TENNESSEE MATHEMATICS TEACHER' ASSOCIATION

SIXTY-FOURTH ANNUAL MATHEMATICS CONTEST

2022

Precalculus

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Scoring formula: 4 x (Number Right) – (Number Wrong) + 40

Directions:

Do not open this booklet until you are told to do so

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you; if you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly eighty minutes to work.

2022 TMTA Pre-Calculus

1. An angle in standard position has a measure of 200°. Identify a corresponding coterminal angle.

   a. $\frac{\pi}{9}$

   b. $\frac{10\pi}{9}$

   c. $\frac{-7\pi}{9}$

   d. $\frac{17\pi}{9}$

   e. $\frac{8\pi}{9}$

2. Which of the following is a solution to the equation $x^4 - 27x^2 - 14x + 120 = 0$?

   a. 1
   b. -2
   c. 3
   d. -4
   e. -5

3. Which of the following is <u>not</u> a solution to $t^4 + 6t^2 - 27 = 0$?

   a. $t = 3$
   b. $t = \sqrt{3}$
   c. $t = -\sqrt{3}$
   d. $t = 3i$
   e. $t = -3i$

4. Find all solutions to the equation $x^4 + 11x^2 + 30 = 0$.

   a. $x = \pm 5, x = \pm 6$
   b. $x = \pm 5i, x = \pm 6i$
   c. $x = \pm i\sqrt{5}, x = \pm i\sqrt{6}$
   d. $x = \pm \sqrt[4]{5}, x = \pm \sqrt[4]{6}$
   e. $x = \pm i\sqrt[4]{5}, x = \pm i\sqrt[4]{6}$

5. Identify all real solutions to $\sqrt[3]{2x+7} = x - 7$.

   a. $x = -14$
   b. $x = \frac{50}{7}$
   c. $x = 10$
   d. $x \approx 7.212$
   e. $x \approx 6.929$

6. Which of the following is not a solution to the equation $x^5 - 10x^4 + 5x^3 + 160x^2 - 156x - 720 = 0$?

   a. $x = -2$
   b. $x = -3$
   c. $x = -4$
   d. $x = 5$
   e. $x = 6$

7. Identify the equation of the horizontal asymptote for the graph of the function $y = -2e^{x+5} + 7$.

   a. $x = -5$
   b. $y = -7$
   c. $x = 5$
   d. $y = 7$
   e. $y = 0$

8. For the function $f(x) = x^3 + 21$, evaluate the expression $\frac{f(x+h)-f(x)}{h}, h \neq 0$.

   a. $3x^2$
   b. h^2
   c. $3x^2 + 3xh + h^2$
   d. $3x^2 + 3xh^2 + h^3$
   e. $\frac{x^4}{4} + 21x$

9. For the function $f(x) = 9x^2 - x$, evaluate the expression $\frac{f(x)-f(a)}{x-a}, x \neq a.$

a. $9x - 1$
b. $9x + 9a - 1$
c. $9x - 9a - 1$
d. $9x + 9a$
e. $9x + 9a + 1$

10. Which of the following is a possible solution to the equation
$x^7 - 35x^6 + 71x^4 - 200x^2 + 97x - 21879 = 0$?

a. 31
b. 32
c. 33
d. 34
e. 35

11. Which of the following best approximates a solution to $x^8 - 9x^4 + 20 = 0$?

a. 1.414
b. 1.516
c. 1.682
d. 2
e. 2.236

12. Which of the following is <u>not</u> a factor of $t^6 - 729$?

a. $t - 3$
b. $t + 3$
c. $t^2 + 3t + 9$
d. $t^2 - 3t - 9$
e. $t^2 - 3t + 9$

13. Determine the value(s) of x that makes the equation true $\begin{vmatrix} x-5 & 5 \\ 17 & x+7 \end{vmatrix} = 0.$

a. x = 12 or x= -10
b. x = -12 or x= 10
c. x = 5 or x= -7
d. x = -5 or x = 7
e. x= -17 or x = -5

14. What conic section is represented by the equation $x^2 - 10x - y^2 + 8y = 400$?

a. Ellipse with center at (5, -4)
b. Ellipse with center at (-5, 4)
c. Hyperbola with center at (-5, -4)
d. Hyperbola with center at (5, 4)
e. Circle with center (5, -4) and radius 21

15. Which of the following does not represent a parabola with a vertex at (-4, 2)?

a. $y = 2x^2 + 16x + 34$
b. $x = 2y^2 - 8y + 4$
c. $y = 3x^2 + 24x + 50$
d. $x = 3y^2 + 12y + 8$
e. $x = 5y^2 - 20y + 16$

16. Determine the coordinates of the focus and equation of the directrix for the parabola given by the equation $x = 4y^2 + 8y + 10$.

a. $\left(5\frac{3}{4}, -1\right)\ y = -1\frac{1}{4}$
b. $\left(6\frac{1}{4}, -1\right)\ y = -1\frac{1}{4}$
c. $\left(6, -1\frac{1}{4}\right)\ y = -\frac{3}{4}$
d. $\left(6, -\frac{3}{4}\right)\ y = -1\frac{1}{4}$
e. $\left(5\frac{3}{4}, -1\right)\ y = -\frac{3}{4}$

17. Identify an equation of an asymptote for the hyperbola given by the equation

$$\frac{(y-4)^2}{9} - \frac{(x+7)^2}{25} = 1$$

a. $3x + 5y = 1$
b. $3x + 5y = -1$
c. $3x + 5y = 71$
d. $3x + 5y = 29$
e. $3x + 5y = -23$

18. For which equation does the corresponding ellipse have a major axis length of 10?

a. $\frac{x^2}{20}+\frac{y^2}{4}=1$

b. $\frac{x^2}{9}+\frac{y^2}{10}=1$

c. $\frac{x^2}{64}+\frac{y^2}{100}=1$

d. $\frac{x^2}{25}+\frac{y^2}{16}=1$

e. $\frac{x^2}{400}+\frac{y^2}{169}=1$

19. Identify the real solutions to the equation $\frac{1}{t^4}-\frac{25}{t^2}+114=0$.

a. $t=\pm\frac{\sqrt[4]{19}}{6589},\ t=\pm\frac{\sqrt[4]{6}}{216}$

b. $t=19,\ t=6$

c. $t=\pm\sqrt{19},\ t=\pm\sqrt{6}$

d. $t=\pm\frac{1}{\sqrt[4]{19}},\ t=\pm\frac{1}{\sqrt[4]{6}}$

e. $t=\pm\frac{1}{\sqrt{19}},\ t=\pm\frac{1}{\sqrt{6}}$

20. Solve the equation $4^x-2^{x+3}+15=0$.

a. $x=5, x=3$

b. $x=\ln 5, x=\ln 3$

c. $x=\log 5, x=\log 3$

d. $x=\log_2 5, x=\log_2 3$

e. no solution

21. What is the coefficient of $x^3y^4z^3$ in the expansion of $(x-y+2z)^{10}$?

a. -120

b. 120

c. 960

d. 33600

e. -33600

22. If $\log_x 10 = 0.362$ and $\log_x 27 = 0.518$, evaluate $\log_x 300$.

a. 2.278
b. 0.354
c. 0.321
d. 0.897
e. 0.934

23. Determine the inverse function, $f^{-1}(x)$, for $f(x) = 2e^{x-3} + 1$.

a. $f^{-1}(x) = 3 + \ln\left(\frac{1}{2}(x-1)\right)$
b. $f^{-1}(x) = \frac{1}{2e^{x-3}+1}$
c. $f^{-1}(x) = \ln 3\sqrt{x-1}$
d. $f^{-1}(x) = \ln\big(2(x-1)\big) - 3$
e. $f^{-1}(x) = \frac{1}{2}e^{x+3} - 1$

24. Identify the polynomial, with integer coefficients, of lowest degree that contains $2 + \sqrt{3}$ and $5i$ as some of the roots of the function.

a. $x^2 - (2 + \sqrt{3} + 5i)x + (10i + 5i\sqrt{3})$
b. $x^3 - (4 + 5i)x^2 + x - 5i$
c. $x^4 - 4x^3 + 26x^2 - 100x + 25$
d. $x^4 - 4x^3 - 26x^2 + 100x + 25$
e. $x^4 - 4x^3 + 24x^2 - 100x - 25$

25. What represents the solution to the system of equations $\begin{cases} 15x - y + 2z = 96 \\ 8x - 2y + z = 53 \\ 7x + 2y + z = 41 \end{cases}$.

a. $\emptyset$
b. A point
c. A line
d. A plane
e. A cube

26. Which statement best describes the end behavior of the graph of the function $f(x) = -4x^8 + 6x^3 - 9$?

a. As $x \to -\infty, f(x) \to \infty$; as $x \to \infty, f(x) \to \infty$
b. As $x \to -\infty, f(x) \to \infty$; as $x \to \infty, f(x) \to -\infty$
c. As $x \to -\infty, f(x) \to -\infty$; as $x \to \infty, f(x) \to \infty$
d. As $x \to -\infty, f(x) \to -\infty$; as $x \to \infty, f(x) \to -\infty$
e. As $x \to -\infty, f(x) \to \infty$; as $x \to \infty, f(x) \to 0$

27. Determine the domain of the function $f(x) = \frac{\ln(2x^2-21x+40)}{\sqrt[4]{3x^2-13x+10}}$.

a. $(-\infty, 1) \cup (8, \infty)$
b. $(-\infty, 1] \cup (8, \infty)$
c. $(-\infty, 1) \cup \left(\frac{10}{3}, \infty\right)$
d. $(-\infty, 1) \cup \left[\frac{10}{3}, \infty\right)$
e. $\left(-\infty, \frac{5}{2}\right) \cup (8, \infty)$

28. The graph of the rational function $f(x) = \frac{x^4+14x^3+59x^2+82x+43}{x^2+9x+11}$ has an asymptote along which of the following functions?

a. $y = x^2 + x + 5$
b. $y = x^2 + 14x + 59$
c. $y = x^2 + 5x + 14$
d. $y = x^2 + 5x + 3$
e. $y = x^2 + 23x + 266$

29. Which statements are true for the graph of the rational function $f(x) = \frac{x^2-x-30}{x^3+x^2-42x}$?

i. The graph does not have a vertical asymptote.
ii. The graph has a vertical asymptote at $x = -7$.
iii. The graph has a hole at $\left(6, \frac{11}{78}\right)$.

a. II only
b. III only
c. I & II only
d. II & III only
e. I, II, & III

30. A triangle has two sides of lengths 5 and 7 with an included angle measurement of 50°. Which of the following is a possible angle measurement for the other angles in the triangle?

a. 5.4°
b. 34.7°
c. 84.7°
d. 95.3°
e. 124.6°

31. For an angle θ in the first quadrant, $\cos\theta = \frac{8}{17}$, determine the value of $\sin(2\theta)$.

a. $\frac{30}{17}$
b. $\frac{120}{289}$
c. $-\frac{120}{189}$
d. $\frac{240}{289}$
e. $-\frac{240}{289}$

32. Which of these expressions simplifies $sin^4\theta - cos^4\theta$?

a. 1
b. $1 + 2cos^4\theta$
c. $1 - 2sin^4\theta$
d. $tan^4\theta$
e. $-\cos(2\theta)$

33. Identify all the angles in the interval $[0,2\pi]$ that make the equation true:

$$3tan^2\theta - 1 = 0.$$

a. $\frac{\pi}{6}, \frac{7\pi}{6}$
b. $\frac{\pi}{6}, \frac{\pi}{3}$
c. $\frac{\pi}{3}, \frac{4\pi}{3}$
d. $\frac{\pi}{6}, \frac{\pi}{3}, \frac{7\pi}{6}, \frac{4\pi}{3}$
e. None of the above

34. Identify the period of the graph of the function $y = 4\csc\left(\frac{1}{3}x\right)$.

a. 3π

b. $\frac{\pi}{3}$

c. $\frac{2\pi}{3}$

d. 6π

e. 4π

35. Describe the graph of the parametric equations $\begin{cases} x = -2t \\ y = \sin(3\pi t) \end{cases}$, on the interval [0,10].

a. Sine curve with period $\frac{4\pi}{3}$, oriented from left to right as t values increase

b. Sine curve with period $\frac{4\pi}{3}$, oriented from right to left as t values increase

c. Sine curve with period $\frac{4}{3}$, oriented from left to right as t values increase

d. Sine curve with period $\frac{4}{3}$, oriented from right to left as t values increase

e. Sine curve with period $\frac{2\pi}{3}$, oriented from left to right as t values increase

36. What graph best represents the polar equation $r = 3\sin(4\theta)$?

a. a rose curve with 4 petals
b. a rose curve with 8 petals
c. a rose curve with 2 petals
d. a rose curve with 3 petals
e. a rose curve with 6 petals

37. Determine the value of x to make the following equation true:

$$\begin{vmatrix} x & 1 & 2 \\ 3 & 4 & 5 \\ 6 & 7 & 8 \end{vmatrix} = \begin{vmatrix} x & 9 \\ 10 & 11 \end{vmatrix}$$

a. $x = \frac{102}{14}$

b. $x = 8$

c. $x = \frac{90}{14}$

d. $x = \frac{51}{4}$

e. $x = \frac{45}{4}$

38. Identify the equation corresponding to the polar function $r = 2\cos\theta$.

a. $(x-1)^2 + y^2 = 1$
b. $(x+1)^2 + y^2 = 1$
c. $x^2 + (y-1)^2 = 1$
d. $x^2 + (y+1)^2 = 1$
e. $x^2 + y^2 = 4$

39. Determine the partial fraction decomposition of $\frac{11x^3-21x^2-20x-150}{x^4-3x^3-5x^2-25x}$.

a. $\frac{6}{x} + \frac{3}{x-5} + \frac{2x-1}{x^2+2x+5}$

b. $\frac{3}{x} + \frac{6}{x-5} + \frac{2x-1}{x^2+2x+5}$

c. $\frac{6}{x} - \frac{3}{x-5} - \frac{2x+1}{x^2+2x+5}$

d. $\frac{6}{x} + \frac{3}{x-5} + \frac{2x+1}{x^2+2x+5}$

e. $\frac{3}{x} + \frac{6}{x-5} + \frac{2x+1}{x^2+2x+5}$

40. Which of the following vectors is orthogonal (perpendicular) to **both** vectors?

$$2\vec{\imath} + 3\vec{\jmath} - 7\vec{k} \text{ and } 5\vec{\imath} - 9\vec{\jmath} + 10\vec{k}$$

a. $-\frac{1}{10}\vec{\imath} + \frac{1}{27}\vec{\jmath} - \frac{1}{70}\vec{k}$
b. $93\vec{\imath} + 15\vec{\jmath} - 3\vec{k}$
c. $9\vec{\imath} + 8\vec{\jmath} + 6\vec{k}$
d. $\vec{\imath} + 5\vec{\jmath} + 4\vec{k}$
e. $-33\vec{\imath} - 55\vec{\jmath} - 33\vec{k}$