FIFTY-EIGHTH ANNUAL MATHEMATICS CONTEST
sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

Precalculus 2014

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Scoring formula: $4R - W + 40$

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft head (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly 80 minutes to work.

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1. Solve $4\,|\,x+8\,|+2<7$.

a) $\frac{27}{4} < x < \frac{37}{4}$ b) $\frac{-37}{4} < x < \frac{-27}{4}$ c) $x > \frac{-27}{4}$

d) $x < \frac{37}{4}$ e) $x < \frac{-37}{4}$ or $x > \frac{-27}{4}$

2. For the three points (1, 13), (2, 5) and (-1, 19), find a quadratic model q through the points.

a) $q(x) = \frac{5x^2}{3} - 3x + \frac{53}{3}$

b) $q(x) = \frac{-5x^2}{3} + 3x + \frac{53}{3}$

c) $q(x) = \frac{5x^2}{3} - 3x - \frac{53}{3}$

d) $q(x) = \frac{-5x^2}{3} - 3x + \frac{53}{3}$

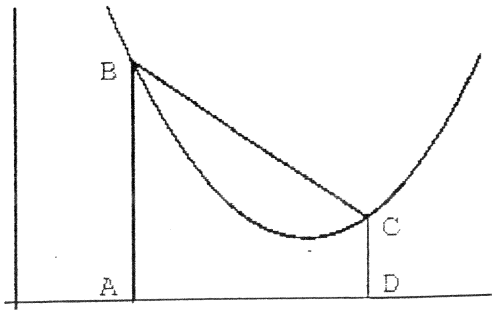
e) $q(x) = -5x^2 - 9x + 53$

3. The period of $f(t) = 5\cos\left(3t + \frac{\pi}{2}\right)$ is:

a) 5 b) $\frac{\pi}{2}$ c) 2π d) $\frac{2\pi}{3}$ e) $\frac{-\pi}{6}$

4. Let $f(x) = 2(x-4)^2 + 12$ be the parabola on the right. ABCD is a right trapezoid such that A = (1,0), D = (5,0) and B and C are on the graph of the parabola. Find the area of the trapezoid ABCD. The graph is not drawn to scale.

a) 89 b) 90 c) 85 d) 88 e) 91

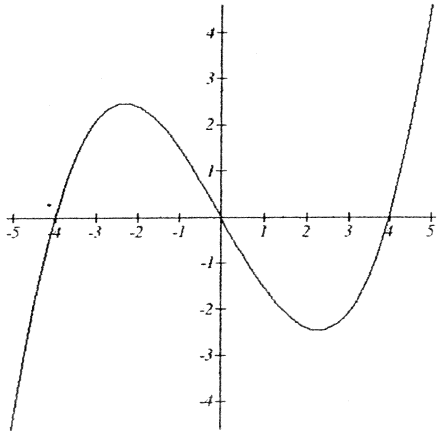


5. If y = sin x is shifted to the left three units, stretched vertically by a factor of 2, and then shifted down 5 units, the resulting function is:

a) $f(x) = 2\sin(x+3) - 5$ b) $f(x) = 2\sin(x+3) + 5$ c) $f(x) = 2[\sin(x+3) - 5]$

d) $f(x) = 2\sin(x-3) - 5$ e) $f(x) = 2[\sin(x+3) + 5]$

6. Identify the function whose graph appears on the right.



a) $f(x) = x(x-4)(x+4)/4$

b) $f(x) = x(x-4)(x+4)/20$

c) $f(x) = x(x-4)(x+4)/10$

d) $f(x) = x(x-4)(x+4)$

e) $f(x) = x(x-4)(x+4)/5$

7. $\dfrac{(11 \times 10^{6})(3 \times 10^{-2})}{2.2 \times 10^{9}} =$

a) 1.5×10^{-4} b) 1.5×10^{-6} c) 15×10^{5} d) 1500 e) 15×10^{3}

8. An expression equivalent to $\log_2\left(\frac{1}{4}\left(2^{-x+5}\right)\right)$ is:

a) $(-x+5)\log_2\left(\frac{2}{4}\right)$ b) $\log_{\frac{1}{2}}\left(2^{x-3}\right)$ c) $\frac{5-x}{4}$

d) $\log_4\left(2^{-x+4}\right)$ e) $\log_2\left(\frac{1}{2^{x+3}}\right)$

9. The graph shown in the figure on the right represents one period of a certain trigonometric function. A correct equation for this function is:

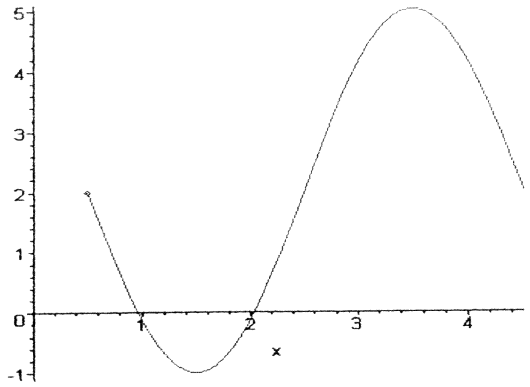
a) $y = -3\sin\left(\frac{\pi}{4}x + \frac{\pi}{2}\right) + 2$

b) $y = 2\sin\left(\frac{\pi}{4}x - \frac{\pi}{2}\right) - 3$

c) $y = -3\cos\left(\frac{\pi}{2}x - \frac{\pi}{4}\right) + 2$

d) $y = 3\cos\left(\frac{\pi}{2}x + \frac{\pi}{4}\right) + 2$

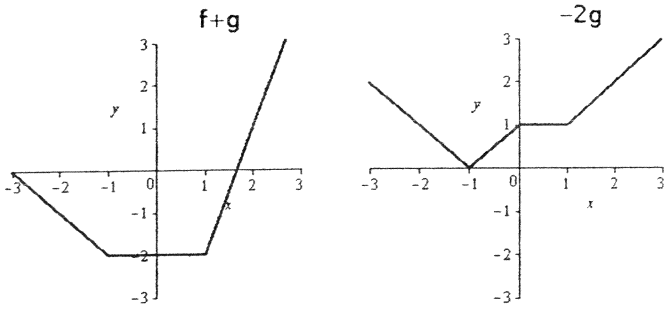
e) $y = 2\sin\left(\frac{\pi}{2}x - \frac{\pi}{4}\right) + 3$



10. Use the graphs to determine the value of f(0).

a) -1.5 b) 1 c) -1

d) -2 e) 1.5



11. The radius of the circle $x^2 + y^2 + 6x - 4y - 3 = 0$ is

a) 4 b) $2\sqrt{2}$ c) 3 d) $\sqrt{3}$ e) 16

12. If $q(x) = (x-3)^2(x+1)$, then q is positive on the interval:

a) $(-1,3)\cup(3,\infty)$ b) $(-\infty,-1)$ c) $(-\infty,-1)\cup(3,\infty)$ d) $(3,\infty)$ e) $(-1,\infty)$

13. Express $\dfrac{3+6i}{1-2i}$ in standard form.

a) $3-3i$ b) $3 + 2.4i$ c) $3 - 4i$ d) $3 + 2.4i$ e) $-1.8 + 2.4i$

14. What is the exact value of $2+\log_3 2$?

a) 2.3 b) $\log_3 4$ c) $\ln 6$ d) $\log_3 18$ e) 2.6

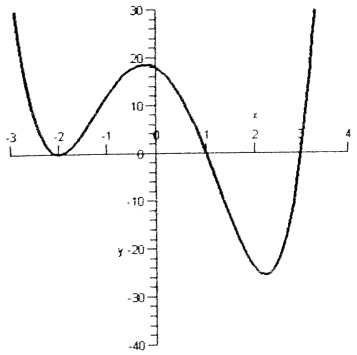
15. Find all solutions to $\sin\theta\cot\theta = -\sin\theta$ on the interval $0 \le \theta \le \pi$.

a) $\dfrac{3\pi}{4}$ b) $0,\dfrac{\pi}{4},\dfrac{3\pi}{4},\pi$ c) $0,\dfrac{\pi}{4},\dfrac{3\pi}{4}$ d) $0,\dfrac{\pi}{4},\dfrac{\pi}{2},\pi$ e) $0,\dfrac{3\pi}{4},\pi$

16. Find sin 2x if $\sin x = \frac{\sqrt{5}}{5}$ and $0 \le x < \frac{\pi}{2}$.

a) $\frac{4}{5}$ b) $\frac{3}{5}$ c) $\frac{\sqrt{2}}{5}$ d) $\frac{2}{\sqrt{5}}$ e) $\frac{1}{5}$

17. If p(x) is the fourth degree polynomial displayed in the graph, what is its leading coefficient?



a) 18 b) 1.5 c) -1.5 d) 3 e) -3

18. Given $f(x) = x^3 - 5x^2 + 7x - 3$, x =1 is a root of the function. What is a factor of f?

a) x-3 b) x+3 c) x+5 d) x-7 e) x+1

19. If $w = \frac{ax+b}{cx+d}$ where a, b, c, d are non-zero constants, the inverse function, w^{-1}, is

a) $\frac{cx+d}{ax+b}$ b) $\frac{cx+b}{ax-d}$ c) $-\frac{dx-b}{cx-a}$ d) $\frac{dx+b}{cx+a}$ e) $-\frac{bx+a}{dx+c}$

20. The trigonometric expression $\frac{1-\sin x}{\cos x}$ is equivalent to

a) sec x tan x b) $\frac{2\sec x}{1-\sin x}$ c) sec x - sin x d) 1 – tan x e) $\frac{\cos x}{1+\sin x}$

21. Suppose x is in the first quadrant and y is in the second quadrant, $\sin x = \frac{4}{5}$ and $\cos y = \frac{-1}{4}$. Where is the terminal side of x+y?

a) Quadrant I b) Quadrant II c) Quadrant III d) Quadrant IV

e) on one of the axes

22. Suppose x is in the first quadrant and y is in the second quadrant, $\sin x = \frac{4}{5}$ and $\cos y = \frac{-1}{4}$. Find the exact value of cos(x+y).

a) $\frac{-3-4\sqrt{15}}{20}$ b) $\frac{-7}{20}$ c) $\frac{-3+4\sqrt{15}}{20}$ d) $\frac{-4+3\sqrt{15}}{20}$ e) $\frac{7}{20}$

23. Find an algebraic expression for $\cos\left(\arccos(2x)+\frac{\pi}{3}\right)$.

a) $\frac{2x+1}{2}$ b) $\frac{2x-\left(1-4x^2\right)\sqrt{3}}{2}$ c) $\frac{2x-\sqrt{1-4x^2}}{2}$ d) $\frac{2x-\sqrt{3\left(1-4x^2\right)}}{2}$ e) $\frac{(4x-1)\sqrt{3}}{2}$

24. A navy pilot is about to land his fighter on the aircraft carrier directly ahead of him. As he starts his descent at an angle of 5.5 degrees with the horizontal, his altimeter reads 5979 feet above the water. How far is the pilot from the deck of the carrier (line of sight) if the deck stands 63 feet above the water?

a) 62,381 ft b) 61,440 ft c) 62,094 ft d) 5943 ft e) 61,724 ft

25. The range of $f(x)=\dfrac{8+2x-x^2}{(x-1)(x+2)}$ is:

a) $(-\infty,\infty)$
b) $(-\infty,-1)\cup(-1,\infty)$
c) $(-\infty,-2)\cup(-2,1)\cup(1,\infty)$
d) $(-\infty,-2)\cup(-2,-1)\cup(-1,\infty)$
e) $(-\infty,8)\cup(8,\infty)$

26. Find the angle between the positive x-axis and the line through the points (-1, 0) and (-3, 4).

a) 63.43° b) 116.57° c) 126.87° d) 53.13° e) 143.13°

27. The value of the determinant $\begin{vmatrix} x & y & z \\ 1 & 2 & 3 \\ 0 & 4 & 0 \end{vmatrix}$ is:

a) 8y b) 4(z – 3x) c) x + 8y + 3z d) 12x - 4z e) -12x

28. The solution of the system $\begin{aligned} x+2y &= 5 \\ -2x-4y+z &= -6 \\ 2x+4y+z &= 14 \end{aligned}$ is

a) no real solution exists
b) infinitely many solutions exist, but x = 5
c) infinitely many solutions exist, but y = 4
d) x = 5, y = 4, z = 0
e) infinitely many solutions exist, but z = 4

29. Given $\cos u = \frac{4}{5}, \ \frac{3\pi}{2} < u < 2\pi$, find the exact value of csc 2u.

a) -25/7 b) 10/3 c) -24/25 d) -25/24 e) 25/7

30. The interval that best describes the domain of the function $\sqrt{x^3+3^x}$ is

a) $(-\infty,\infty)$ b) $[0,\infty)$ c) $(-1.5,\infty)$ d) $(-0.76,\infty)$ e) $\left[\sqrt[3]{3},\infty\right)$

31. A certain radioactive substance decays exponentially. A scientist had a 500 gram sample 10 years ago that currently weighs 392 grams. What is the half-life of the substance in years (the amount of time for half of the radioactive substance to decay)?

a) 28.5 b) 23.2 c) 20.5 d) 30.2 e) 25.2

32. The function given by $g(x) = k(3 - 2x + x^4)$ has an inverse function for $x \geq 1$, and $g^{-1}(-3) = 1$. Find k.

a) 1/90 b) 2/3 c) -3/2 d) 1/78 e) no such k exists

33. Given $\tan u = \frac{12}{5}$, $\pi < u < \frac{3\pi}{2}$, find the exact value of $\cos\frac{u}{2}$.

a) $\frac{3\sqrt{13}}{13}$ b) $\frac{4\sqrt{13}}{26}$ c) $\frac{-3\sqrt{13}}{13}$ d) $\frac{-2\sqrt{13}}{13}$ e) $\frac{5}{26}$

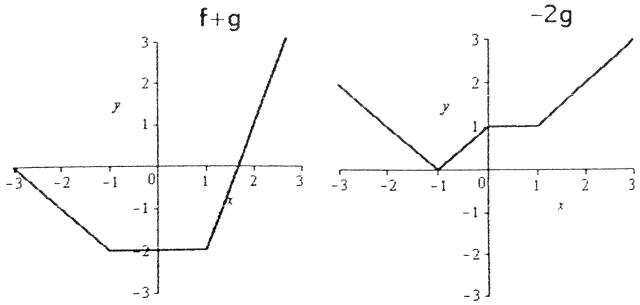
34. Solve $\cos 2z = \sin z$ for $0 \leq z \leq 2\pi$.

a) $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$ b) $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$ c) $\frac{\pi}{6}, \frac{3\pi}{2}$ d) $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{3\pi}{2}$ e) $\frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}$

35. The sides of a right triangle have lengths in inches of three consecutive multiples of 5. Find the area of the triangle.

a) 6 sq inches b) 25 sq inches c) 150 sq inches d) 187.5 sq inches e) 250 sq inches

36. Use the graphs to determine the value of f(g(2)).



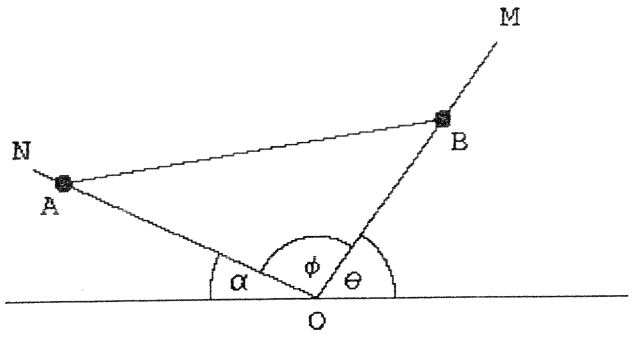
a) -1.5 b) 1 c) -1

d) -2 e) 1.5

37. Suppose that the population of Funny Town has a fixed yearly percentage increase. If the population was 500 in 2005 and grew to 600 in five years, the annual fixed percentage increase is:

a) 20% b) 4% c) 1.2% d) 3.7% e) 12%

38. Two ships leave port O at the same time. Ship A travels in the direction ON with velocity of 24 mph. Ship B travels in the direction OM with velocity of 14 mph. If $\theta = 40°$ and $\alpha = 23°$, find the distance between the two ships after 6 hours. Give your answer to the nearest integer. The picture is not drawn to scale.



a) 196 b) 195 c) 197 d) 200 e) 194

39. The range of $p(x)=\begin{cases}2x^2+4x-2 & x\le 1\\ 2x-8 & x>1\end{cases}$ is

a) $(-\infty,\infty)$ b) $[-4,\infty)$ c) $(-\infty,4]$ d) $(-6,\infty)$ e) $(-6,4]$

40. If $\log_b 20 = 2.161$, then $\log_b 80^2 =$

a) $4.322\left(\log_b 4\right)$ b) 18.679 c) 6.322 d) $2.161\left(\log_b 16\right)$ e) 74.719