1. Find the y-intercept of the line defined by the points $(\frac{2}{3}, \frac{1}{7})$ and $(\frac{2}{5}, \frac{1}{2})$.
   (a) $(0, \frac{29}{28})$
   (b) $(0, \frac{11}{14})$
   (c) $(0, \frac{25}{28})$
   (d) $(0, 1)$
   (e) $(0, \frac{-75}{56})$

2. Find the domain of the real-valued function
$$f(x) = \frac{(x-1)\sqrt{8-x}}{x^2-1}$$
   (a) $(-\infty, -1) \cup (-1, 8)$
   (b) $(-\infty, -1) \cup (8, \infty)$
   (c) $(-\infty, -1) \cup (-1, 1) \cup (8, \infty)$
   (d) $(-1, 1)$
   (e) $(-\infty, -1) \cup (-1, 1) \cup (-1, 8)$

3. Consider the piecewise-defined function
$$f(x) = \begin{cases} (x-2)^2 - 1, & x < 1 \\ 0, & 1 \le x < 4 \\ |2x - 10|, & x \ge 4. \end{cases}$$
   Where is f decreasing?
   (a) $(-\infty, 1)$
   (b) $(-\infty, 4)$
   (c) $(-\infty, 1] \cup [4, 5]$
   (d) $(-\infty, 1) \cup (4, \infty)$
   (e) All real numbers.

4. If $\log_b 3 = 1.585$, $\log_b 5 = 2.322$, and $\log_b 7 = 2.810$, find $\log_b \left(\frac{125}{21}\right)$.
   (a) 8.191
   (b) 2.571
   (c) -0.71463
   (d) 0.82633452
   (e) 5.741

5. Suppose $f(x)$ is a function defined for all real numbers which is not injective (one-to-one). Only one of the following *must* be true. Which is it?
   (a) The range of f can not be all real numbers.
   (b) The domain of the inverse function is all real numbers.
   (c) The range of the inverse function can not be all real numbers.
   (d) The inverse of f fails the horizontal line test.
   (e) The inverse of f is not a function of x.

6. Solve the following system of equations.

$$\begin{aligned} x - 2y + 3z &= 11 \\ 4x + 2y - 3z &= 4 \\ 3x + 3y - z &= 4 \end{aligned}$$

   (a) $x = 3$, $y = 7$, $z = -2$
   (b) $x = 3$, $y = -1$, $z = 2$
   (c) $x = -5$, $y = 2$, $z = -3$
   (d) $x = 3$, $y = 2$, $z = 4$
   (e) $x = 4$, $y = -4$, $z = -4$

7. Suppose you wish to demonstrate to a friend that matrix multiplication is *not* commutative. Which of the following pairs would provide a particularly bad example?
   (a) $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$
   (b) $A = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$
   (c) $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$
   (d) $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
   (e) $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$

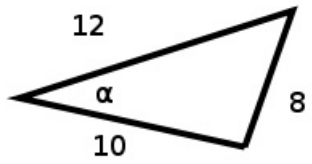
8. Divide $1 + 3i$ by $2 - i$. Give your answer in the form $a + bi$.
   (a) $\frac{1}{5} + \frac{7}{5}i$
   (b) $\frac{-1}{5} + 2i$
   (c) $\frac{-1}{5} + \frac{7}{5}i$
   (d) $-1 + 2i$
   (e) $\frac{-1}{5} - 2i$

9. In order to render $4x^2 - 7x + C$ a perfect square, C must be...
   (a) $\frac{-7}{4}$
   (b) $\frac{7}{4}$
   (c) $\frac{-49}{16}$
   (d) $\frac{49}{16}$
   (e) There is no real number C which can make $4x^2 - 7x + C$ a perfect square.

10. A real-valued function which is symmetric with respect to the y-axis is called an *even function* while a real-valued function symmetric with respect to the origin is called *odd*. Which of the following is true?
    (a) All even functions are polynomials of even degree, while all odd functions are polynomials of odd degree.
    (b) No even function ever takes on a negative value, while odd functions may assume negative values.
    (c) Even functions only take on even values and odd functions only odd values
    (d) The product of two even functions or the product of two odd functions is always even, while the product of an even function and an odd function is always odd.
    (e) The sum of two even functions or the sum of two odd functions is always even, while the sum of an even function and an odd function is always odd.

11. Write $12,340_{10}$ in base 7.
    (a) 1763_7
    (b) 50656_7
    (c) 2401_7
    (d) 100656_7
    (e) 46065_7

12. Given the information in the following figure, find the measure of α in radians to the nearest ten-thousandth.



(a) 1.3009

(b) 0.1823

(c) 0.7227

(d) 0 7411

(e) 0.5000

13. The expression $\sec x\,(\sec x - \cos x)$ is equal to

(a) $\tan^2 x$.

(b) $\frac{1}{\cos x}$.

(c) $\sec^2 x$.

(d) $\cot^2 x$.

(e) $\csc x\,(\csc x - \sin x)$.

14. The polar equation $r = 2 + 3\sin t$ defines a limaçon. If this equation models the position of a particle at time t and we begin observing the particle at time $t = 0$, how many complete circuits of the limaçon has the particle made at time $t = 7\pi$?

(a) 1

(b) 3

(c) 4

(d) 7

(e) 14

15. Consider the system of parametric equations

$$x = \sec t,\ y = \tan^2 t.$$

Select the rectangular equation that best corresponds to this system.

(a) $y = \tan^2 x$

(b) $y = \sec^{-1} x$

(c) $y = x^2 - 1$

(d) $y = x^4 - 1$

(e) $y = \tan^2 x - 1$

16. A ninth degree polynomial may have a maximum of how many local extrema?

(a) 0

(b) 1

(c) 8

(d) 9

(e) 10

17. Using the graph of $f(x) = \sqrt{x}$ as a reference, describe the graph of $g(x) = \frac{1}{2}\sqrt{3-x} + 2$.

(a) A reflection about the y-axis, followed by a shift left 3 units, a vertical compression by $\frac{1}{2}$, and a shift up by 2 units.

(b) A reflection about the y-axis, followed by a shift left 3 units, a vertical stretch by 2, and a shift up by 2 units.

(c) A reflection about the x-axis, followed by a shift left 3 units, a vertical compression by $\frac{1}{2}$, and a shift up by 2 units.

(d) A reflection about the y-axis, followed by a shift right 3 units, a vertical compression by $\frac{1}{2}$, and a shift up by 2 units.

(e) A reflection about the x-axis, followed by a shift right 3 units, a vertical compression by $\frac{1}{2}$, and a shift up by 2 units.

18. Dividing $3x^7 + 5x^6 + 2x^4 - 7x^3 + x^2 + 4x + 9$ by $2x + 4$ leaves a remainder of...

(a) 125

(b) 61

(c) -11

(d) -107

(e) 29

19. Suppose $a > 1$. Given the equation $\log_a b + 2\log_b a = 4$, choose the statement that is always true.
    (a) There are two real solutions for b in the interval $[1, a^4]$.
    (b) There is no more than one real solution for b in the interval $[\frac{1}{a^3}, 1]$.
    (c) There are two real solutions for b in the interval $[\frac{1}{a}, a^2]$.
    (d) There are no real solutions for b in the interval $[1, a^3]$.
    (e) There are no real solutions for b in the interval $[\frac{1}{a}, a^2]$.

20. Find the solution set for the inequality
$$\frac{(2x-1)(x^2-3x+2)}{(4x+5)\sqrt{8-x}} \geq 0.$$
    (a) All real numbers.
    (b) $\left(-\infty, \frac{1}{2}\right) \cup \left(\frac{1}{2}, 1\right] \cup \left[\frac{5}{4}, 8\right)$
    (c) $\left(-\infty, \frac{-5}{4}\right) \cup \left[\frac{1}{2}, 1\right] \cup \left[\frac{5}{4}, 8\right)$
    (d) $\left(-\infty, \frac{-5}{4}\right) \cup \left[\frac{1}{2}, 1\right] \cup [2, \infty)$
    (e) $\left(-\infty, \frac{-5}{4}\right) \cup \left[\frac{1}{2}, 1\right] \cup [2, 8)$

21. Find the formula of a hyperbola with foci $(0, 5)$ and $(0, -5)$, and vertices $(0, 3)$ and $(0, -3)$.
    (a) $\frac{y^2}{9} - \frac{x^2}{16} = 1$
    (b) $\frac{y^2}{16} - \frac{x^2}{9} = 1$
    (c) $\frac{x^2}{16} - \frac{y^2}{9} = 1$
    (d) $\frac{x^2}{9} + \frac{y^2}{16} = 1$
    (e) $\frac{y^2}{9} + \frac{x^2}{16} = 1$

22. When $(3x + 2)^{13}$ is expanded, what is the coefficient of x^7?
    (a) 160,123,392
    (b) 240,185,088
    (c) 7,413,120
    (d) 115,288,842,240
    (e) 1,716

23. Solve

$$1 + 2\sec\theta = 2\tan\theta$$

List values of θ in radians and round all values to the nearest ten-thousandth.

(a) 2.4981

(b) -2.4981

(c) 0.6435

(d) -0.6435

(e) 1.3452

24. Find the area of a triangle with sides of length 4, 6, and 8. All measurements are in inches.

(a) 12 in^2

(b) $12\sqrt{2}$in^2

(c) $\sqrt{15}$ in^2

(d) $3\sqrt{15}$ in^2

(e) $\sqrt{45}$ in^2

25. A team is building a solar car for a competition. Given their engine and transmission, optimal efficiency is produced when the driving wheels are rotated at 2000 rotations per minute. All else remaining the same, how much faster will the car be if the 16 inch diameter wheels are replaced with 22 inch diameter wheels?

(a) $12,000$ in/min

(b) $18,850$ in/min

(c) $37,699$ in/min

(d) $704,000$ in/min

(e) $2,211,681$ in/min

26. Disregarding friction, find the force necessary to pull a 200 pound weight up an incline that makes a 20 degree angle with the horizontal. Round to the nearest hundredth of a pound.

(a) 68.40

(b) 182.59

(c) 11.11

(d) 71.37

(e) 154.66

27. Suppose a given investment guarantees a 1.5% annual percentage rate, with interest compounded continuously. What is the doubling time of the investment?

    (a) 4.6 yrs.

    (b) 46.2 yrs.

    (c) 20.1 yrs.

    (d) 462.1 yrs.

    (e) 201.0 yrs.

28. Consider the following three statements.
    Statement A: "If chimpanzees can write plays then Shakespeare was a chimpanzee."

    Statement B: "If Shakespeare was a chimpanzee then chimpanzees can write plays."

    Statement C: "If Shakespere was not a chimpanzee then chimpanzees cannot write plays."
    Which of the following statments is true?

    (a) Statements A and B are logically equivalent.

    (b) Statements A and C are logically equivalent.

    (c) Statements B and C are logically equivalent.

    (d) Statement B is the inverse of statement A.

    (e) Statement C is the inverse of statement A.

29. 215 winter sports enthusiasts are surveyed. Every person surveyed likes at least one of the following sports: skiing, figure skating, and hockey. 55 of the people who like skiing don't like figure skating and 20 people who like figure skating don't like hockey. 50 of the people who like hockey either like both of the other sports, or neither of the others. How many people like both hockey and figure skating, but not skiing?

    (a) 55

    (b) 75

    (c) 80

    (d) 90

    (e) 100

30. Radu has seven pairs of shoes, one of each color of the visible spectrum. Likewise, he has seven shirts and seven pairs of slacks. What is the probability that on any given day, having selected articles of clothing at random, Radu is wearing one article that is red, one that is yellow, and one that is blue?

    (a) $\frac{6}{343}$
    (b) $\frac{1}{343}$
    (c) $\frac{27}{343}$
    (d) $\frac{9}{343}$
    (e) $\frac{3}{343}$

31. Suppose we know that a certain fifth degree polynomial with rational coefficients and no constant term has at least one irrational root and at least one imaginary root. List all of this polynomial's rational roots.

    (a) -1, 1, 0
    (b) 1
    (c) 0
    (d) This polynomial has no rational roots.
    (e) There is not enough information to find the rational roots.

32. If $\log\left(x^2y^3\right) = a$ and $\log\left(\frac{x}{y}\right) = b$ then $\log x =$...

    (a) $\frac{a-2b}{5}$.
    (b) $a - 2b$.
    (c) $\frac{a+3b}{5}$.
    (d) $3b - a$.
    (e) 10^{ab}.

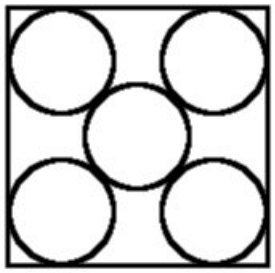
33. Suppose a, b, x, and y represent distinct positive real numbers. Select the true statement.
    (a) $(ab + xy)(ax + by) > 4abxy$
    (b) $(ab + xy)(ax + by) = 4abxy$
    (c) $(ab + xy)(ax + by) < 4abxy$
    (d) Either (a) or (b) is possible.
    (e) Either (b) or (c) is possible.

34. An infectious disease spreads in a given population according to the formula

$$P(t) = \frac{P_0 e^{2t}}{P_0 - 1 + e^{2t}}.$$

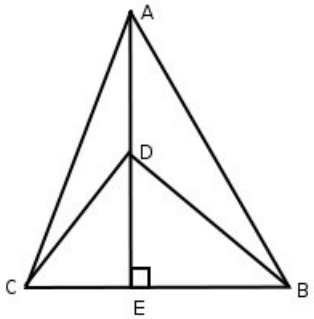
    where $P(t)$ denotes the number of people infected, P_0 the initial population all healthy, and t the time in days. If the outbreak begins on $t = 0$, when will over half the population be infected?
    (a) $t = \frac{\ln(P_0 - 1)}{2}$
    (b) $t = \frac{\ln(P_0)}{2}$
    (c) $t = \frac{\ln(1 - P_0)}{2}$
    (d) $t = \frac{1}{2}P_0$
    (e) $t = \frac{-\ln 2}{2}$

35. The drawing below depicts an optimal packing of five circles into a square and is accurate to the extent that (1) the circles that look to lie on a diagonal do, (2) all circles that appear tangent to one another are, and (3) the four circles in the corners are tangent to the surrounding square in two places. If the circles all have radius 1, what is the length of one side of the square?



(a) 6
(b) $2+2\sqrt{2}$
(c) $4\sqrt{2}$
(d) 4
(e) $5+\sqrt{2}$

36. In the figure below, the line segment $\overline{AB}$ has length 7, the line segment $\overline{BD}$ has length 5, and the line segment $\overline{CD}$ has length 4. Use this information and the Pythagorean Theorem to find the length of $\overline{AC}$.



(a) $2\sqrt{10}$
(b) 6
(c) 2π
(d) $\frac{63}{10}$
(e) $3\sqrt{2}$

37. Sum the first 10,000 odd numbers.
    (a) 20,001
    (b) 49,995,000
    (c) 99,990,000
    (d) 100,000,000
    (e) 199,990,000

38. The n^{th} roots of unity are the complex solutions to the equation $z^n = 1$. The fifth roots of unity, listed counterclockwise around the unit circle, are: 1, $\cos\frac{2\pi}{5} + i\sin\frac{2\pi}{5}$, $\cos\frac{4\pi}{5} + i\sin\frac{4\pi}{5}$, $\cos\frac{6\pi}{5} + i\sin\frac{6\pi}{5}$, and $\cos\frac{8\pi}{5} + i\sin\frac{8\pi}{5}$. Find $(\cos\frac{2\pi}{5} + i\sin\frac{2\pi}{5})^{2702}$.
    (a) 1
    (b) -1
    (c) $\cos\frac{2\pi}{5} + i\sin\frac{2\pi}{5}$
    (d) $\cos\frac{8\pi}{5} + i\sin\frac{8\pi}{5}$
    (e) $\cos\frac{4\pi}{5} + i\sin\frac{4\pi}{5}$

39. An airplane flys a round trip between Hermannstadt and Temesvar. On the trip out there is a headwind and the average speed is 360 mph. On the trip back there is a tailwind- average speed: 440 mph. What is the average speed for the entire trip?
    (a) 415 mph
    (b) 400 mph
    (c) 396 mph
    (d) 200 mph
    (e) 198 mph

40. Anne, Bob, Carol, Dan, and Edward are the volunteer staff of the Cumberland community garden. Each can only work one day a week, and the garden must be open Monday through Friday. Anne, who was scheduled on Monday, trades days with Bob. She is now scheduled for Thursday. Carol trades days with Edward, and will now work Friday, and then Edward trades with Dan, who was scheduled for Wednesday. Who is working on Tuesday?
    (a) Anne
    (b) Bob
    (c) Carol
    (d) Dan
    (e) Edward

Extra Questions

1. Where is $x^2 + 2x + 1 > 5x - 1$?
   (a) $(-\infty, -7) \cup (0, \infty)$
   (b) $(-\infty, 1) \cup (2, \infty)$
   (c) $(-\infty, -2) \cup (-1, \infty)$
   (d) $(-\infty, 0) \cup (7, \infty)$
   (e) All real numbers.

2. Find the area of an equilateral triangle that circumscribes a circle with radius 3.
   (a) $27\sqrt{3}$
   (b) $27\sqrt{2}$
   (c) $9\sqrt{3}$
   (d) $9\sqrt{2}$
   (e) $3\sqrt{2}$