FORTY-NINTH ANNUAL MATHEMATICS CONTEST
sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

Statistics 2005

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Scoring formula: 4R - W + 40

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft head (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly 80 minutes to work.

Contributors to TMTA for the Annual Mathematics Contest:

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TMTA STATISTICS 2005

Department of Mathematical Sciences
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1. You read a report of a survey that states that based on a sample of 1000 Tennessee residents, a 95% confidence interval estimate (in thousands of dollars) of increase in income from year 2003 to year 2004 is [0.2, 16.75]. You can conclude that

   a. 95% of all residents of Tennessee received an increase in income in year 2004

   b. The largest increase in income received by Tennessee residents in year 2004 was $16,750.

   c. An estimate of the average income received in year 2004 by Tennessee residents is between $200 and $16,750.

   d. If 10,000 researchers perform the same survey independently and each computes a 95% confidence interval estimate of the increase in income among Tennessee residents for the same period, 95% of the confidence intervals will be expected to contain a common point.

   e. If 10,000 researchers perform the same survey independently and each computes a 95% confidence interval estimate of the increase in income among Tennessee residents for the same period, 95% of them will get exactly the same numbers.

2. Let G represent the number of goals scored by a local soccer team in a season. Let P(G = g) be the probability that the team scores g goals, and suppose the distribution of G is

G	0	1	2	3	4
P(G = g)	0.1	0.30	0.25	0.25	0.10

These are empirical probabilities, calculated over the course of a season.
Then, in any given game, the team can be expected to score

a) one goal b) two goals c) 1.9 goals d) 1.35 goals e) 1.95 goals

3. The power of a test of a null hypothesis H_0 versus an alternative hypothesis H_1 is

a) the probability of rejecting H_0 when it is true.
b) the probability of rejecting H_1 when it is true.
c) the probability of accepting H_0 when it is true.
d) the probability of accepting H_1 when it is true.
e) the probability of making the correct decision about which hypothesis to accept and which to reject.

4. A box contains 2 red marbles and 3 blue marbles. You select two marbles randomly with replacement. That is, you select a marble record its color and return it into the box. Then you select the second marble and record its color also. The probability that you select a red marble at least once is

a) 5/6 b) 0 .64 c) 0.3 d) 0.1 e) 2/3

5. Suppose that the heights of NBA players follow a normal distribution. You select a random sample of 16 players and calculate a sample mean of 80 inches and a sample standard deviation of 6.4 inches. Using the table of t-distribution, find a 90% confidence interval for the average height of NBA players.

a) (77.37, 82.63) b) (73. 6, 86.4) c) (78. 25, 81.75) d) (68.83, 91.74) e) (77.20, 82.81)

6. X and Y are independent normal random variables with means 5 and 7 respectively and variances 9 and 16 respectively. Let $W = 4X + 3Y - 6$. Then

a) W has a normal distribution with mean 12 and standard deviation 5.
b) W has a normal distribution with mean 41 and standard deviation 16.97
c) W has a normal distribution with mean 41 and standard deviation 288.
d) W has a normal distribution with mean 35 and standard deviation 16.97
e) W has a normal distribution with mean 35 and standard deviation 288.

7. In a class of 154 students, 143 students score less than Joan on a test. Based on this test, what is Joan's percentile ranking in the class?

a) 75 b) 90 c) 89 d) 93 e) 7

8. Akim is shooting 60% from two-point range. If he takes 5 shots from two-point range in a game what is the probability that he will make at least 3 of them?

a) 0.6 b) 0.4 c) 0.3174 d) 0.6827 e) 0.2304

9. A four letter code word is to be formed using the letters A, B, C, D. If the code word is formed at random using all four letters, what is the probability that it spells ABCD?

a) ¼ b) 1/6 c) 1 / 12 d) 1/ 256 e) 1/24

10. If the correlation coefficient between x, the heights of men and y, the heights of their wives is -0.3 then
    a) The taller the man, the taller his wife
    b) The taller the man, the shorter his wife
    c) There is a no relationship the two heights
    d) There is a straight line relationship between the heights of men and their wives
    e) It is highly likely that tall men marry short women.

11. Suppose the standard deviation of some data set is equal to 10 and suppose each observation in the data set is multiplied by 6. The standard deviation of the new data set would be

a) 60 b) 360 c) 46 d) 16 e) 10

12. Suppose the range of some data set is equal to 20 and suppose each observation in the data set is increased by 5. The range of the new data set would be

a) 25 b) 30 c) 20 d) 15 e) 10

13. A balanced die is rolled until 'five' occurs. The expected number of trials would be

a) Greater than 20 b) 5 c) 6 d) 3 e) 10

14. Two cards are chosen at random without replacement from a regular deck of 52 cards. The probability that the second card is an ace would be

a) 1/13 b) 4/51 c) 11/12 d) 4/50 e) None of the given choices

15. Suppose Z has standard normal distribution. Find x, correct to two decimal places, such that $P(Z > x)=.9940$.

a) 2.75 b) -2.75 c) 2.51 d) -2.51 e) None of the given choices

16. If the odds in favor of some event A are 1 to 5 then P(Not A) would be

a) 4/5 b) 1/6 c) 5/6 d) 1/5 e) None of the given choices

17. If a 95% confidence interval for population mean μ is (45.6, 80.5) which one of the following statements is **not necessarily** true?

a) We can reject the null hypothesis H_0: μ=90 at 5% significance level.
b) We can reject the null hypothesis H_0: μ=90 at 1% significance level.
c) We fail to reject the null hypothesis H_0: μ=60 at 5% significance level.
d) We can reject the null hypothesis H_0: μ=90 at 10% significance level.
e) We fail to reject the null hypothesis H_0: μ=70 at 1% significance level.

18. Which one of the following statements concerning hypothesis testing is **not** correct if we increase the sample size?

a) We can decrease the probability of a Type I error.
b) We can decrease the probabilities of both Types I and II error.
c) We can increase the probability of rejecting the null hypothesis when it is not true.
d) We can increase the probability of rejecting the null hypothesis when it is true.
e) We can decrease the probability of rejecting the null hypothesis when it is true.

19. These data sets represent samples. Which of them has the highest variance?

a) 6, 8, 10 b) 7, 7, 9, 9 c) 2,3,3,3,7,7,9,9 d) 2, 10 e) 2,2,10,10

20. Suppose the variance of some data set is equal to 9 and suppose each observation in the data set is multiplied by 3. The variance of the new data set would be

a) 27 b) 9 c) 3 d) 12 e) 81

21. The union of all simple events of an experiment is called:

a) a compound event
b) a sample space
c) a population
d) a random sample
e) an independent event

22. If events A and B are mutually exclusive, then the probability of both events occurring simultaneously is equal to

a) 0
b) -1
c) 1
d) 0.5
e) Any value between 0 and 1.

23. If $P(A \mid B) = P(A)$, or $P(B \mid A) = P(B)$, then events A and B are said to be:

a) mutually exclusive
b) disjoint
c) independent
d) dependent
e) complementary

24. Let X denote the weight gain in pounds per month for a calf. The probability distribution of x is shown below:

X	0	5	10	15
P(x)	0.1	0.5	0.3	0.1

Find the average weight gain in pounds per month for a calf.

a) 0
b) 7
c) 4
d) 6
e) 9

25. Two hundred single-sport athletes were cross-classified according to gender as follows:

Gender	Swimmer	Runner	Cyclist	Total
Male	25	60	25	110
Female	15	60	15	90
Total	40	120	40	200

An athlete is selected at random. If the athlete is known to be a runner, then the probability that the athlete is female is:

a) 0.60
b) 0.45
c) 0.50
d) 0.55
e) 0.20

26. Studies show that gasoline use for compact cars sold in the United States is normally distributed, with a mean of 25.5 miles per gallon (mpg) and a standard deviation of 4.5 mpg. The percentage of compacts which get 30 mpg or more is:

a) 84.13 %
b) 13.68 %
c) 10.26 %
d) 15.87 %
e) 66.32 %

27. If the random variable X is normally distributed with a mean of 88 and a standard deviation of 10, then $P(80 \leq X \leq 96)$ is

a) 0.2486
b) 0.1243
c) 0.4972
d) 0.7881
e) 0.5763

28. A soft drink distributor was interested in examining the relationship between the number of ads (x) for his product during prime time on a local television station and the number of sales per week (y) in 1000's of cases. He compiled the figures for 20 weeks and computed the following summary information:

$$n = 20 \qquad \sum x = 92, \qquad \sum y = 177 \qquad \sum xy = 884, \quad s_x = 1.3917, \; s_y = 2.9069$$

The fitted least squares line is given by

a) $y = 1.8966 + 0.1256x$
b) $y = 4.2308 + 1.8966x$
c) $y = 0.1256 + 1.8966x$
d) $y = 1.2347 + 2.5187x$
e) $y = 9 + 3.5167x$

29. The expected value, E (X), of a binomial probability distribution with n trials and probability p of success is:

a) n / p
b) np (1-p)
c) np
d) np – 1
e) np + 1

30. The standard deviation of a binomial distribution for which n = 50 and p = 0.15 is:

a) 50.15
b) 7.082
c) 6.375
d) 2.525
e) 5.134

31. If 'r' is the correlation coefficient between two variables X and Y then 'r' always lies between:

a) - 2 and + 2
b) -1 and + 1
c) - 3 and + 3
d) 0 and + 1
e) - 1 and 0

32. If X is a binomial random variable with parameters 'n' and 'p' then:

a) Mean of X is always greater than variance of X
b) Mean of X is always less than variance of X
c) Mean of X is always equal to variance of X
d) Mean of X is always equal to two times variance of X
e) Mean of X is always equal to three times variance of X

33. The lifespan of a light bulb is random with a probability distribution given by the table below.

Hours	0–999	1000–1999	2000–2999	3000–3999	≥ 4000
Probability	0.11	0.26	0.22	0.17	??

What is the probability of a light bulb lasting at least 3000 hours?

a) 0.17
b) 0.76
c) 0.26
d) 0.41
e) 0.35

34. A bag of 30 M&M's contains 10 red, 10 green, and 10 yellow M&M's. If you pick two M&M's from the bag at random, what is the probability that they will be the same color?

a) 27/87
b) 4/87
c) 9/87
d) 12/87
e) 6/30

35. A bank allocates PIN numbers consisting of 4 digits from the set of digits {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}, with no digits repeated, each such number occurring with equal probability. What is the probability that a PIN does not contain the number 3?

a) $(4/10)^4$
b) 3/5
c) $(9/10)^4$
d) $1 - (9/10)^4$
e) None of the above answers.

36. In an opinion poll, 360 out of a simple random sample of 600 people answer Yes to a question. Give a 95% confidence interval for the percentage of the population which would answer yes.

a) (.360, .600)
b) (.561 , .639
c) (.404, .796)
d) (.435, .765)
e) (.567, .633)

37. Suppose you know that the mean income in a population is $30,000 and the standard deviation of income is $10,000. Then

a) 90% of the population has income between $10,000 and $50,000.
b) 100% of the population has income between $10,000 and $50,000.
c) 97.5% of the population has income between $10,000 and $50,000.
d) 95% of the population has income between $10,000 and $50,000.
e) None of the above is correct

38. In a game costing \$9 to play, two dice are rolled. If Die 1 rolls a 5 or a 6 and Die 2 rolls a 5 or 6, you receive \$72. Otherwise you lose. How much can you expect to gain or lose if you play one game?

a) $-\$1.00$
b) \$0.00
c) \$ 7.00
d) \$8.00
e) None of the above answers

39. Let A and B be independent events such that $P(A) = 0.5$ and $P(B) = 0.6$. Then $P(A \cup B)$ is

a) 1.1
b) 0.3
c) 0.1
d) 0.5
e) 0.8

40. The maximum safe load allowed in an elevator is 3500 lbs. Assume that the weight of people is approximately normally distributed with mean 130 lbs and standard deviation 25 lbs. Then the probability that 25 people will overload the elevator is

a) 0.0114
b) 0.0228
c) 0.0456
d) 0.0500
e) none of the above

CRITICAL VALUES OF "STUDENT'S T" DISTRIBUTION

Critical values t_p satisfy $p = P(t \geq t_p)$.

d.f.	$t_{.250}$	$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$	$t_{.005}$	$t_{.0025}$	$t_{.001}$	$t_{.0005}$
1	1.000	3.078	6.314	12.706	31.821	63.657	127.32	318.31	636.62
2	.816	1.886	2.920	4.303	6.965	9.925	14.089	22.327	31.598
3	.765	1.638	2.353	3.182	4.541	5.841	7.453	10.214	12.924
4	.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.767
24	.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
60	.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
inf.	.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

STANDARD NORMAL PROBABILITY DISTRIBUTION

Tabulated values are $P(0 < z < c)$, rounded to four places.

c	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998

Forty-ninth Annual Mathematics Contest
Tennessee Mathematics Teachers Association
STATISTICS 2005

Notation:

P(A) represents the probability of the event A.

The letter z always represents a quantity having a standard normal (i.e. Gaussian) distribution.

Some possibly useful formulas:

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$\bar{x} \pm t^* \frac{s}{\sqrt{n}}$$

$$\bar{x}_1 - \bar{x}_2 \pm t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

Only two statistical tables are needed and provided for this contest exam: the 'standard normal' table of probabilities and the t-table of critical values. The form of the standard normal table provided has probabilities of the form $P(0 < z < c)$, where c is a constant ranging from 0 to 3.49. *Each problem on this contest exam has an ordinary solution not requiring any other statistical tables.*