# Pre-Calculus

1. If the mean, median, and mode for the set of numbers $\{10, 20, 30, 40, 50, X\}$ are equal, determine the value of X.
    A. 10   B. 20   C. 30   D. 40   E. 50

2. Determine the domain of the function $f(x) = \dfrac{\sqrt{x+4}}{x-5}$.

    A. $[-4, \infty)$ B. $[-4, 5) \cup (5, \infty)$ C. $(-\infty, 5) \cup (5, \infty)$
    D. $(-4, 5] \cup [5, \infty)$ E. $(-\infty, 5] \cup [5, \infty)$

3. Determine the value of the expression $\dfrac{(3-i)(10+50i)}{3+i}$.

    A. $38 + 34i$ B. $10 + 50i$ C. $-10 - 50i$ D. $40 - 460i$ E. $60i$

4. If $x^5 - 2x^3 + 3x - 6$ is divided by $x + 2$, the result is
    $x^4 - 2x^3 + 2x^2 - 4x + 11 + \dfrac{n}{x+2}$. What is n?
    A. -7   B. 20   C. 8   D. -8   E. -28

5. If $h(x) = \left(x^5 - 1\right)^{\frac{1}{3}}$, then $h^{-1}(x)$ is

    A. $\sqrt[3]{x^5 - 1}$ B. $\sqrt[5]{x^3 + 1}$ C. $\sqrt[5]{\sqrt[3]{x} + 1}$ D. $\sqrt[5]{(x+1)^3}$ E. h has no inverse

6. Determine the set of real solutions of the following equation.
    $$\begin{vmatrix} 1 & x & 1 \\ x & 1 & 2 \\ 1 & 0 & 1 \end{vmatrix} = 0$$
    A. $\{1\}$ B. $\{0, 2\}$ C. $\{1, 2\}$ D. $\{0\}$ E. $\emptyset$

7. Determine the values of k such that $x^2 + kx + 1 = 0$ has imaginary roots.
    A. $k < 2$ B. $|k| \leq 2$ C. $-2 < k < 2$ D. $k > 2$ E. $|k| \geq 2$

8. Let a, b, c, and d be real numbers such that $a > b$ and $c > d$. Which of the following is guaranteed to be true?

    A. $a - c > b - d$ B. $ac > bd$ C. $\dfrac{a}{b} > \dfrac{c}{d}$ D. $a + c > b + d$ E. Both A. and D.

9. The point (- 3, k) is on the line that passes through (- 6, 4) and (3, $\frac{5}{2}$). What is k?

   A. 2   B. 3.5   C. 6   D. 6.5   E. 0

10. The $\cos\left(\cot^{-1}\frac{x}{2}\right)$ is

    A. $\dfrac{2}{\sqrt{4-x^2}}$ B. $\dfrac{x}{\sqrt{4+x^2}}$ C. $\dfrac{\sqrt{4+x^2}}{2}$ D. $\dfrac{2}{\sqrt{4+x^2}}$ E. $\dfrac{\sqrt{4+x^2}}{x}$

11. Determine the solution set of $|1-2x|<3$.

    A. $-1<x<2$ B. $-4<x<2$ C. $-2<x<-1$ D. $-2<x<4$
    E. $-2<x<1$

12. The expression $(\sin x - \cos x)^2 + \sin 2x$ simplifies to

    A. $1+\sin 2x$ B. $\sin^2 x - \cos^2 x$ C. $2\sin 2x$ D. 1   E. $3\sin x\cos x +1$

13. If two fair dice are tossed, what is the probability that the total is greater than 3?

    A. $\dfrac{4}{9}$ B. $\dfrac{11}{12}$ C. $\dfrac{35}{36}$ D. $\dfrac{5}{6}$ E. $\dfrac{3}{4}$

14. The equation $x^3 - 9 = 9x - x^2$ has three solutions. One solution is -1. What is the quotient of the other two solutions?

    A. -9   B. -1   C. 0   D. -6   E. 9

15. A rectangle is drawn in the xy-plane so that its bottom edge lies on the x-axis. Its top corners lie on the graph of $y = 5 - x^2$. What is the maximum area of such a rectangle to two decimal places?

    A. 8.25   B. 8.49   C. 8.59   D. 8.61   E. 8.73

16. Determine the value of y that maximizes the objective function $z = x + 2y$ over the feasible region defined by the constraints:

$$\begin{cases} x+y\le16 \\ x+y\ge10 \end{cases} \text{ and } \begin{cases} 0\le x\le16 \\ 0\le y\le8 \end{cases}$$

    A. 0   B. 2   C. 6   D. 8   E. 10

17. If all denominators are non-zero, in simplest form $\dfrac{\dfrac{1}{a^2}+\dfrac{2}{ab}+\dfrac{1}{b^2}}{\dfrac{1}{a^2}-\dfrac{1}{b^2}}$ equals

A. $a-b$ B. $\dfrac{b^2+2ab+a^2}{b^2-a^2}$ C. $b-a$ D. $\dfrac{b+a}{b-a}$ E. -1

18. Over the real numbers, what is the domain of $f(x)=\sin\big(\cos\big(\log(x-\pi)\big)\big)$?

A. $(1,\infty)$ B. $(-\infty,\infty)$ C. $(-\infty,\pi)$ D. $(10\pi,\infty)$ E. (π,∞)

19. Determine the center of the circle $x^2-6x+10y+y^2=-18$.
    A. $(4,0)$ B. $(-3,5)$ C. $(0,4)$ D. $(3,-5)$ E. $(-6,10)$

20. If θ is an angle in quadrant IV such that $\cos\theta=a$, then $\tan\theta$ is equal to

A. $-\sqrt{1-a^2}$ B. $\dfrac{-a}{\sqrt{1-a^2}}$ C. $\dfrac{-1}{\sqrt{1-a^2}}$ D. $\dfrac{-1}{a}$ E. $\dfrac{-\sqrt{1-a^2}}{a}$

21. Which of the following statements is true for $h(x)=\dfrac{x-5}{x^3-2x}$?

   I $h(x)$ is negative when $-\sqrt{2}<x<0$

   II $h(x)$ is negative when $0<x<\sqrt{2}$

   III $h(x)$ is negative when $x\geq 5$

   IV $h(x)$ is negative when $\sqrt{2}<x<5$

A. I only   B. I and III   C. II and IV   D. IV only   E. I and IV

22. A bag holds eight marbles, all of which are identical except for color. One of the marbles is green, two are red, and the remaining five are blue. If you do not replace any marbles you pick from the bag and the first marble you pick is red, what is the probability that the next two marbles you pick are both blue?

A. $\dfrac{5}{7}$ B. $\dfrac{5}{42}$ C. $\dfrac{10}{21}$ D. $\dfrac{6}{49}$ E. $\dfrac{5}{16}$

23. What is the solution (rounded to the nearest thousandth, if necessary) of the equation $5^{x+1}=4^{2x-1}$?

A. 2 B. ≈ 0.700 C. $-\dfrac{1}{2}$ D. ≈ 0.181 E. ≈ 2.576

24. If the graph of $f(x)$ is shown below.
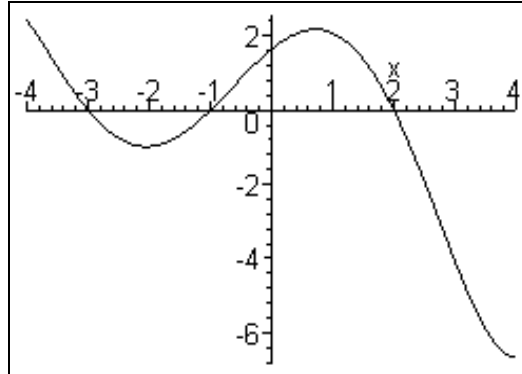
Which of the graphs below is the graph of $g(x) = 2 + f(2 - x)$?

A.

D.

B.

E.

C.
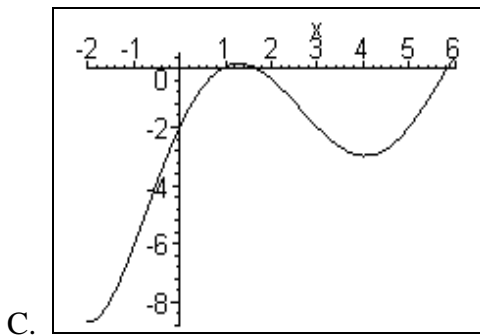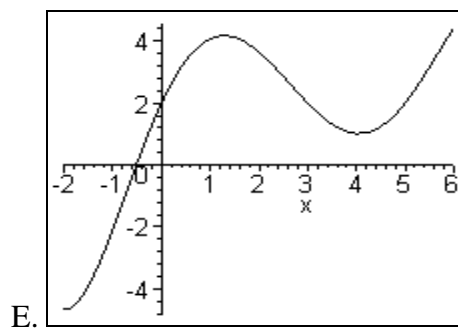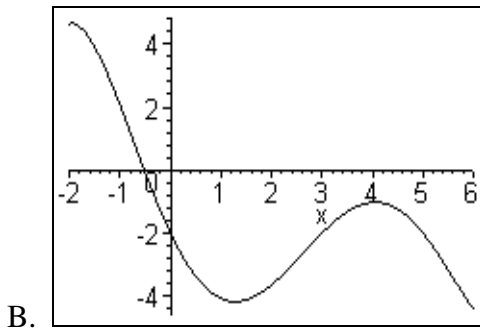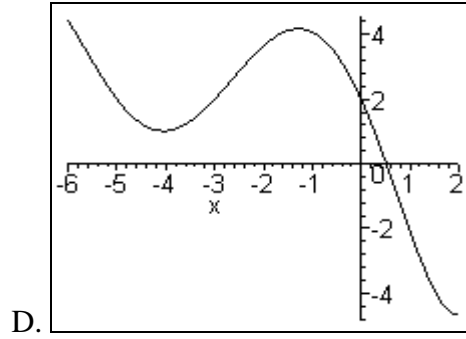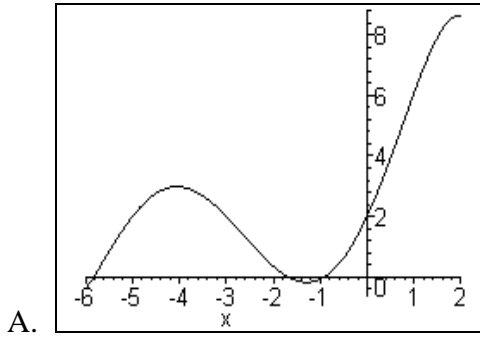
25. Simplify $\dfrac{6}{x+3} - \dfrac{x+4}{9-x^2} + \dfrac{3x-1}{x^2-6x+9}$.

A. $\dfrac{10x^2 - 35x + 63}{(x-3)^2(x+3)}$

B. $\dfrac{8x^2 - 21x + 39}{(x-3)^2(x+3)}$

C. $\dfrac{8x^2 - 29x + 63}{(x-3)^2(x+3)}$

D. $\dfrac{10x^2 - 27x + 39}{(x-3)^2(x+3)}$

E. $\dfrac{4x^2 - 43x + 45}{(x-3)^2(x+3)}$

26. If $f(x, y, z) = \dfrac{2^x + 2^z}{2^y}$, then $f(2,3,2)$ is equal to

A. 2   B. 3   C. 8   D. 1   E. 0

27. Solve $\log(x+4) + \log(x-4) = 2$ for x. The solution set is which of these?

A. $\left\{-2\sqrt{29}, 2\sqrt{29}\right\}$ B. $\left\{2\sqrt{29}\right\}$ C. $\left\{0, -2\sqrt{29}\right\}$

D. $\{4, 2\}$ E. $\left\{2\sqrt{29}, 4\right\}$

28. Simplify $\dfrac{\ln x^2}{\log_c x}$.

A. $\sqrt{x}$ B. x C. $\ln c^2$ D. $\ln c$ E. $\dfrac{2}{\ln c}$

29. A jar of coins contains only dimes and half dollars. The total value of the coins in dollars is v and the total number of coins is n. Determine the number of dimes in the jar.

A. $\dfrac{5v}{2} + \dfrac{5n}{4}$ B. $\dfrac{v}{2} - \dfrac{5n}{4}$ C. $\dfrac{5v}{2} - \dfrac{n}{4}$ D. $\dfrac{5n}{4} - \dfrac{5v}{2}$ E. $\dfrac{5v}{2} - \dfrac{5n}{4}$

30. A car travels 100 miles at 60 miles per hour and then 50 more miles at 70 miles per hour. What is the car's average speed for the entire trip?

A. 62.5 mph   B. 63 mph   C. 64.5 mph   D. 65 mph   E. 68 mph

31. Shown is the graph of $q(x)$. Which number is the closest to $\dfrac{q(1.01)-q(1)}{.01}$?
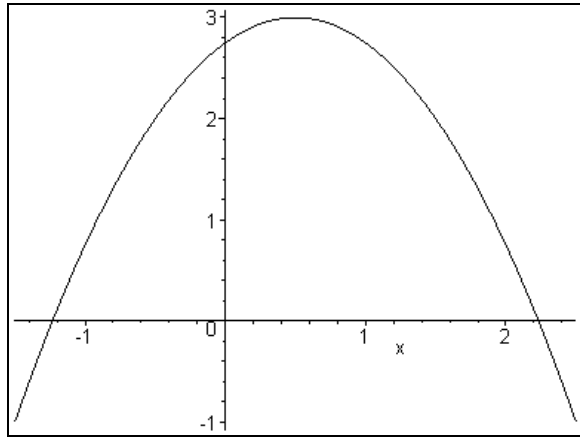
A. -3   B. -1   C. 0   D. 1   E. 3

32. A wheel with a 30 cm radius is rotating at a rate of 3.2 radians per sec. What is the linear speed of a point on its rim, in meters per minute?

A. 1.024 m/min   B. 576 m/min   C. 5.76 m/min
D. 57.6 m/min   E. 614.4 m/min

33. Determine the fourth term of the expansion of $(x+6)^{12}$.

A. $47520x^9$ B. $47520x^8$ C. $47520x^{10}$ D. $641520x^8$ E. $641520x^9$

34. The graph of a parabola has x-intercepts (-2, 0) and (5, 0) with the y-intercept (0, -20). If the equation for the parabola is expressed as $y=ax^2+bx+c$, what is the product of a, b, and c?

A. -30   B. -240   C. 60   D. 240   E. 30

35. Consider the matrices A=$\begin{pmatrix} 6 & 1 \\ 4 & 9 \end{pmatrix}$ and B=$\begin{pmatrix} -4 & 6 \\ 0 & x \end{pmatrix}$.

Determine the value of x such that matrix C = A + B will not have an inverse.

A. -2   B. 0   C. 2   D. 5   E. 8

36. The number of points of intersection of the graphs of the equations $y=x^2-2$ and $x=y^2+8y+12$ is

A. 0   B. 1   C. 2   D. 3   E. 4

37. Uranium has a half life of 4,560 years. For the exponential function in the form $P(t) = P_0 e^{-kt}$, determine the value of k to describe the amount of Uranium after t years.

 A. $\{.000152\}$

 B. $\{-.00152\}$

 C. $\{-.000152\}$

 D. $\{.00152\}$

 E. $\{-.0000152\}$

38. The third term of a geometric sequence is 12 and the fifth term is 75. Determine the nth term of the sequence if all its terms are positive.

 A. $\dfrac{48}{25}\left(\dfrac{5}{2}\right)^{n}$ B. $\dfrac{48}{25}\left(\dfrac{5}{2}\right)^{n-1}$ C. $-51+\dfrac{63}{2}(n-1)$ D. $-51+\dfrac{63}{2}n$ E. $12+63n$

39. When the radius of the base of a cylinder was multiplied by five, the total surface area of the new cylinder was thirteen times as large as the original. What was the ratio of the height to the radius of the original cylinder?

 A. 1:3   B. 3:2   C. 5:13   D. 2:3   E. 5:1

40. Let $f(x)=9-x^2$ and $g(x)=\dfrac{1}{\sqrt{x}}$. The domain of $f \circ g$ is

 A. $(0,\infty)$ B. $(-\infty,-3]\cup[3,\infty)$ C. $[-3,3]$

 D. $(-3,3)$ E. $(-\infty,0)\cup(0,\infty)$