FORTY-NINTH ANNUAL MATHEMATICS CONTEST
sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

Precalculus 2005

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Scoring formula: 4R - W + 40

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft head (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly 80 minutes to work.

Contributors to TMTA for the Annual Mathematics Contest:

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TMTA 2005 Precalculus Test

1. Express $-\frac{11\pi}{6}$ radians in degrees.

   (A) -660° (B) -330° (C) -630° (D) -300° (E) None of these.

2. Simplify: $e^{3\ln x - 2\ln y}$

   (A) $x^3 - y^2$ (B) $3x - 2y$ (C) $\frac{y^2}{x^3}$ (D) $\frac{3x}{2y}$ (E) $\frac{x^3}{y^2}$

3. Determine the value of the expression $\frac{4+3i}{3-4i} + \frac{4-3i}{3+4i}$.

   (A) $\frac{24}{25}$ (B) 0 (C) $\frac{4}{3}$ (D) $\frac{48}{25}$ (E) $\frac{8}{3} - \frac{3}{2}i$

4. In ΔABC, a = 5, b = 9, and c = 7 where a, b and c are the lengths of the sides opposite $\angle A$, $\angle B$, and $\angle C$, respectively. Find the measure of the largest angle in ΔABC correct to the nearest tenth of a degree.

   (A) 60.9° (B) 95.7° (C) 44.4° (D) 84.3° (E) None of these.

5. A stock loses 60% of its value. What must the percent of increase be to recover all of its lost value?

   (A) 60% (B) 120% (C) 150% (D) 200% (E) 300%

6. Find the domain of $f(x) = \frac{x^2+9}{x^2+25}$

   a) $(-\infty, -5i) \cup (-5i, -3i) \cup (-3i, 3i) \cup (3i, 5i) \cup (5i, \infty)$

   b) $(-\infty, -5i) \cup (-5i, 5i) \cup (5i, \infty)$

   c) $(-\infty, -3i) \cup (-3i, 3i) \cup (3i, \infty)$

   d) $[-5, -3] \cup [3, 5]$

   e) $(-\infty, \infty)$

7. How many revolutions per second does a tire having a 32 inch diameter make if it is traveling at a speed of 60 miles per hour?

(A) 21.3 rev/sec (B) 10.5 rev/sec (C) 34.3 rev/sec (D) 17.8 rev/sec

8. Which of the following is always true for any three real numbers m, n, and p?

(A) If $m < n$, then $mp < np$

(B) If $m < n$, then $m^2 < n^2$

(C) If $m < n$, then $m^3 < n^3$

(D) If $mp < np$, then $m < n$

(E) If $m > n$, then $\frac{1}{m} > \frac{1}{n}$

9. Of 11 marbles in a bag, 3 are red and 8 are green. If 2 different marbles are chosen, what is the probability of choosing one red and one green marble?

(A) $\frac{21}{121}$ (B) $\frac{24}{121}$ (C) $\frac{12}{55}$ (D) $\frac{24}{55}$ (E) $\frac{4}{5}$

10. Solve for x: $3^{1-\sqrt{x}} = \frac{3}{9^8}$

(A) -14 (B) 7 (C) 16 (D) 49 (E) 256

11. Find the domain of $\log_4\left(\frac{x^2-9}{x+1}\right)$

(A) $(-3,-1)\cup(3,\infty)$ (B) $[-3,-1)\cup[3,\infty)$

(C) $(-\infty,-1)\cup(-1,\infty)$ (D) $[-3,3]$ (E) $(4,\infty)$

12. Find the largest possible perimeter for a rectangle with area 100 square inches.

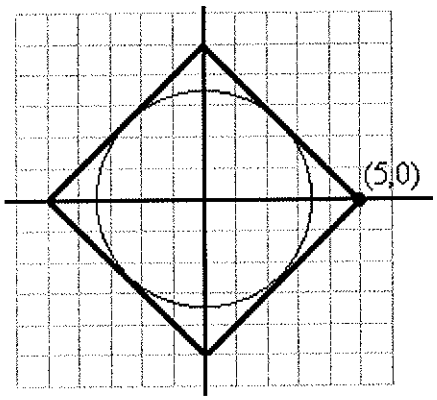
(A) 202 inches (B) 100 inches (C) 50 inches

(D) 40 inches (E) no solution

13. If 3 fair dice are tossed, what is the probability that the total tossed is greater than 12 (between 13 and 18)?

(A) $\frac{7}{72}$ (B) $\frac{1}{6}$ (C) $\frac{1}{3}$ (D) $\frac{7}{27}$ (E) $\frac{3}{8}$

14. Determine the equation of the circle inscribed in the square pictured here.



(A) $x^2 + y^2 = 25$

(B) $x^2 + y^2 = 12.5$

(C) $x^2 + y^2 = 2.5$

(D) $x^2 + y^2 = 50$

(E) $x^2 + y^2 = 13$

15. Solve for x: $\frac{7x}{3-\sqrt{5}} = \frac{\sqrt{3}+5}{7}$

(A) -2 (B) $\frac{-\sqrt{15}-5\sqrt{5}+3\sqrt{3}+15}{49}$

(C) $-\frac{2}{49}$ (D) $\frac{3\sqrt{3}-5\sqrt{5}}{49}$ (E) $15-5\sqrt{5}$

16. Given the one-to-one function $f(x) = x^3 + 2x^2 + 5x + 9$, determine the value of $f^{-1}(-15)$.

(A) $f^{-1}(-15) = -3$ (B) $f^{-1}(-15) = -2991$

(C) $f^{-1}(-15) = 5$ (D) $f^{-1}(-15) = -710$

17. Teams A and B play a series of games; whoever wins two games first wins the series. If Team A has a 70% chance of winning any single game, what is the probability that Team A wins the series?

(A) 0.616 (B) 0.637 (C) 0.657 (D) 0.700 (E) 0.784

18. State the exact domain of the function $f(x) = \ln\left(\ln\left(\ln(x-1)\right)\right)$ in interval notation.

(A) $(1,\infty)$ (B) (e^2,∞) (C) (e,∞) (D) $(e+1,\infty)$ (E) $(-\infty, \infty)$

19. A tricimal is like a decimal, except the digits represent fractions with powers of 3 instead of 10. For example, 16/27 = 1/3 + 2/9 + 1/27 = 0.121 as a tricimal. How is 77/81 expressed as a tricimal?

(A) 0.950617 (B) 0.2012 (C) 0.1211 (D) 0.1111 (E) 0.2212

20. Simplify $(\log_a b)(\log_b a)$.

(A) ab (B) $\frac{a}{b}$ (C) $\frac{b}{a}$ (D) 1 (E) a^2b^2

21. The function $f(x) = \frac{1}{4-x^2}$ can be written as $g(h(x))$. One possibility for $g(x)$ and $h(x)$ is:

(A) $g(x) = \frac{1}{4x}; h(x) = -x^2$ (B) $g(x) = \frac{1}{4} - \frac{1}{x}; h(x) = x^2$

(C) $g(x) = \frac{1}{4-x}; h(x) = x^2$ (D) $g(x) = \frac{1}{x^2}; h(x) = 4-x$

(E) $g(x) = x^2; h(x) = \frac{1}{4-x}$

22. Evaluate ${}_mP_3$.

(A) 1 (B) m (C) $m^2 - m$ (D) $m^3 - 3m^2 + 2m$ (E) $(m-3)!$

23. A bag holds 5 cards identical except for color. Two are red on both sides, two are black on both sides, and one is red on one side and black on the other. If you pick a card at random and see that the only side you can see is red, what is the probability that the other side is also red?

(A) 1/2 (B) 2/3 (C) 3/4 (D) 4/5 (E) 5/6

24. Find the domain of $f(x) = \log(\ln(\sin x))$ for $0 \le x \le 6\pi$

(A) No real numbers

(B) $\left\{\frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}\right\}$

(C) $\left\{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi, \frac{5\pi}{2}, 3\pi, \frac{7\pi}{2}, 4\pi, \frac{9\pi}{2}, 5\pi, \frac{11\pi}{2}, 6\pi\right\}$

(D) $[0,\pi] \cup [2\pi, 3\pi] \cup [4\pi, 5\pi]$

(E) $[0, 6\pi]$

25. Find the 200^{th} term of the sequence -1, 2, 5, 8, 11, …

(A) 593 (B) 596 (C) 599 (D) 602 (E) 605

26. The spread of a strep throat virus can be modeled by

$$y = \frac{1000}{1 + 990e^{-0.7t}}$$

where y is the total number of people infected after t days. In how many days will 800 people be infected?

(A) 7 days (B) 9 days (C) 10 days (D) 12 days (E) 13 days

27. Enrique walks along a level road and then up a hill. At the top he immediately turns and walks back to his starting point. He walks 4 mph on level ground, 3 mph uphill, and 6 mph downhill. If the entire walk takes 6 hours, how far does he walk?

(A) 16 mi (B) 20 mi (C) 24 mi (D) 28 mi (E) 32 mi

28. Evaluate $(\log_5 6)\cdot(\log_6 7)\cdot(\log_7 8)\cdot(\log_8 9) \ldots (\log_{78123} 78124)\cdot(\log_{78124} 78125)$

(A) – 6.5 (B) 7 (C) 8.25 (D) 9 (E) 1

29. A sphere contains points A (8, -2, 3) and B (4, 0, 7) as endpoints of the diameter. What is the equation of the sphere?

(A) $(x-6)^2+(y+1)^2+(z-5)^2=9$

(B) $(x-2)^2+(y-1)^2+(z+2)^2=86$

(C) $(x-8)^2+(y+2)^2+(z-3)^2=6$

(D) $(x-4)^2+(y+2)^2+(z+4)^2=6$

(E) $(x+6)^2+(y-1)^2+(z+5)^2=9$

30. Find the inverse of the function $f(t)=a+be^{-kt}$ where a, b and k are constants.

(A) $f^{-1}(t)=\frac{-1}{k}\ln(\frac{t-a}{b})$ (B) $f^{-1}(t)=\ln(\frac{t-a}{kb})$ (C) $f^{-1}(t)=\frac{-1}{k}e^{\frac{t-a}{b}}$

(D) $f^{-1}(t)=\frac{e^{-kt}-ab}{b}$ (E) $f^{-1}(t)=\frac{1}{be^{kt}-a}$

31. Find the sum of the geometric series: $1+\frac{1}{2}+\frac{1}{4}+...$

(A) -2 (B) 2 (C) -1 (D) 1 (E) 0

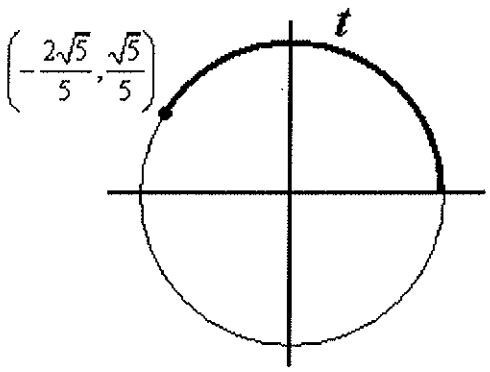
32. Find the distance between the centers of the two circles with equations $x^2+y^2+4x-6y=17$ and $3x^2+3y^2-x-10y-40=0$.

(A) $\frac{\sqrt{23}}{10}$ (B) $\sqrt{58}$ (C) $\frac{\sqrt{905}}{3}$ (D) $\frac{\sqrt{233}}{6}$ (E) $\frac{5\sqrt{6}}{6}$

33. Find the fifth term of the expansion of $(x+2)^9$.

(A) $2016x^5$ (B) $4032\ x^4$ (C) $5376x^3$ (D) $126x^5$ (E) $1008x^5$

34. Given the value of the arc (or angle) t corresponds to the point $\left(-\frac{2\sqrt{5}}{5}, \frac{\sqrt{5}}{5}\right)$ on the unit circle… what would be the x-coordinate at the point associated at the arc $t - \frac{\pi}{4}$?



(A) $x = \frac{\sqrt{2} - 2\sqrt{5}}{10}$

(B) $x = -\frac{\sqrt{5}}{5}$

(C) $x = -\frac{\sqrt{10}}{10}$

(D) $x = \frac{\sqrt{5} - \sqrt{2}}{5}$

(E) $x = \frac{\sqrt{10} - \sqrt{5}}{5}$

35. Find the equation of the ellipse having foci (-1,0) and (7,0) if 10 is the sum of the focal radii.

(A) $\frac{(x-4)^2}{25} + \frac{y^2}{9} = 1$ (B) $\frac{(x-3)^2}{25} + \frac{y^2}{16} = 1$

(C) $\frac{(x-4)^2}{64} + \frac{y^2}{16} = 1$ (D) $\frac{(x-4)^2}{64} + \frac{y^2}{9} = 1$

(E) $\frac{(x-3)^2}{25} + \frac{y^2}{9} = 1$

36. If one is a zero of $f(x) = 8x^4 + 20x^3 - 78x^2 - 125x + 175$, then factor $f(x)$.

(A) $(x+1)(x-5)(2x+5)(4x-7)$

(B) $(x+1)(x-5)(x+7)(2x-5)$

(C) $(x-1)(2x+5)(2x+7)(2x-5)$

(D) $(x-1)(x+1)(2x-25)(4x+7)$

(E) $(x-1)(2x+5)(2x+5)(2x-7)$

37. For $f(x)=\dfrac{\tan(2x)}{(x^2-8x+7)(x^2-5x+6)}$, find the number of values of x on the interval $[0,2\pi)$ for which $f(x)=0$ or $f(x)$ is undefined.

(A) 7 (B) 8 (C) 11 (D) 12 (E) 13

38. The second term of an arithmetic sequence is 12 and the seventh term is 67. Find the nth term of the sequence.

(A) $3+11(n-1)$

(B) $1+11(n-1)$

(C) $1+7(n-1)$

(D) $2+4(n-1)$

(E) $3+9(n-1)$

39. Given the system $\begin{array}{rcrcrcrcl} 8A & + & 10B & - & C & + & 12D & = & 52 \\ A & - & B & + & C & + & 2D & = & -7 \\ 10A & - & 3B & - & 5C & + & 8D & = & 29 \\ -3A & - & 2B & + & C & + & 18D & = & 32 \end{array}$, $(A,B,C,D)=$

(A) $(0,-10,12,1)$ (B) $(8,1,6,-3)$

(C) $(-1.5,3,-7,2.25)$ (D) $(1.5,-3,-10,5)$ (E) no solution

40. Find the set of all real values of x for which $f(x)$ is a real number if

$$f(x)=7+\left(x^3-7x^2-4x+28\right)i-\frac{i^3}{(x^2-7x+10)\,i}$$

(A) $\varnothing$ (B) $\{2,5\}$ (C) $\{-2,2,7\}$ (D) $\{-2,7\}$ (E) $\{2,5,7\}$