TENNESSEE MATHEMATICS TEACHERS ASSOCIATION

FIFTY-NINTH ANNUAL MATHEMATICS CONTEST

2015

Precalculus

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Scoring formula: 4 x (Number Right) – (Number Wrong) + 40

DIRECTIONS

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you; if you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly eighty minutes to work.

Tennessee Mathematics Teachers' Association
Precalculus Exam 2015

1. If $\tan\theta < 0$, and $\cos\theta = \frac{1}{4}$ then what is the exact value of $\csc\theta$?

   (a) $\csc\theta = \sqrt{15}$ (b) $\csc\theta = \frac{\sqrt{15}}{4}$ (c) $\csc\theta = 4$ (d) $\csc\theta = -\frac{\sqrt{15}}{4}$ (e) $\csc\theta = -\frac{4\sqrt{15}}{15}$

2. The Fine Bank of Nashville pays 2.5% interest, compounded monthly, on all of its savings accounts. Three years after making a deposit, Tina has \$13,500 in the account. Assuming she did not add to or take away from the account during those years, what was her initial deposit to the nearest dollar?

   (a) \$12,488 (b) \$12,526 (c) \$12,536 (d) \$13,416 (e) \$13,023

3. Suppose that a is a positive real number. Which of the following functions is the same as $f(x) = a^{-3x}$?

   (a) $g(x) = \left(\frac{1}{a^3}\right)^x$

   (b) $g(x) = \left(\frac{1}{\sqrt[3]{a}}\right)^x$

   (c) $g(x) = \frac{1}{a^3} \cdot a^x$

   (d) $g(x) = -a^{3x}$

   (e) $g(x) = a^{-3} \cdot a^x$

4. Samuel starts with a positive number x and takes the square root, followed by the cube root, followed by the fourth root, followed by the fifth root. Ramona tells Samuel that he could have simply raised x to one power and gotten the same answer. What power is Ramona talking about?

   (a) He could have raised x to the $\frac{1}{60}$ power.

   (b) He could have raised x to the $\frac{60}{77}$ power.

   (c) He could have raised x to the $\frac{77}{60}$ power.

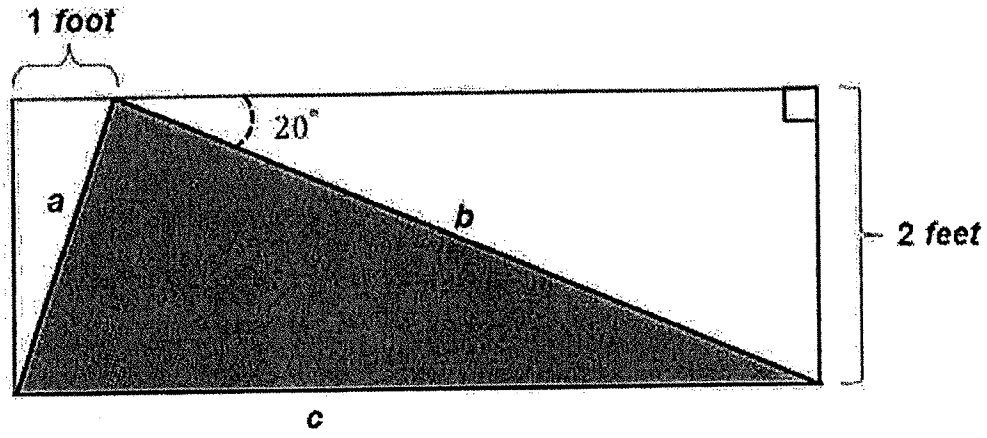
   (d) He could have raised x to the $\frac{1}{120}$ power.

   (e) He could have raised x to the -120 power.

5. The solutions to the equation $\log_2(x) + \log_2(x+2) = 3$ will be

   (a) $x = -4,\ x = 2$ (b) only $x = -4$ (c) only $x = 2$ (d) only $x = \frac{1}{2}$ (e) $x = 2,\ x = -2$

6. What is the perimeter of the shaded triangle below to the nearest hundredth?



(a) 15.26 feet (b) 6.09 feet (c) 14.58 feet (d) 6.58 feet (e) 9.82 feet

7. What is the domain of the function $f(x) = \dfrac{\sqrt{3-x}}{\ln(2x)}$?

(a) $(0, +\infty)$ (b) $\left(0, \frac{1}{2}\right) \cup \left(\frac{1}{2}, 3\right]$ (c) $(-\infty, 3)$ (d) $(0, 3]$ (e) $(0, 1) \cup (1, 3]$

8. Jose plants a seven millimeter tall tomato seedling in his garden, and its height (in milimeters) increases 3% per day for the next thirty days. Which of the following functions gives the height H of the plant as a function of the number d of days since Jose planted it?

(a) $H(d) = 7 + .03d$

(b) $H(d) = 7 + 1.03d$

(c) $H(d) = 7 \cdot (0.03)^d$

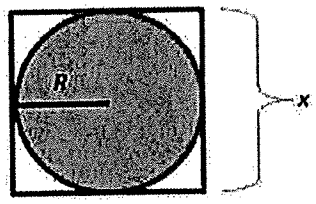
(d) $H(d) = 7.21^d$

(e) $H(d) = 7 \cdot 1.03^d$

9. Lorraine has a collection of fifty-two nickels, dimes, and quarters with a total value of \$4.75. If she has twice as many nickels as dimes, then how many quarters must Lorraine have?

(a) 11 quarters (b) 16 quarters (c) 10 quarters (d) 7 quarters (e) 28 quarters

10. Jacob wants to plant a circular flower bed centered in a square plot of land as shown below. Jacob plans on filling in the unshaded portions of the square plot with grass. Which of the formulas below gives the number of square feet of grass he will need as a function of the flower bed's radius R? Assume R is measured in feet.



(a) $A = (2\pi - 2)^2 R$

(b) $A = (4 - \pi) R^2$

(c) $A = \dfrac{R^2}{4} - \pi R^2$

(d) $A = \pi R^2 - 4R^2$

(e) $A = (4\pi - 4) R^2$

11. Suppose the population of deer in a state forest is given by the function $D(t) = 750 + 150 \cos\left[\frac{\pi}{2}(t-3)\right]$, where t is measured in years since 2000. How many years will pass between one maximum value of the deer population and the next maximum value?

(a) One year (b) Two years (c) Three years (d) Four years (e) Five years

12. If $f(x) = \log_4(\log_3(\log_2(x)))$, then what is $f^{-1}\left(\frac{1}{4}\right)$ to the nearest ten-thousandth?

(a) $f^{-1}\left(\frac{1}{4}\right) \approx 26.516$

(b) $f^{-1}\left(\frac{1}{4}\right) \approx 167.289$

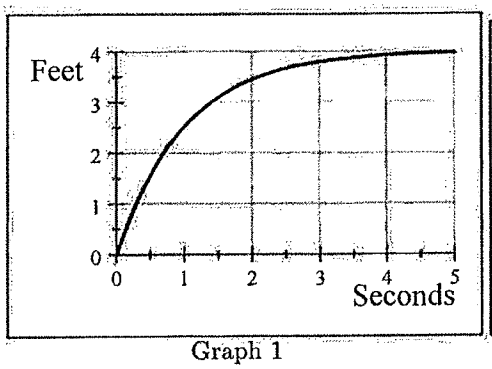
(c) $f^{-1}\left(\frac{1}{4}\right) \approx 2.006$

(d) $f^{-1}\left(\frac{1}{4}\right) \approx .5500$

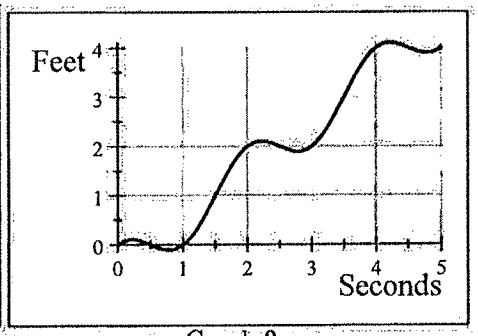
(e) $f^{-1}\left(\frac{1}{4}\right)$ is undefined

13. The graphs below show the position function for three people walking along the same straight road. The time and distance scales are the same, and distance is measured relative to the same reference point

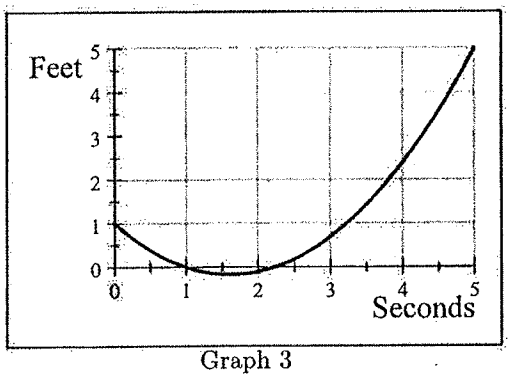
on all three graphs. Which graphs have the same average velocity over the time interval $0 \leq t \leq 5$ shown?



Graph 1



Graph 2



Graph 3

(a) None of the graphs have the same average velocity.

(b) Only Graph 1 and Graph 2 have the same average velocity.

(c) Only Graph 2 and Graph 3 have the same average velocity.

(d) Only Graph 1 and Graph 3 have the same average velocity.

(e) All graphs have the same average velocity.

14. Suppose that a and b are nonzero. The function $f(x) = \ln(b - ax)$ will have a vertical asymptote at

(a) $x = 0$ (b) $x = \frac{b}{a}$ (c) $x = \frac{a}{b}$ (d) $x = a$ and $x = b$ (e) $x = \frac{b}{a}$ and $x = \frac{a}{b}$

15. Compared to the standard cosine function, the horizontal translation for $f(x) = 3\cos[2x - 1] + 4$ will be

(a) one unit right (b) one unit left (c) $\frac{1}{2}$ unit right (d) two units left (e) π units right

16. Suppose that the vertex of a parabola is (a, b). If this vertex is a maximum, which of the following functions could represent this parabola?

(a) $f(x) = (x + a)^2 - b$

(b) $f(x) = -(x + a)^2 + b$

(c) $f(x) = -3(x - a)^2 + b$

(d) $f(x) = 5(x - a)^2 + b$

(e) $f(x) = -5(x - b)^2 + a$

17. Which of the following points lies *inside* the circle $x^2 + y^2 - 2x + 6y = -1$?

(a) $(1, 3)$ (b) $\left(-\frac{1}{2}, \frac{1}{3}\right)$ (c) $\left(\frac{1}{2}, -\frac{1}{2}\right)$ (d) $\left(\frac{1}{4}, -\frac{1}{15}\right)$ (e) $\left(-\frac{1}{3}, \frac{1}{20}\right)$

18. What is the solution set for the inequality $3 - \frac{x}{x+1} \geq 0$?

(a) $\left(-\infty, -\frac{3}{2}\right] \cup (-1, +\infty)$

(b) $(-\infty, -1] \cup \left[-\frac{1}{2}, +\infty\right)$

(c) $\left[-\frac{3}{2}, -1\right)$

(d) $\left[-1, -\frac{1}{2}\right]$

(e) The solution set is empty.

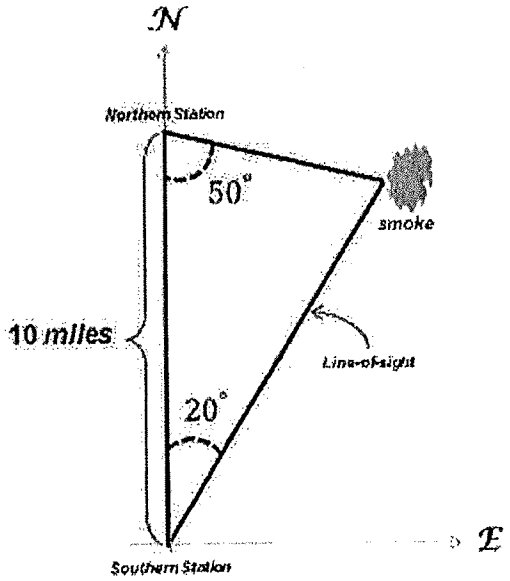
19. If we add up the coefficients of the terms in the polynomial $(x - y)^6$ we obtain the number

(a) 2^6 (b) 2^5 (c) 20 (d) 22 (e) 0

20. The coefficient of the a^3b^2 term in the polynomial $(3a - 2b)^5$ will be

(a) 1,080 (b) -720 (c) 540 (d) 10 (e) -60

21. Two fire towers are located ten miles apart, one due south of the other. The northern station spots smoke rising from a fire on a bearing of 50° east of due south, while the southern station spots the same smoke on a bearing of 20° east of due north. (See diagram below.) To the nearest hundredth, what is the approximate line-of-sight distance from the southern station to the fire?



(a) 8.15 miles (b) 12.27 miles (c) 3.64 miles (d) 27.47 miles (e) 22.40 miles

22. The model that describes the number of bacteria in a culture after t days has just been updated from $P(t) = 7 \cdot 2^t$ to $Q(t) = 7 \cdot 3^t$. What implication can you draw from this information?

(a) The final number of bacteria is three times as much as the initial population instead of two times as much.

(b) The initial number of bacteria is 3 instead of 2.

(c) The number of bacteria triples every day instead of doubling every day.

(d) The growth rate of the bacteria is 30% per day instead of 20% per day.

(e) None of the above.

23. Solve the equation $3^{u^2-3u} = 9^{u-3}$ for the variable u.

(a) The solutions will be $u = 2$ and $u = 3$.

(b) The solutions will be $u = 1$ and $u = 3$.

(c) The solutions will be $u = 2$ and $u = 1$.

(d) The solutions will be $u = 1$ and $u = \frac{3}{2}$.

(e) The solutions will be $u = 2$ and $u = \frac{3}{2}$.

24. Suppose p is a fifth-degree polynomial with real coefficients. If p is known to have at least one complex root, which of the following statements must be true?

(a) The polynomial has exactly two real roots.

(b) The polynomial has at least one real root.

(c) The polynomial has exactly three real roots.

(d) The polynomial has four real roots.

(e) The polynomial has exactly four complex roots.

25. The rational function f below is *improper* because the degree of its numerator is greater than or equal to the degree of its denominator.

$$f(x) = \frac{3x^3 - 2x^2 - 10x + 7}{x^2 - 4}$$

If we rewrite f as a *polynomial* plus a *proper* rational function, then we have

(a) $f(x) = x^2 - 4 + \dfrac{3}{x-2}$

(b) $f(x) = 3x + 2 + \dfrac{2x+15}{x^2-4}$

(c) $f(x) = 3x - 2 + \dfrac{2x-1}{x^2-4}$

(d) $f(x) = 3x + 10 + \dfrac{30x+7}{x^2-4}$

(e) $f(x) = 3x^2 - 12 + \dfrac{x-3}{x+2}$

26. What is the center of the circle $18x^2 + 18y^2 - 18x + 12y - 143 = 0$?

(a) $(1,3)$ (b) $\left(-\frac{1}{2}, \frac{1}{3}\right)$ (c) $\left(\frac{1}{2}, -\frac{1}{2}\right)$ (d) $\left(\frac{1}{2}, -\frac{1}{3}\right)$ (e) $(-3,1)$

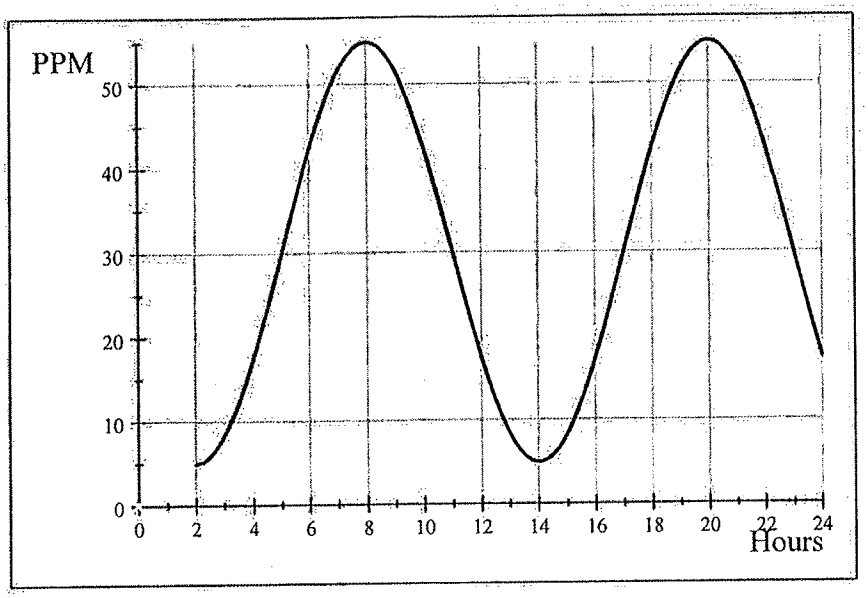
27. A small college has $1,000$ students. A virus spreads through the student population according to the formula

$$s(t) = \frac{1000e^{kt}}{499 + e^{kt}}$$

where time is measured in days since the virus first appeared on campus, and s denotes the number of students sick on day t. If it takes fourteen days for half of the student population to become sick, what is the value of k?

(a) $\dfrac{14}{1000}$ (b) $\dfrac{\ln(499)}{14}$ (c) $\dfrac{\ln(500)}{2\ln(14)}$ (d) $\dfrac{\ln(500) - \ln(2)}{14}$ (e) $\dfrac{1000}{499}$

28. The graph below shows the concentration (in parts per million) of an antibiotic in a person's bloodstream as a function of time (measured in hours since midnight on the first day of the treatment). After the first dose, nurses must give additional doses of the antibiotic when the concentration is about 20 ppm and decreasing. Based on the graph, at what approximate times should the antibiotic be given on the first day?



(a) When $t \approx 2$ and $t \approx 14$ hours

(b) When $t \approx 12$ and $t \approx 24$ hours

(c) When $t \approx 12$ and $t \approx 16$ hours

(d) When $t \approx 4$ and $t \approx 16$ hours

(e) When $t \approx 4$, $t \approx 12$, $t \approx 16$ and $t \approx 24$ hours

29. When the system below is solved, the formula for x will be

$$\begin{cases} x + 5y + 5z + 9w = 0 \\ x + 10y + 15z + 27w = 0 \\ 2x + 15y + 20z + 36w = 0 \end{cases}$$

(a) $x = 15$ (b) $x = z + w$ (c) $x = -15z - 27w$ (d) $x = 5z + 9w$ (e) $x = 10w$

30. Which of the following is the correct value of the geometric series $\sum_{k=0}^{\infty}(-1)^k \frac{2^k}{3^{k-1}}$?

(a) 9 (b) $\frac{9}{5}$ (c) $\frac{1}{45}$ (d) $\frac{2}{3}$ (e) $\frac{3}{2}$

31. Which formula gives the average rate of change between $x = 2$ and $x = 2 + b$ for the function $f(x) = \dfrac{x^2 - 1}{x}$? (Assume $b > 0$.)

(a) Average rate of change is $\dfrac{b^2 + 4b - 3}{4 + 2b}$

(b) Average rate of change is $\dfrac{5}{4}$

(c) Average rate of change is $\dfrac{b^2 + 4 - 3}{b}$

(d) Average rate of change is $\dfrac{b^2 + 4b + 4}{b}$

(e) Average rate of change is $\dfrac{5 + 2b}{4 + 2b}$

32. Suppose that a rational function has a removable singularity at $x = a$ and a vertical asymptote at $x = b$. Which of the following formulas could represent this function?

(a) $f(x) = \dfrac{x - a}{(x - b)(x - a)}$

(b) $f(x) = \dfrac{x - b}{(x - b)(x - a)}$

(c) $f(x) = \dfrac{x - a}{(x - b)(x - a)^2}$

(d) $f(x) = \dfrac{(x - b)^2}{(x - b)(x - a)}$

(e) $f(x) = \dfrac{(x + a)}{(x - b)(x - a)}$

33. Simplify the complex expression $\left(\dfrac{1}{2} + \dfrac{i\sqrt{3}}{2}\right)^4$.

(a) $-\dfrac{1}{2} + \dfrac{i\sqrt{3}}{2}$ (b) $\dfrac{1}{2} - \dfrac{i\sqrt{3}}{2}$ (c) $-\dfrac{1}{2} - \dfrac{i\sqrt{3}}{2}$ (d) -1 (e) 1

34. What are the roots of the polynomial $p(x) = x^3 - 1$?

(a) Only $x = 1$

(b) $x = 1$, and $x = \dfrac{1}{2}\left(1 \pm i\sqrt{3}\right)$

(c) $x = 1$, and $x = \dfrac{1}{2}\left(i\sqrt{3} - 1\right)$

(d) $x = 1$, and $x = \pm\dfrac{1}{2}\left(i\sqrt{3} + 1\right)$

(e) $x = 1$, and $x = \dfrac{1}{2}\left(-1 \pm i\sqrt{3}\right)$

In the following problem, $C(a,b)$ stands for the combination of a objects chosen b at a time. In symbols,

$$C(a,b) = \frac{a!}{b!(a-b)!}$$

35. A pet store owner has seven identical fishbowls lined up in a window. She wants to display four bettas and three goldfish in these bowls. If each fish must be in a bowl by itself, and there must be a goldfish between each betta, how many different displays can be made?

(a) $3!4!$ (b) $9 \cdot 3!4!C(4,3)$ (c) $3!4!C(7,3)$ (d) $7 \cdot 4!3!C(7,4)$ (e) $2^3 \cdot 4!$

36. Nell is riding on a ferris wheel with a radius of twelve feet. The ferris wheel is rotating in the counterclockwise direction; and it stalls while Nell's seat is rising, leaving her stranded. If the origin of a rectangular grid is placed at the center of the ferris wheel, the beam connecting Nell's seat to the center has slope $m = 3.14$. If the lowest point on the ferris wheel is three feet off the ground, how far off the ground (to the nearest hundredth foot) is Nell at her stranded position?

(a) 11.43 feet (b) 14.43 feet (c) 4.67 feet (d) 26.43 feet (e) 19.67 feet

37. Anne is a DJ at a local honky-tonk. She has twelve songs about truckers, fourteen songs about folks in honky-tonks, and fourteen songs about cowboys. A patron asks her how many songs she has about cowboy truckers who are not in honky-tonks. Anne is annoyed and tells the patron that she has

- Seven songs about truckers who are not cowboys
- Three songs about truckers in honky-tonks
- Four songs about cowboys in honky-tonks
- One song about cowboy truckers in honky-tonks

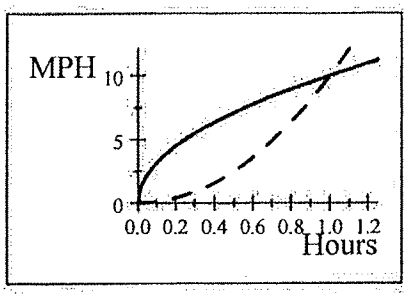
What is the answer to the patron's question?

(a) Four songs (b) Two songs (c) One song (d) Five songs (e) No songs

38. What is the algebraic formula that is equivalent to $\sec\left(\sin^{-1}\left(\frac{b}{\sqrt{10}}\right)\right)$?

(a) $\frac{\sqrt{10}}{b}$ (b) $\frac{b}{\sqrt{10}}$ (c) $\sqrt{\frac{10}{10-b^2}}$ (d) $\frac{\sqrt{10-b^2}}{b}$ (e) $\frac{\sqrt{10+b^2}}{10}$

39. The diagram below shows the velocity graphs for two cars traveling along straight roads. What is the relationship between the position of Car A and the position of Car B at one hour? The dashed graph refers to Car B.



(a) Car A and Car B are colliding.

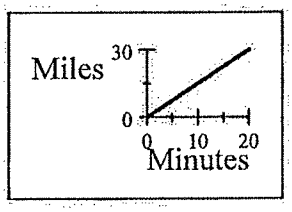
(b) Car A is ahead of Car B.

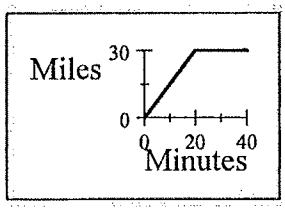
(c) Car B is ahead of Car A.

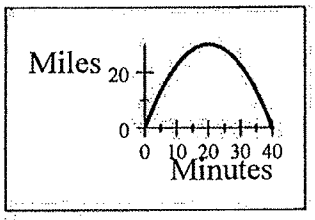
(d) Car B is passing Car A.

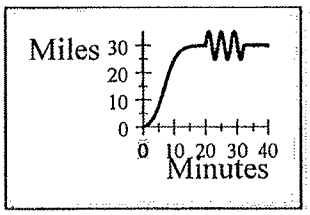
(e) Car A is passing Car B.

40. A small engine plane leaves the Murfreesboro airport and flies to the Nashville airport thirty miles away while some aerospace majors use radar to measure its distance from the Murfreesboro Airport. The plane arrives at the Nashville airport after about twenty minutes, but is put in a holding pattern because the airport is busy. The plane must circle the airport before it is allowed to land. Which of the following graphs best represents the plane's distance from the Murfreesboro airport as a function of time?

(a)


(b)


(c)


(d)


(e)