TMTA Pre-Calculus Test Spring 2019

1. Identify the negative-number solution to the equation $\sqrt{2x + 21} = x + 3$
   a. $x = -2$
   b. $x = -6$
   c. $x = -3$
   d. $x = -5$
   e. No real solution

2. In a right triangle, if $\cos \theta = \frac{5}{7}$, evaluate $\tan \theta$
   a. $\frac{\sqrt{24}}{5}$
   b. $\frac{\sqrt{74}}{7}$
   c. $\frac{\sqrt{74}}{5}$
   d. $\frac{5\sqrt{24}}{24}$
   e. $\frac{\sqrt{24}}{7}$

3. If $f(x) = \sqrt{x}$, and $g(x) = 5x - 3$, and $h(x) = x^2 + 4$, evaluate $f\left(g(h(1))\right)$.
   a. $\sqrt{22}$
   b. 8
   c. $2\sqrt{2}$
   d. $5\sqrt{5} - 3$
   e. 6

4. Find all real solutions to the equation $x^6 - 35x^3 + 216 = 0$:
   a. $x = 8, x = 27$
   b. $x = -8, x = -27$
   c. $x = 2, x = 3$
   d. $x = -2, x = -3$
   e. $x = -6, x = 6$

5. Identify all solutions to the equation $t^3 = 64$
   a. 4
   b. $-4, 4$
   c. $4, -2 \pm 2i\sqrt{3}$
   d. $-4, -2 \pm 2i\sqrt{3}$
   e. $4, 2 \pm 2i\sqrt{3}$

6. Identify the value of A and B that will make the following equation true:
$$\frac{A}{x-5} + \frac{B}{x+4} = \frac{10x-23}{x^2-x-20}$$

   a. $A = 3$ and $B = 7$
   b. $A = 7$ and $B = 3$
   c. $A = 4$ and $B = 6$
   d. $A = 6$ and $B = 4$
   e. $A = 5$ and $B = 5$

7. Describe the horizontal phase shift from the parent graph $f(x) = \sin x$ to the transformed graph $g(x) = -3\sin(3x + \pi) - 3$

   a. The phase shift is a horizontal shift of π units to the right.

   b. The phase shift is a horizontal shift of $\frac{\pi}{3}$ units to the right.

   c. The phase shift is a horizontal shift of π units to the left.

   d. The phase shift is a horizontal shift of $\frac{\pi}{3}$ units to the left.

   e. The phase shift is a horizontal shift of 3π units to the left.

8. What is an equation of the line that is the horizontal asymptote for the graph of
$$f(x) = -2e^{4(x-3)} + 5 ?$$
   a. $y = 5$
   b. $y = -2$
   c. $y = 3$
   d. $y = -5$
   e. $y = 2$

9. For the equation $x^2 + 6x + y^2 + 10y = 66$, identify the center and radius of the corresponding circle:
   a. Center at $(3,5)$, radius of 10

   b. Center at $(-3,-5)$, radius of 10

   c. Center at $(3,5)$, radius of $\sqrt{66}$

   d. Center at $(-3,-5)$, radius of $\sqrt{66}$

   e. Center at $(-3,-5)$, radius of 100

10. The formula for wind chill is based on temperature and wind speed. For temperature T (in Fahrenheit) and wind speed W (in miles per hours), the wind chill (also reported in degrees Fahrenheit) can be found by using the following expression:

$$35.74 + 0.6215T - 35.75W^{0.16} + 0.4275TW^{0.16}$$

What is the wind chill reading, to the nearest degree Fahrenheit, if the outdoor temperature is 30°F with a breeze of 12 mph?

a. −10°F
b. 20°F
c. 2°F
d. −17°F
e. −18°F

11. The number of bacteria in a certain colony doubles every 5 days. At the same rate, how long will the colony need to triple in number?
a. 7.925 days
b. 10.000 days
c. 3.808 days
d. 5.152 days
e. 15.000 days

12. According to the Rational Roots Theorem, which of these can NOT be a root of
$f(x) = 8x^5 - x^4 + 10x^3 - 63x^2 + 8x - 120$?

a. $-\dfrac{5}{6}$

b. $\dfrac{3}{4}$

c. $-\dfrac{5}{2}$

d. $-\dfrac{15}{8}$

e. $\dfrac{15}{2}$

13. How much of a 40% concentration liquid and how much 25% concentration liquid must be combined to make 10L of 30% concentration?

    a. $3\frac{1}{3}$L of 40%, $6\frac{2}{3}$L of 25%

    b. $3\frac{1}{3}$L of 25%, $6\frac{2}{3}$L of 40%

    c. 2.414L of 40%, 7.586L of 25%

    d. 7.586L of 40%, 2.414L of 25%

    e. 5 L of 40%, 5 L of 25%

14. Simplify $\dfrac{2+5i}{3-4i}$

    a. $\dfrac{26+23i}{25}$

    b. $\dfrac{-14+23i}{25}$

    c. $\dfrac{-14+23i}{-7}$

    d. $\dfrac{26+23i}{7}$

    e. $\dfrac{26-7i}{-7}$

15. Simplify $(2+2i)^6$

    a. $512i$

    b. -512

    c. $-512i$

    d. 512

    e. $256\sqrt{2}+256i\sqrt{2}$

16. If $0 \le \theta \le 2\pi$ and $\cos\theta = \frac{3}{5}$, what is the value of $\cos(2\theta)$?

    a. -0.96

    b. 0.96

    c. 1.2

    d. -0.28

    e. 0.28

17. What is the remainder when $x^4 - 3x^3 + 5x^2 - 7x + 9$ is divided by $2x - 1$?
   a. 5
   b. 7
   c. $\dfrac{133}{16}$
   d. $\dfrac{227}{16}$
   e. $\dfrac{103}{16}$

18. Identify the number of points of intersection for the system of equations
$$\begin{cases} y = x^2 - 5 \\ x^2 + y^2 = 25 \end{cases}$$
   a. 0
   b. 1
   c. 2
   d. 3
   e. 4

19. What fraction can be represented by the repeating decimal $0.2\overline{15}$?
   a. $\dfrac{1}{5}$
   b. $\dfrac{1}{22}$
   c. $\dfrac{15}{99}$
   d. $\dfrac{71}{330}$
   e. $\dfrac{215}{999}$

20. On the interval $[0, \infty)$ and the function $f(x) = \sqrt[3]{x^2 + 6x + 11}$, determine $f^{-1}(6)$
   a. 2.224
   b. 3.317
   c. 4.291
   d. 4.362
   e. 11.629

21. The first 10 terms of the sequence of decagonal numbers are 1, 10, 27, 52, 85, 126, 175, 232, 297, and 370. What formula could be used to determine the -nth decagonal number, where n is an element of the natural numbers?
   a. $d_n = 4n^2 - 3n$
   b. $d_n = 4n^2 + 5n + 1$
   c. $d_n = 4n^2 + 53n + 175$
   d. $d_n = 4n^2 + 45n + 126$
   e. $d_n = 10n^2 - 21n + 12$

22. How many ways can one rearrange all of the letters in the word TENNESSEE?
   a. 362880
   b. 15120
   c. 3780
   d. 45360
   e. 22680

23. If $\log_x 2 = 1.351$ and $\log_x 3 = 1.611$, evaluate $\log_x 12$.
   a. 2.940
   b. 3.436
   c. 4.313
   d. 4.353
   e. 4.573

24. Which of these is NOT a solution to $8\cos^4 x - 10\cos^2 x + 3 = 0$?
   a. $\dfrac{7\pi}{6}$
   b. $\dfrac{7\pi}{4}$
   c. $\dfrac{5\pi}{6}$
   d. $\dfrac{5\pi}{4}$
   e. $\dfrac{5\pi}{3}$

25. Identify all solutions to the equation $\sin^4 x - \cos^4 x = 0$ on the interval $[0,2\pi]$
   a. $x = \dfrac{\pi}{4}, \dfrac{5\pi}{4}$
   b. $x = \dfrac{3\pi}{4}, \dfrac{7\pi}{4}$
   c. $x = \dfrac{\pi}{4}, \dfrac{3\pi}{4}, \dfrac{5\pi}{4}, \dfrac{7\pi}{4}$
   d. $x = 0, \dfrac{\pi}{2}, \pi, \dfrac{3\pi}{2}, 2\pi$
   e. $x = 0, \dfrac{\pi}{2}, \pi, \dfrac{3\pi}{2}$

26. The vertices of a kite are formed by the vertices and intersection points of the quadratic functions $f(x) = -x^2 + 6x - 4$ and $g(x) = 3x^2 - 18x + 16$. What is the area of the kite?
    a. 16 square units
    b. 32 square units
    c. 64 square units
    d. 128 square units
    e. 256 square units

27. Identify the polynomial of lowest degree that contains $2i$ and $-4i$ as roots.
    a. $x^2 + 8$
    b. $x^2 - 2x + 8$
    c. $x^4 - 20x^2 + 64$
    d. $x^4 + 20x^2 + 64$
    e. $x^4 - 2x^2 - 8$

28. An isosceles triangle has two congruent sides of length 5 with an included angle of $40°$. What is the length of the third side?
    a. 3.420
    b. 7.071
    c. 9.129
    d. 6.214
    e. 4.698

29. An 8-inch by 10-inch piece of paper has four corners cut out of it, one at each corner, to create an open box by folding the flaps upward. What is the side length of the cut-out square need to be to maximize the volume of the box?
    a. 2.944 in
    b. 1.916 in
    c. 0.958 in
    d. 1.472 in
    e. Not possible to maximize volume

30. For the graph of the rational function $(x) = \frac{x^4-15x^2-10x+24}{x^2+7x+14}$, identify the parabolic asymptote.

a. $f(x) = x^2 + 7x + 14$
b. $f(x) = x^2 + 7x + 48$
c. $f(x) = x^2 - 22$
d. $f(x) = x^2 - 8$
e. $f(x) = x^2 - 7x + 20$

31. Which of these is not a solution to $e^{4x} - 19e^{3x} + 124e^{2x} - 324e^x + 288 = 0$?

a. $\ln 9$
b. $\ln 8$
c. $\ln 6$
d. $\ln 3$
e. $\ln 2$

32. For the parametric equations $\begin{cases} x = 2t + 3 \\ y = (t - 4)^2 \end{cases}$, identify the equation of the curve in rectangular coordinates

a. $f(x) = \frac{1}{4}x^2 - \frac{11}{2}x + \frac{121}{4}$

b. $f(x) = \frac{1}{4}x^2 - 7x + 49$

c. $f(y) = 2\sqrt{y} + 3$

d. $f(y) = 2\sqrt{y} + 8$

e. $f(x) = \frac{1}{4}x^2 - \frac{5}{2}x + \frac{25}{4}$

33. For the function $f(x) = x^3 + x$, simplify the expression $\frac{f(x+\Delta x)-f(x)}{\Delta x}$

a. $x^2 + 2x(\Delta x) + (\Delta x)^2 + 1$

b. $3x^2$

c. $3x^2 + 3x(\Delta x) + (\Delta x)^2 + 1$

d. $3x^2 + 3x(\Delta x)^2 + \Delta x + 1$

e. $x^2 + x(\Delta x) + (\Delta x)^2$

34. Identify the domain of the function: $f(x) = \ln \sqrt{\dfrac{x-6}{x^3-9x^2+18x}}$

    a. $(-\infty, 0) \cup (3, \infty)$

    b. $(-\infty, 0] \cup [3, \infty)$

    c. $(-\infty, 0] \cup (3,6) \cup (6, \infty)$

    d. $(-\infty, 0) \cup [3, 6) \cup (6, \infty)$

    e. $(-\infty, 0) \cup (3,6) \cup (6, \infty)$

35. Determine the average rate of change for the function $f(x) = \sqrt{x^2 + x + 5}$ on the interval $[2, 4]$.

    a. 4.158

    b. 1.386

    c. 7.000

    d. 0.561

    e. 0.842

36. Which of these lines represents an asymptote of the hyperbola $\dfrac{(x-3)^2}{25} - \dfrac{(y+2)^2}{16} = 1$?

    a. $y + 2 = \dfrac{5}{4}(x - 3)$

    b. $y - 2 = \dfrac{5}{4}(x - 3)$

    c. $y + 2 = \dfrac{5}{4}(x + 3)$

    d. $y + 2 = \dfrac{4}{5}(x - 3)$

    e. $y - 2 = \dfrac{4}{5}(x - 3)$

37. Identify the coefficient of $x^3 y^4 z^2$ in the expansion of $(2x - 3y + z)^9$

    a. 1260

    b. -1260

    c. 816480

    d. -816480

    e. -7560

38. Using the formula $\cos \theta = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\|\|\vec{v}\|}$ to determine the measure of θ, rounded to the nearest tenth of a degree, between the vectors $\vec{u} = (-8,15)$ and $\vec{v} = (9,40)$

   a. 40.8°

   b. 139.2°

   c. 15.4°

   d. 0.3°

   e. 0.7°

39. Determine a three-dimensional vector that is perpendicular to both (2,3,6) and $(-3,-1,-4)$.

   a. $(-6,-10,7)$

   b. $(-18,26,-11)$

   c. $(-6,10,7)$

   d. $(18,-26,11)$

   e. $(-6,10,-7)$

40. Determine the value of x that would make the determinant equal 20.

$$\begin{bmatrix} x & 0 & 0 & 0 \\ 2 & 3 & -5 & 4 \\ 6 & 1 & 7 & -10 \\ -3 & 4 & 2 & -1 \end{bmatrix}$$

   a. $x = 2$

   b. $x = \frac{2}{13}$

   c. $x = 1$

   d. $x = \frac{13}{2}$

   e. No solution