on the bull is normally distributed with a mean of 4.8 seconds and a standard deviation of 1.9 seconds. What is the probability that for a randomly selected bull ride the rider stays on the bull for more than 8 seconds (the required time for a qualifying ride)?

- (A) 0.898
- (B) 0.102
- (C) 0.046
- (D) 0.954
- (E) 0.454

13) The prices of university textbooks are normally distributed with a mean of \$93.07 and a standard deviation of \$52.87. How much do the most expensive 10% of all textbooks cost?

- (A) \$109.77
- (B) \$160.74
- (C) \$172.00
- (D) \$25.40
- (E) \$128.00

14) In an effort to construct a confidence interval for the mean weight of an adult breed of dog, a simple random sample of 41 adult dogs of this breed was found to have a mean weight of 50.5 lbs. and standard deviation of 9.8 lbs. Select the correct 95% confidence interval.

- (A) $47.4 \leq \mu \leq 53.6$
- (B) $47.5 \leq \bar{x} \leq 53.5$
- (C) $47.5 \leq \mu \leq 53.5$
- (D) $47.4 \leq \bar{x} \leq 53.6$
- (E) Not enough information is given in the problem to find the solution.

15) What is the definition of Alpha (the significance level)?

- (A) The probability of rejecting the alternative hypothesis when it is true
- (B) The probability of rejecting the alternative hypothesis when it is false
- (C) The probability of rejecting the null hypothesis when it is false
- (D) The probability of rejecting the null hypothesis when it is true
- (E) The probability of rejecting the significance level when it is false

16) Which of these best describes what the standard deviation tells you?

- (A) The average distance from the mean
- (B) The relative size of a data value compared to the other values in the sample
- (C) The distance between the minimum and maximum values in a data set
- (D) The average squared distance from the mean
- (E) The value that is bigger than half the values and smaller than the other half of the values

17) A researcher has two samples of the same size. The values in Sample B are exactly twice as big as the values in sample A. How does the sample variance of sample B compare to that of sample A?

- (A) The variance of Sample B is 4 times as big as the variance of Sample A
- (C) The variance of Sample A is twice as big as the variance of Sample B
- (B) The variance of Sample A is 4 times as big as the variance of Sample B
- (D) The variance of Sample B is twice as big as the variance of Sample A
- (E) The two variances are equal

18) Assume that the average time for a beginning runner to run one mile is 10 minutes with a standard deviation of 1.15 minutes. Also assume that the one mile times for all beginning runners have a bell-shaped distribution. Applying the Empirical Rule, one could say:

- (A) About 99% of the one mile times fall within [6 min 55 sec, 13 min 45 sec]
- (B) About 99% of the one mile times fall within [6.55 min, 13.45 min]
- (C) About 95% of the one mile times fall within [6.55 min, 13.45 min]
- (D) About 99% of the one mile times fall within [7.7 min, 12.3 min]
- (E) About 95% of the one mile times fall within [6 min 55 sec, 13 min 45 sec]

19) A fair six sided die is rolled five times. What is the probability that you get at least one three, but they are not all threes?

- (A) $\frac{3125}{7776}$ (B) $\frac{7775}{7776}$ (C) $\frac{4651}{7776}$ (D) $\frac{1}{7776}$ (E) $\frac{4650}{7776}$

20) If you roll 3 fair dice what is the probability that when you multiply the three numbers showing on the faces you get 30?

- (A) 0 (B) $\frac{1}{216}$ (C) $\frac{6}{216}$ (D) $\frac{12}{216}$ (E) $\frac{1}{30}$

21) Suppose that there are 4 people in a room. What is the probability that all 4 people were born on a different day of the week?

- (A) $\frac{360}{2041}$ (B) $\frac{1029}{2041}$ (C) $\frac{840}{2041}$ (D) $\frac{1}{2041}$ (E) $\frac{1372}{2041}$

22) A survey of 302 people asked the following two questions and their responses are in the table below. 1) Which do you like better, plain or peanut m&m's? 2) Given the choice between Coke, Pepsi and Dr.Pepper, which would you chose?

	Coke	Pepsi	Dr.Pepper	TOTAL
Plain m&m's	47	25	38	110
Peanut m&m's	98	54	40	192
TOTAL	145	79	78	302

If one person is randomly selected, what is the probability they prefer peanut m&m's given the selected person prefers Coke?

- (A) $\frac{98}{302}$ (B) $\frac{98}{192}$ (C) $\frac{98}{145}$ (D) $\frac{145}{192}$ (E) $\frac{239}{302}$

23) The number of surfers on Hawaii's Banzai Pipeline beach varies greatly and is definitely not normally distributed. The average number of surfers per day is 397 with a standard deviation of 254 surfers. If you randomly select 41 days, what is the probability that the average number of surfers is less than 300?

- (A) 0.993
- (B) 0.007
- (C) The solution can't be found since the population is not normally distributed.
- (D) 0.024
- (E) 0.476

24) Assume that about 15% of the U.S. population has blue eyes. If 30 people from the U.S. population are randomly selected, what is the probability that at least 6 of them have blue eyes?

- (A) ≈ 0.153
- (B) ≈ 0.863
- (C) ≈ 0.137
- (D) ≈ 0.289
- (E) ≈ 0.711

25) The Tennessee Lottery has a Cash 3 lottery game where you can pay \$1 to select a sequence of 3 digits, such as 123. If you select the EXACT sequence of 3 digits that are drawn, you win \$250. If an individual played this game a large number of times, on average, what amount of money could they expect to gain or lose each time?

- (A) Lose \$1
- (B) Gain \$0.75
- (C) Gain \$0.749
- (D) Lose \$0.75
- (E) Lose \$0.749

26) Australia's most famous beach is Bondi beach near Sydney. A random sample of 61 days in 2012 had an average high temperature of 21.85 degrees Celsius and a standard deviation of 3.65 degrees Celsius. Furthermore, only 11 of those days had measurable rainfall at the beach. Find the margin of error for a 90% confidence interval for the mean high temperature at Bondi beach in 2012.

- (A) $1.671\left(\frac{3.65}{\sqrt{61}}\right)$ (B) $11\left(\frac{3.65}{\sqrt{61}}\right)$ (C) $.90\left(\frac{3.65}{\sqrt{61}}\right)$ (D) $1.96\left(\frac{3.65}{\sqrt{61}}\right)$ (E) $21.85\left(\frac{3.65}{\sqrt{61}}\right)$

27) In a simple random sample of 365 high school seniors, 203 said they have smoked a cigarette before. Which expression below correctly represents the 95% confidence interval for the proportion of high school seniors who have never smoked a cigarette before?

- (A) $0.5562 \pm .0510$
(B) $0.7980 \pm .0552$
(C) $0.4438 \pm .0428$
(D) $0.5562 \pm .0428$
(E) $0.4438 \pm .0510$

28) A recent study of 1500 U.S. teenagers between 13 and 17 years of age who use the internet found that 75% of them have a Facebook account. Sources report that 69.02% of all internet users in the U.S. population have a Facebook account. Does it appear, with 99% confidence, that a larger percentage of U.S. teens between the ages of 13-17 who are internet users have Facebook accounts than the general, U.S. population of internet users?

- (A) Yes, because 69.02% is included in the confidence interval for these teens.
(B) No, because 69.02% is not included in the confidence interval for these teens.
(C) No, because the information is based on a sample.
(D) Yes, because 69.02% is not included in the confidence interval for these teens.
(E) No, because 69.02% is included in the confidence interval for these teens.

29) There are five pairs of identical twins of varying ages. Each pair of twins contains one brother and one sister. Clearly, the mean and variance of the ages will be the same for the set of male twins as it is for the set of female twins. How will the mean and variance of the ages for the set of male twins compare to the mean and variance of the ages of all 10 twins? (Keep in mind this is a sample of twins)

(A) The variances will be the same for the group of 10 as it is for the males, but the mean will be lower for the males.

(B) The variances will be the same for the group of 10 as it is for the males, but the mean will be higher for the males.

(C) The means and variances will be the same for the group of 10 as it is for the males.

(D) The means will be the same for the group of 10 as it is for the males, but the variance will be higher for the males.

(E) The means will be the same for the group of 10 as it is for the males, but the variance will be lower for the males.

30) What is the best definition for P-Value when performing a hypothesis test?

(A) P-Value is the probability of getting the test statistic or a less convincing test statistic, assuming the null hypothesis is true.

(B) P-Value is the probability of rejecting the null hypothesis when it is false.

(C) P-Value is the probability that the test statistic is greater than the critical value.

(D) P-Value is the probability of getting the test statistic or a more convincing test statistic, assuming the null hypothesis is true.

(E) P-Value is the probability of rejecting the null hypothesis when it is true.

31) Which of the following best describes how you decide whether you are supposed to be doing a right tailed, left tailed, or a two tailed hypothesis test?

(A) It is independent of the test statistic equation you are using and completely depends on the form of the alternative hypothesis. If it is (1) a parameter is greater than a number then it is a right tailed test; (2) a parameter is less than a number then it is a left tailed test; (3) a parameter is not equal to a number then it is a two tailed test.

(B) If it's a t-test or a Z test then it's two tailed, but if it's a Chi-Squared test then it's a right tailed test because the Chi-squared distribution doesn't have a left tail.

(C) It depends only on which parameter you are testing.

(D) You consider what sample values would convince you that the alternative hypothesis is true and see what effect that would have on the value of the test statistic.

(E) If your test statistic is in the right tail then it's a right tailed test, if your test statistic is in the left tail then it's a left tailed test, if you have two test statistics then it's a two tailed test.

32) Regarding statements I, II, and III below, which conclusion is true?

I. If a null hypothesis is rejected with a significance level of 0.05, it is also rejected with a significance level of 0.01.

II. If a null hypothesis is rejected with a significance level of 0.01, it is also rejected with a significance level of 0.05.

III. We can never reject the null hypothesis.

(A) Only II is true

(B) None of these are true

(C) Both I and II are true

(D) Only III is true

(E) Only I is true

33) A hypothesis test (with $\alpha = 0.05$) is performed to test that the standard deviation is less than 100 for a normally distributed population. A sample size of 50 is used in determining a sample standard deviation of 98. Which χ^2 test statistic is appropriate?

- (A) 52.9404
- (B) -1.645
- (C) -52.9404
- (D) 47.0596
- (E) -47.0596

34) In March the heights of waves on Lighthouse beach on the west coast of Tasmania are normally distributed with a mean of 8.3 feet and a standard deviation of 2.7 feet. What is the probability that a randomly selected March wave on Lighthouse beach is less than 6 feet tall or more than 10 feet tall?

- (A) 0.462
- (B) 0.302
- (C) 0.067
- (D) 0.538
- (E) 0.264

35) The best fast food burgers in the world are found at In-N-out Burger in southern California. Their most popular burger is the Double-Double, the weights of which are normally distributed with a mean of 8.13 ounces and a standard deviation of 0.05 ounces. If you randomly select 2170 Double-Doubles about how many would you expect to weigh less than 8 ounces?

- (A) .9953
- (B) 2160
- (C) 1075
- (D) 10
- (E) 0.0047

36) A barbeque restaurant let's people choose 1 meat and 1 side item from their menu for the lunch special. The number of times people chose each item last week can be found in the table below. If four different customers are chosen at random, what is the probability that at least one of them chooses Chicken?

	Pork	Beef	Chicken	Total
Fries	64	87	25	176
Cole slaw	48	37	7	92
Baked beans	34	29	11	74
Total	146	153	43	342

- (A) 0.417258
- (B) 0.000220
- (C) 0.000216
- (D) 0.874269
- (E) 0.415774

37) The composition of a high school band is given below. If one student from the band is randomly chosen, which probability statement given below is false?

	Male	Female	Totals
Sophomore	8	10	18
Junior	14	9	23
Senior	11	10	21
Totals	33	29	62

- (A) $P(\text{Junior} \mid \text{Female Student}) \neq P(\text{Female Student} \mid \text{Junior})$
- (B) $P(\text{Junior} \mid \text{Male Student}) = \frac{14}{33}$
- (C) $P(\text{Senior and Male Student}) = \left(\frac{21}{62}\right)\left(\frac{33}{62}\right)$
- (D) $P(\text{Sophomore or Female}) = \frac{37}{62}$
- (E) $P(\text{Senior and Male Student}) = \frac{11}{62}$

38) Using the forecasted and actual high temperatures from a consecutive, 7 day period below, if one hears the forecasted high temperature for the next (8th) day is 60 degrees, what is the best estimate for the actual high temperature?

Day	1	2	3	4	5	6	7
x , Forecasted High Temperature ($^{\circ}F$)	69	68	75	71	68	69	56
y , Actual High Temperature ($^{\circ}F$)	66	62	71	68	66	71	58

- (A) $60^{\circ}F$
- (B) $58^{\circ}F$
- (C) $59^{\circ}F$
- (D) $61^{\circ}F$
- (E) $66^{\circ}F$

39) Given the claim: "More than 75% of graduating college students land a job in their degree field within 1 year after graduation."

If you were testing the given statement above and you found a test statistic of $z = 2.46$ and were using a significance level of 0.01, what should you conclude?

- (A) There is sufficient evidence to support the claim because the test statistic is less than the critical value.
- (B) There is insufficient evidence to support the claim because the P-Value is not statistically significant.
- (C) There is sufficient evidence to support the claim because the test statistic is greater than the significance level.
- (D) There is sufficient evidence to support the claim because the P-Value is statistically significant.
- (E) There is insufficient evidence to support the claim because the test statistic is greater than the significance level.

40) When we consider all possible samples of size n , taken from a population with mean, μ ; standard deviation, σ ; and variance, σ^2 , which of the following is true?

(A) $\mu_{s^2} = \mu$, $\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n}$, $\mu_{\bar{x}} = \mu$

(B) $\mu_{\hat{p}} = \mu$, $\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n}$, $\mu_{s^2} = \sigma^2$

(C) $\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n}$, $\mu_{s^2} = \mu$, $\sigma_{\hat{p}}^2 = \frac{p(1-p)}{n}$

(D) $\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n}$, $\sigma_{\hat{p}}^2 = \sqrt{\frac{p(1-p)}{n}}$, $\mu_{\bar{x}} = \mu$

(E) $\mu_{\bar{x}} = \mu$, $\mu_{s^2} = \sigma^2$, $\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n}$