FORTY-SEVENTH ANNUAL MATHEMATICS CONTEST
sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

Statistics 2003

Prepared by:

Department of Mathematics and Computer Science
  Bryan College
  Dayton, TN

Coordinated by: Phil Lestmann

Reviewed by:

Mathematics Faculty
  Austin Peay State University
  Clarksville, TN 37044

Scoring formula: 4R − W + 40

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the <u>best</u> answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft head (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase <u>completely</u>. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly 80 minutes to work.

1. You are using a computer to conduct a test of hypothesis at the 0.05 level of significance. The reported P-value is 0.0612. What should you do?

   a. conclude that the sample results are significant enough to reject the null hypothesis
   b. perform a test using the t-distribution
   c. perform a test using the normal distribution
   d. conclude that the sample results are not significant enough to reject the null hypothesis
   e. collect additional data for further testing

2. A random sample of 9 killer bee hives were subjected to colder and colder temperatures until all bees died. The sample mean killing temperature was $\bar{x} = 37$ F with sample standard deviation $s = 4$ F. Suppose that we want to test the claim that the population mean killing temperature $\mu = 36°$ F against the claim $\mu \neq 36°$ F. At a 1% level of significance, what critical (cut-off) value should be used from the t-distribution?

   a. 3.250 b. 2.896 c. 2.821 d. 1.860 e. 3.355

3. Researchers have hypothesized that the proverbial rose-colored glasses might indeed help lessen depression caused by light deprivation. If we wanted to test the claim that rose-colored glasses reduce the reading on the depression inventory exam, we would take for the null hypothesis that the population mean of those not wearing rose-colored glasses minus the population mean of those wearing rose-colored glasses is

   a. positive
   b. zero
   c. negative
   d. different for each sample
   e. dependent upon the level of significance

4. The probability of an adverse reaction to a flu shot is 0.10. Suppose flu shots that have been given to a group of 6 people.
   What is the expected number of people that will have an adverse reaction to the shot?

   a. 0 b. 0.6 c. 1 d. 2 e. cannot tell from the information given

5. The following is a list of values from the Consumer Price Index (CPI) from the last 15 years:

   111 126 114 130 109 121 115 102 118 118 121 125 138 119 119

   Find the sample median of this data set.

   a. 118   b. 119.07   c. 102   d. 118.5   e. 119

6. Consider a normal distribution with mean μ and standard deviation σ. The point on the graph of the distribution where $x = \mu + \sigma$ is

   a. a relative minimum   b. a point of inflection   c. a point of discontinuity

   d. a point of symmetry   e. a point on the line $y = x$

7. A sample correlation coefficient of -0.893 would indicate

   a. The data lie fairly close to a falling straight line.
   b. The data are scattered randomly, with no discernable pattern.
   c. The data lie fairly close to a horizontal line.
   d. The data lie fairly close to a rising straight line.
   e. The data are clustered to the left of the vertical axis.

8. The inaugural use of Piffle's Weight Loss Center had 10 participants. The weight loss for this group was 11, 14, 10, 9, 15, 20, 16, 8, 12, and 13 pounds. What were the mean and variance of the weight losses?

   a. 13, 13   b. 130, 130   c. 13, 3.8   d. 3.8, 13   e. 14, 3.8

9. A semilog plot of a collection of data produced a plot that was very close to a straight line. This was strong evidence that the original data were close to a

   a. straight line   b. random scatter   c. parabola   d. exponential curve

   e. circular arc

10. There are several ways to plot frequency distributions. A plot which plots cumulative frequencies as the class boundaries is called a

a. Frequency polygon   b. Histogram   c. Scatter plot

d. Probability scale   e. Ogive

11. Approximately what percentage of values from any normal distribution will fall within one standard deviation on either side of the mean?

a. 32%   b. 49%   c. 68%   d. 83%   e. 95%

12. In a random sample of eight military contracts involving cost overruns, the following information was obtained. In this problem x = bid price of contract (in millions of dollars) and y = cost overrun (also in millions of dollars). The least-squares line for this data is $y = 42.99 - 1.68x$ and the correlation coefficient is 0.981. Predict the cost overrun for a contract worth 12 million dollars.

a. $20,160,000   b. $18,446,400   c. $22,830,000   d. $7,142,830

e. not enough information is given

13. A random sample of 40 retired professional football players showed that the length of their career had sample mean $\overline{x}$ = 5.2 years with sample standard deviation s = 2.3 years. Suppose we want to test the hypothesis that the population mean μ = 4.3 years against an alternative $\mu > 4.3$ years. If we want to run a test by converting to the standard normal distribution for comparison, what is the value of the test statistic we should use?

a. 2.475   b. .3913   c. 15.65   d. 4.3   e. .9

14. *Let's Go Hamburger Restaurant* does not want to display the nutritional contents of its offerings, claiming that only 50% of its customers read them. A consumer group surveyed a random sample of 92 people and asked whether they read nutritional labels. Of this group, 50 said that they do read labels. To test the claim that more than 50% of the people read nutritional labels, using the standard normal distribution, we might use the test statistic

a. .8341   b. .5435   c. .0435   d. 1.645   e. .8377

15. The P-value of a test of a null hypothesis is

   a. the probability, assuming the null hypothesis is true, that the test statistic will take a value less extreme than that actually observed.
   b. the probability, assuming the null hypothesis is false, that the test statistic will take a value at least as extreme as that actually observed.
   c. the probability that the null hypothesis is true.
   d. the probability that the null hypothesis is false.
   e. the probability, assuming the null hypothesis is true, that the test statistic will take a value at least as extreme as that actually observed.

16. Lewis earned 85 on his biology midterm and 81 on his history midterm. However, in the biology class the mean score was 79 with standard deviation 5. In the history class the mean score was 76 with standard deviation 3. Lewis converted each score to standard (normal) units for comparison and then subtracted the smaller standardized score from the larger. What answer did he get?

   a. 4   b. 1   c. .0102   d. 0.47   e. 0

17. The Snack Pack of potato chips is advertised to weigh 3.5 ounces. The weights are normally distributed with mean 3.5 oz and standard deviation 0.2 oz. For a snack pack selected at random find the probability that it weighs less than 3.0 oz.

   a. 0.5   b. .4987   c. .0062   d. .4938   e. .1915

18. The Twister Roller Coaster always has a long line of thrill seekers waiting to take a ride on it. A random sample of 60 riders waited in line a mean time of 42 minutes with a standard deviation of 12 minutes. Using a critical value from the normal distribution, find a 90% confidence interval for the population mean waiting time in the Twister line.

   a. (40.45,43.55)   b. (39.45, 44.55)   c. (40.02,43.98)   d. (38.96,45.04)

   e. (41.67,42.33)

19. The weights of hybrid grapefruit follow a normal distribution. A random sample of 12 new hybrid grapefruit had a mean weight of 1.7 pounds with a standard deviation of 0.24 pounds. Using a critical value from the t-distribution, find a 95% confidence interval for the mean weight of the population of the new hybrid grapefruit.

   a. (1.6560,1.7440)   b. (1.1718,2.2282)   c. (1.5475, 1.8525)

   d. (1.4490,1.9510)   e. (1.5756,1.8244)

20. In order to guage the opinion of the American public about Janet Reno's job performance, a poll is commissioned. A random sample of Americans are asked, "Do you think that Janet Reno is doing a good job?" The margin of error is found to be ± 4 percentage points with a 95 percent level of confidence. Which of the following statements most accurately describes the meaning of the margin of error and the level of confidence?

   a. If 95 people are questioned, we would expect all but 4 of them to show a positive opinion of Janet Reno.
   b. Between 91% and 99% of the population thinks that Janet Reno is doing a good job.
   c. If 1000 samples are taken, we would expect no more than 50 of them to differ from the true population proportion by more than 4 percentage points.
   d. If 100 samples were taken, we would expect all but 4 of them to show a positive opinion of Janet Reno.
   e. If 100 samples are taken, we would expect no more than 4 of them to differ from the true population proportion by more than 5 percentage points.

21. If Z is the standard normal random variable, then the variance of $3Z + 5$ is

   a. 5   b. 3   c. 1   d. 9   e. 25

22. A sample of 64 data were taken from a population with mean 176 with standard deviation 16. What are the mean and standard deviation of the sample mean $\bar{x}$?

   a. 176, 16   b. 22, 16   c. 176, 2   d. 22, 2   e. 176, 4

23. A poll of 200 voters was taken about an upcoming election between Smith and Jones. The gender of the voter was noted. The results were tabulated in the following table. A voter is chosen at random. What is the probability that the voter will either vote for Smith or is a woman?

	Smith	Jones
Men	75	30
Women	55	40

a. 0.275   b. 0.650   c. 0.350   d. 0.475   e. 0.850

24. A basketball player usually hits 63% of his free throws. He shoots six times. What is the probability that he misses at least one of the six shots?

a. 0.0367   b. 0.9974   c. 0.00256   d. 0.9375   e. 0.0625

25. It is reported that 74% of adult Americans favor government regulation of firearms. A random sample of 250 adult Americans is chosen. Let r be the number of people in the sample who favor this control. Calculate the mean and standard deviation for r.

a. 185, 9.250   b. 185, 6.935   c. 65, 6.935   d. 185, 48.1   e. 65, 9.250

26. Of the senior class, 18% are athletes and student leaders and 48% are athletes. What proportion of the athletes are student leaders?

a. 0.24   b. 0.375   c. 0.425   d. 0.535   e. 0.16

27. If X is a random variable whose mean is 83 and standard deviation is 6, then what is the mean of the random variable $5X - 12$?

a. 403   b. 30   c. 168   d. 180   e. 888

28. On a standardized test in Beth's school, 17 out of 142 students scored higher than Beth. What was Beth's percentile ranking among the students in her school who took the test?

a. 12   b. 11   c. 20   d. 86   e. 88

29. Assume the probability that Team A will win the National League pennant is 0.32, and that Team B will win it is 0.26. Also, the probability that the National League team will win the World Series is 0.34, 0.49, or 0.41, depending on whether Team A, Team B, or another team wins the National League pennant. What is the probability that a National League team will win the World Series?

    a. 0.58   b. 0.83   c. 0.45   d. 0.19   e. 0.41

30. The lifetimes of a sample of fluorescent light tubes fit a binomial distribution. The probability that any one tube will last 5000 hours is 0.80. Given a sample size of 10, what is the probability that *at least* 9 out of 10 such tubes will last at least 5000 hours?

    a. 0.268   b. 0.80   c. 0.376   d. 0.624   e. 0.20

31. Assume the lifetimes of a sample of 10 fluorescent light tubes fit a binomial distribution. What is the probability that at most 8 of the light tubes will last 5000 hours, given the probability that any one tube will last at least 5000 hours is 0.80?

    a. 0.268   b. 0.80   c. 0.376   d. 0.624   e. 0.20

32. A class of 14 students are considered to represent a subset of a normal distribution. Rather than computing grades in the familiar method of 90-100 = A, 80-89 = B, and so forth, the instructor chooses to assign points to each problem and compute the grades statistically. The instructor uses a method that makes the mean score a middle C, and the range of C grades is from half a standard deviation less than the mean to half a standard deviation above the mean. The B range is from half a standard deviation above the mean to one and a half standard deviations above the mean. Every score above one and a half standard deviations above the mean is an A. D and F are computed in the same scheme below the mean. The following 14 scores are given:

    57 34 53 47 71 51 33 61 44 49 53 38 42 34

    What is the grade assigned to a score of 57?

    a. A   b. B   c. C   d. D   e. F

33. The calorie content of Speedy Supper Chunky Beef dinners is normally distributed with mean $\mu = 580$ calories and standard deviation $\sigma = 35$ calories. If you had a random sample of 10 Speedy Supper Chunky Beef dinners, what is the probability that the mean calorie count $\bar{x}$ is 600 or less?

    a. .9649   b. .7157   c. .2157   d. .4649   e. .2843

34. The life of a Corn Delight popcorn popper is normally distributed with mean 20 months and standard deviation 2 months. The manufacturer will replace a Corn Delight popper if it breaks down within the guarantee period. How long should the guarantee period be if the manufacturer does not want to replace more than 3% of the poppers? Round your answer to the nearest month.

    a. 13 months   b. 14 months   c. 15 months   d. 16 months   e. 17 months

35. A poll was taken of families in Metropolis with the result that, of 200 families asked, 140 owned a house. Using a critical value from the normal distribution, find a 95% confidence interval for the proportion of families in Metropolis that own a house.

    a. (0.636, 0.764)   b. (0.678, 0.732)   c. (0.625, 0.775)   d. (0.605, 0.795)
    e. (0.695, 0.705)

36. Suppose that you draw two cards from a standard 52-card deck without replacing the first card. Find the probability that they will both be the same color, either red or black.

    a. 0.2451   b. 0.4902   c. 0.5000   d. 0.1225   e. 0.3745

37. Three people are in a room. What is the probability, to the nearest hundredth, that the birth date of *at least* two of them will fall on the same day of the month (not necessarily the same month or same year)? Assume for simplicity that all months have 30 days.

    a. 0.05   b. 0.07   c. 0.10   d. 0.14   e. 0.17

38. Suppose X and Y are independent normal random variables with means 30 and 50, respectively, and standard deviations of 6 and 4, respectively. What are the mean and standard deviation of $W = 2X + 5Y$?

    a. 80, 10   b. 80, $\sqrt{52}$ c. 310, $\sqrt{544}$ d. 310, 32   e. 250, $\sqrt{544}$

39. A process for manufacturing push rods is under control if the lengths of the rods have a mean of 3.4 inches. What is the t-value and the conclusion with a .05 level of significance if a sample of 10 rods has a mean of 3.5 inches and a standard deviation of 0.07 inches? "In" means "In Control" and "Out" means "Out of Control".

    a. 4.52, In   b. 4.52, Out   c. 4.28, In   d. 4.28, Out   e. 1.195, In

40. For any given sample of data from a normal distribution, what is the value of $\Sigma(a_i - \bar{a})$, where the summation is from $i = 1$ to n and n is the sample size.?

    a. 0   b. sum of all the data   c. n d. $\bar{a}$ e. cannot be determined

Forty-seventh Annual Mathematics Contest
Tennessee Mathematics Teachers Association
STATISTICS 2003

Notation:

P(A) represents the probability of the event *A*

z always represents a quantity having a standard normal (i.e., Gaussian) distribution

Some possibly useful formulas:

$$\hat{p} \pm z * \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$\overline{x} \pm t * \frac{s}{\sqrt{n}}$$

$$(\overline{x}_1 - \overline{x}_2) \pm t * \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

Only two statistical tables are needed and provided for this contest exam: the "standard normal" table of probabilities and the *t*-table of critical values. The form of the standard normal table provided has probabilities of the form $P(0 < z < c)$ where *c* is a constant ranging from 0 to 3.49. *Each problem on this contest exam has an ordinary solution not requiring any other statistical tables.*

STANDARD NORMAL PROBABILITY DISTRIBUTION
Tabulated values are $P(0 < z < c)$, rounded to four places.

c	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998

CRITICAL VALUES OF "STUDENT'S T" DISTRIBUTION
Critical values t_p satisfy $p = P(t \geq t_p)$.

d.f.	$t_{.250}$	$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$	$t_{.005}$	$t_{.0025}$	$t_{.001}$	$t_{.0005}$
1	1.000	3.078	6.314	12.706	31.821	63.657	127.32	318.31	636.62
2	.816	1.886	2.920	4.303	6.965	9.925	14.089	22.327	31.598
3	.765	1.638	2.353	3.182	4.541	5.841	7.453	10.214	12.924
4	.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.767
24	.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
60	.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
inf.	.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291