FORTY-SIXTH ANNUAL MATHEMATICS CONTEST
sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

Precalculus 2002

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Scoring formula: $4R - W + 40$

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft head (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly 80 minutes to work.

Contributors to TMTA for the Annual Mathematics Contest:

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Precalculus

1. If the mean, median, and mode are all equal for the set of data{60, 10, 20, 30, X }, find the value of X.

   a. 10

   b. 15

   c. 20

   d. 30

   e. Does not exist

2. Identify the third term of the expansion $(3x-5)^8$

   a. $27x^3$

   b. $510300x^6$

   c. $18225x^6$

   d. $27x^5$

   e. $3{,}937{,}500\ x^2$

3. What is the x-coordinate of the simultaneous solution of $y - 3x = 1$ and $y = e^{0.4(x-1)}$ to the nearest thousandth?

   a. –0.118

   b. –0.106

   c. 0.638

   d. –0.120

   e. –0.123

4. Solve for x and y.

$$\begin{bmatrix} 4 & x \\ 1 & 0 \end{bmatrix} \begin{bmatrix} y & 3 \\ -1 & -3 \end{bmatrix} = \begin{bmatrix} 3 & 9 \\ 1 & 3 \end{bmatrix}$$

a. (9, 3)

b. (1, 3)

c. (1, 1)

d. (1, 2)

e. (2, 1)

5. The graph of $f(x) = 3 + \ln(x - 2)$ can be determined by one or more of the following operations on the function:

i. shift the graph of $g(x) = 3 + \ln x$ two units to the right

ii. shift the graph of $g(x) = 3 + \ln x$ two units to the left

iii. shift the graph of $h(x) = \ln x$ two units to the right and 3 units up

iv. shift the graph of $h(x) = \ln x$ two units to the right and 3 units down

a. *i* only

b. *ii* only

c. *iii* only

d. *i* and *iii*

e. *i* and *iv*

6. Solve for x:

$$\begin{vmatrix} x & 14 & 15 \\ 0 & x-2 & 20 \\ 0 & 0 & x-3 \end{vmatrix} = 0$$

a. {0, 2, -3}

b. {14, 20, 0}

c. {3, -2, 0}

d. {0, 2, 3}

e. {-3, -2, 0}

7. Three vertices of a parallelogram are P_1 (0,0); P_2 (a, 0); P_3 (b, c). P_2P_3 is a diagonal. What are the coordinates of the fourth vertex P_4 ?

a. ((a + b)/2, c)

b. (b – a, c)

c. ((b – a)/2, c)

d. (a + b, c)

e. (a – b, c)

8. Find the equation of the line passing through (2, 1) that is perpendicular to the line $2X + Y - 6 = 0$.

a. $2X - Y - 3 = 0$

b. $2Y - X = 0$

c. $2X + Y - 5 = 0$

d. $2Y + X = 0$

e. $2X - Y = 0$

9. A line has a positive slope and the X-value for the X-intercept is positive. What do we know about the Y- value for the Y- intercept?

a. It is positive.

b. It is negative.

c. It is zero.

d. It has no Y- intercept.

e. It cannot be determined.

10. If the parabola $y = ax^2 + bx + c$ opens upward and has vertex $(-1,4)$, how many zeros will the equation $y = ax^2 + bx + c$ have?

a. 0

b. 1

c. 2

d. 3

e. Cannot be determined

11. What is the solution of $|9 - 3x| < 4$?

a. $5 < x < 13$

b. $-\frac{5}{3} > x > -\frac{13}{3}$

c. $-\frac{5}{3} < x < \frac{13}{3}$

d. $-5 < x < -13$

e. $\frac{5}{3} < x < \frac{13}{3}$

12. Consider the function $f(x) = x^3 - ax^2 + 5$. If $f(2) = 1$, what is the value of a ?

a. 3

b. -3

c. -1

d. 1

e. $\frac{13}{2}$

13. An elevator with two passengers stops at the second, third, and fourth floors. If a passenger must get off at one of the three floors and if he is equally likely to get off at any of the three, what is the probability that the passengers get off at different floors?

a. 1/2

b. 2/3

c. 3/4

d. 1/3

e. 1/4

14. Find the value of x given that the sum of the infinite geometric series equals $\frac{6}{7}$.

$\frac{6}{7} = 1 + 3x + 9x^2 + \ldots$

a. $-\frac{13}{18}$

b. $-\frac{1}{18}$

c. $-\frac{7}{6}$

d. $\frac{13}{18}$

e. $-\frac{6}{7}$

15. Decompose into partial fractions, solving for the parts A, B, C, and D.

$$\frac{14x-32}{x^2-5x+4}=\frac{A}{B}+\frac{C}{D}$$

a. $\frac{8}{x-1}+\frac{6}{x-4}$

b. $\frac{6}{x^2-5x+4}+\frac{8}{x^2-5x+4}$

c. $\frac{6}{x-1}+\frac{8}{x-4}$

d. $\frac{14x}{x-1}-\frac{32}{x-4}$

e. None of the above

16. A car dealership has 2 dealership locations and a warehouse. He can minimize his shipping cost between the warehouse and the dealerships using the function

$$C=1220-5x-6y$$

x = number of cars shipped from the warehouse to dealership A
y = number of cars shipped from the warehouse to dealership B

The feasible region is described by the constraints:

$$\begin{cases} x+y\le 14 \\ x+y\ge 10 \\ 0\le x\le 8 \\ 0\le y\le 10 \end{cases}$$

Find the value of x so that his shipping cost is minimized.

a. 0

b. 4

c. 8

d. 6

e. 2

17. Given ΔABC with sides a = 10, b = 12 and c = 15 units, the sum of angles A and B, correct to the nearest hundredth is

a. 79.16°

b. 85.46°

c. 94.54°

d. 102.67°

e. 86.54°

18. For any integers a and b, define $a \Delta b = ab - (a + b) + 2$ and $a \square b = a + b - 1$. Find $(3 \Delta 4) \square (-1 \Delta 0)$.

a. 0

b. −12

c. 9

d. −1

e. 4

19. If a parabolic function passes through the points $(1,-2.25),(2,-4)$ and $(-1,-3.25)$, what is the equation of the function?

a. $y = -.75x^2 + .5x - 2$

b. $y = -.75x^2 + .5x + 2$

c. $y = -.75x^2 - .5x + 2$

d. $y = -.75x^2 - .5x - 2$

e. $y = .75x^2 + .5x + 2$

20. Montreal, Canada is approximately due north of Santiago, Chile. The latitude of Montreal is 45° N, while the latitude of Santiago is 35° S. Find the distance between the two cities.to the nearest mile. (The radius of the earth is 3960 miles)

a. 691 miles

b. 39,600 miles

c. 316,800 miles

d. 5,529 miles

e. 1,260.51 miles

21. If B is the vertex angle of an isosceles triangle with equal sides of length a, the law of cosines will show that the length of side b (opposite angle B) equals

a. $4a^2 \cos B$

b. $a\sqrt{2(1-\cos B)}$

c. $2a\sqrt{\cos B}$

d. $a\sqrt{2(1+\cos B)}$

e. $2a^2\sqrt{\cos B}$

22. If $(2x - 1)$ is a factor of $4x^3 + kx^2 - 2x - 1$, then the value of k is

a. -6

b. 4

c. 2

d. -4

e. 6

23. Let $A = 0.4x^2 + 10x + 5$ and $B = 0.5x^2 + 2x + 101$. For what value of x is A – B a maximum?

a. $x = 40$

b. $x = -40$

c. $x = -12.5$

d. $x = 12.5$

e. $x = -2$

24. Let $f(x) = \dfrac{x^3 - 2}{x^3 + 1}$. If (f o g)(x) = x, then g(x) equals:

a. $\dfrac{x^3 + 1}{x^3 - 2}$

b. $\sqrt[3]{\dfrac{2 - x}{x - 1}}$

c. $\sqrt[3]{\dfrac{2x + 1}{x - 1}}$

d. $(x^3 + 1)^3 - 2$

e. $\sqrt[3]{\dfrac{x + 2}{1 - x}}$

25. If the average price of a loaf of bread is \$1.50 and the inflation rate is 4% per year, how many years, correct to the nearest year, will it take for the price of a loaf of bread to double?

a. 15 years

b. 16 years

c. 17 years

d. 18 years

e. 20 years

26. A car travels from one city to another at 40 miles per hour. On the return trip, the rate of the car is 60 miles per hour. What is its average rate for the entire trip?

a. 48 mph

b. 50 mph

c. 52 mph

d. 55 mph

e. 58 mph

27. Of the 354 surveys returned by a group of college students, 294 indicated they liked mathematics, 188 liked English, 196 liked history, 148 liked both mathematics and English, 92 liked all three subjects, 175 liked both history and mathematics; and 9 only liked history and disliked both mathematics and English. How many disliked English or history?

a. 175

b. 250

c. 176

d. 11

e. 74

28. A basket contains 100 snakes of which ten are mean and always bite their handlers. In a survivors game, each participant must reach into the basket three times and pick up a snake to release on the ground each time. If the 90 nice snakes never bite when handled, what is the probability that the second participant does not get bitten?

a. 0.521

b. 0.727

c. 0.996

d. 0.591

e. 0.027

29. Determine the expected value for the following problem. If a gambler rolls two dice and gets a sum of 2 or 12, he/she will win $25. If the gambler gets a 7, he/she will win $4. The cost to play this game is $5.

a. $2.94

b. $294

c. -$2.94

d. -$294

e. $7

30. Write the n^{th} term of the series $\sum_{k=3}^{150}\left(k^2-6k\right)$

a. n^2-2n-8

b. $n^2-6n+10$

c. $n^2+10n+6$

d. n^2+6n+2

e. $n^2-10n+16$

31. What curve is defined by these parametric equations?

$x = \cos t$
$y = \sin t$
$0 \leq t \leq 2\pi$

a. ellipse

b. line

c. hyperbola

d. circle

e. parabola

32. Find the distance between the parallel lines $3X + 4Y - 8 = 0$ and $6X + 8Y + 3 = 0$ to the nearest tenth of a unit.

a. 2.4

b. 1.9

c. 1.4

d. 5.0

e. 1.3

33. Find the percent increase (to the nearest whole number) in the volume of a sphere when the surface area of the sphere is increased by 10%.

a. 1%

b. 5%

c. 10%

d. 15%

e. cannot be determined

34. Solve $4\cos^2 x - 4\cos x + 1 = 0$ in the interval $[0, \pi]$.

a. 0

b. $\pi/4$

c. $\pi/3$

d. $2\pi/3$

e. $\pi/6$

35. Solve for all real values of x in $[0, 2\pi)$. (Leave in radians rounded to 3 decimal places.)
$\sec^2 x + 3 \tan x - 5 = 0$

a. 0.785 , 3.927

b. 1.816 , 4.957 , -0.785 , -3.927

c. 1.816 , 4.957 , 0.785 , 3.927

d. 1.816 , 4.957 , 0.785

e. 4.957 , 1.816

36. If $f(X, Y, Z) = \dfrac{\text{LOG } X + \text{LOG } Z}{\text{LOG } Y}$, then $f(4, 2, 2)$ equals:

a. 6

b. 4

c. 3

d. 2

e. 16

37. Solve the system of equations:

$$\frac{5}{x} + \frac{2}{y} - 1 = 0$$
$$\frac{1}{x} - \frac{3}{y} - 7 = 0$$

a. (1, -2)

b. (1, -1/2)

c. (-1/2, 1/3)

d. (-1/3, 2)

e. (-2, -1)

38. Simplify $\left[(x+3)^2(x-1)^{-\frac{2}{5}} - 4(x-1)^{\frac{3}{5}}\right]^{-1}$

a. $\dfrac{(x-1)^{\frac{2}{5}}}{x^2+2x+13}$

b. $\dfrac{x^2+2x+13}{(x-1)^2}$

c. $\dfrac{x^2+2x+13}{(x-1)^{\frac{2}{5}}}$

d. $(x-1)^{\frac{2}{5}}(x^2+2x+13)$

e. $(x-1)^2(x^2+2x+13)$

39. The inverse of the function $y = \text{LOG}_3\left(\dfrac{4x-1}{3}\right)$ is:

a. $y = \dfrac{3^{(x+1)}+1}{4}$

b. $y = \text{LOG}_3\left(\dfrac{3}{4x-1}\right)$

c. $y = \text{LOG}_3\left(\dfrac{3x+1}{4}\right)$

d. $y = \dfrac{3^{(x+1)}}{4}$

e. $y = \dfrac{3x+1}{4}$

40. If R_1 and R_2 are roots of the quadratic equation $ax^2 + 3x + c = 0,\ a \neq 0,\ c \neq 0,$ then what is the value of $\frac{1}{R_1} + \frac{1}{R_2}$?

a. 3c

b. $-3a$

c. $\frac{-3}{c}$

d. $\frac{-c}{3}$

e. $\frac{3}{c}$