Key

TWENTIETH ANNUAL MATHEMATICS CONTEST

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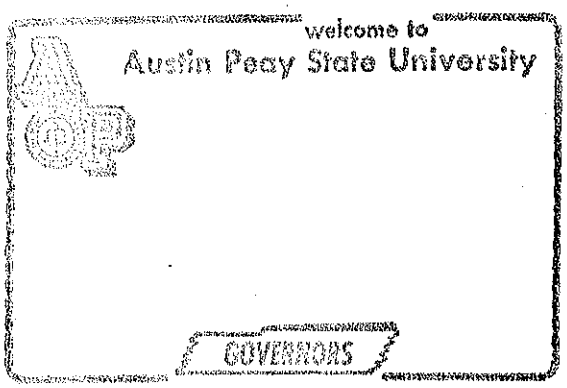


ALGEBRA II TEST

1976

Scoring Formula: 4R - W

This test was prepared from a list of Algebra II questions submitted by Southwestern at Memphis.

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school algebra. For each problem there are listed 5 possible answers; one and only one is correct. You are to work each problem, determine the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. You must use a pencil with soft lead (No. 2 lead or softer). A sample problem follows:

1. If $2x = 3$, then x equals

(a). 2/3. (b). 3. (c). 6.

(d). 3/2 (e). none of these

	A	B	C	D	E
1.	☐	☐	☐	■	☐

The correct answer for the sample problem is 3/2, which is answer (d); so you would answer this problem by making a heavy black mark under space D as indicated above.

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers so it is advisable not to do much wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet.

The answer sheets will be used for a statistical compilation and will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet to page 1 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

1. If $i^2 = -1$, then $\frac{1}{i}$ is:

(a) i

(b) $-i$

(c) $\sqrt{i}$

(d) $i + 1$

(e) $i - 1$

2. Which of these proves that the subset $\{0, -1\}$ of the integers is <u>not</u> closed under multiplication?

(a) $0 \times 0 = 0$

(b) $0 \times (-1) = 0$

(c) $0 \times (-1) = (-1) \times 0$

(d) $(-1) \times (-1) = 1$

(e) none of these

3. What is the distance between the points $(2, 3)$ and $(-5, -2)$?

(a) $\sqrt{74}$

(b) $2\sqrt{6}$

(c) $\sqrt{10}$

(d) 12

(e) $\sqrt{7} + \sqrt{5}$

4. Which of these is a polynomial?

(a) $\frac{2}{3-x} + 2x + x^2$

(b) $5x + 2\sqrt{x} - 4$

(c) $\frac{2}{3} x^3 + 5x^2 - \sqrt{2}\ x$

(d) $\frac{2}{3} x^{-1} + x + 7x^2$

(e) $5x^{1/3} + 2x + 5$

5. What is the value of $\sum_{k=1}^{3} (2k + 1)$?

(a) 3

(b) 7

(c) 15

(d) 21

(e) none of these

$= 2(1)+1 + 2(2)+1 + 2(3)+1$

$= 3 + 5 + 7$

$= 15$

6. Which of these is equal to $\dfrac{a^3 - b^3}{a^2 - b^2} \cdot \dfrac{a^2 + 2ab + b^2}{a^2 + ab + b^2}$ for all real numbers a and b, with $a \neq b$, $a \neq -b$?

(a) $a + b$

(b) $a - b$

(c) $\dfrac{a^2 + ab + b^2}{(a - b)(a + b)}$

(d) $\dfrac{(a - b)(a + b)}{a^2 + ab + b^2}$

(e) $\dfrac{a + b}{a - b}$

$\dfrac{(a-b)(a^2+ab+b^2)}{(a+b)(a-b)} \cdot \dfrac{(a+b)^2}{a^2+ab+b^2}$ $a \neq b$, $a \neq -b$

7. Which of these is a root of $3x^2 - 2x - 2 = 0$?

(a) $1/3 + 2i$

(b) 5

(c) $\dfrac{1 - \sqrt{7}}{3}$

(d) $\dfrac{3 + \sqrt{28}}{6}$

(e) none of these

$x = \dfrac{+2 \pm \sqrt{4 - 4(3)(-2)}}{2(3)}$

$= \dfrac{2 \pm \sqrt{28}}{6}$

$= \dfrac{1 \pm \sqrt{7}}{3}$

8. What is the value of $\sqrt{9^{16}}$? $= 9^8$

(a) 9^4

(b) 9^8

(c) 3^8

(d) 3^4

(e) none of these

9. What is the largest possible subset of real numbers which can be the domain of a function defined by

$f(x) = \sqrt{12 - x}$? $\quad 12 \geq x$

(a) The set of non-negative real numbers

(b) The set of positive real numbers

(c) The set of real numbers greater than or equal to 12

(d) The set of real numbers less than or equal to 12

(e) none of these

10. What is the value of x if $\log_3 x = -2$?

$3^{-2} = x$

$\frac{1}{9} = x$

(a) -6

(b) -1/6

(c) 1/9

(d) -1/9

(e) none of these

11. Suppose E and F are events in a probability space, $P(E) = 0.4$, $P(F) = 0.5$, and $P(E \cap F) = 0.2$. What is $P(E \cup F)$?

$P(E \cup F) = P(E) + P(F) - P(E \cap F)$
$= .4 + .5 - .2$
$= .7$

(a) 0.9

(b) 0.15

(c) 0.7

(d) 0

(e) none of these

12. What is the term involving b^{12} in the binomial expansion of $(2a + b^2)^{20}$?

(a) $\frac{20!}{6!\ 14!}\ 2^{14}\ a^{14}\ b^{12}$

(b) $\frac{20!}{6!}\ 2^{14}\ a^{14}\ b^{12}$

(c) $\frac{20!}{8!\ 12!}\ 2^{8}\ a^{8}\ b^{12}$

(d) $\frac{20!}{12!}\ 2^{8}\ a^{8}\ b^{12}$

(e) none of these

13. If $\frac{1}{x^2 - 1} = \frac{a}{x + 1} - \frac{a}{x - 1}$ for all $x (x \neq 1, x \neq -1)$, then a is:

$x+1$ $\quad x-1$

$1 = a(x-1) - a(x+1)$
$1 = ax - a - ax - a$
$1 = -2a$
$-\frac{1}{2} = a$

(a) 1/2

(b) -1/2

(c) $\sqrt{2}$

(d) $-\sqrt{2}$

(e) none of these

14. The solution set for $\frac{2}{x-1} > -1$ is:

(a) $\{x \mid x > 1 \text{ and } x < -1\}$

(b) $\{x \mid x < 1 \text{ and } x > -1\}$

(c) $\{x \mid x < -1\}$

(d) $\{x \mid x > 1 \text{ or } x < -1\}$

(e) none of these

15. A function f is defined on the set of real numbers by $f(x) = \frac{x-1}{x+1}$. The value of $f\left(\frac{a-1}{a+1}\right)$ (for $a \neq -1$, $a \neq 0$) is:

(a) $\frac{a-1}{a+1}$

(b) a

(c) 1/a

(d) -1/a

(e) none of these

16. What is the solution set of $\frac{1}{x+1} = \frac{x-1}{2x^2+2x}$

(a) {0,1}

(b) {-1}

(c) the empty set

(d) {-1,1}

(e) none of these

17. For real numbers x and y, $2xy - x^2 - y^2$ is:

(a) never negative

(b) never positive

(c) always irrational

(d) always rational

(e) sometimes positive and sometimes negative

18. The solution set of $\begin{vmatrix} x & 0 & 0 \\ 1 & x & 0 \\ y & 1 & x \end{vmatrix} = x$ is:

(a) {0}

(b) {0,1}

(c) {0,1,-1}

(d) {2,-2}

(e) {y}

$x \begin{vmatrix} x & 0 \\ 1 & x \end{vmatrix} = x$

$x[x^2] = x$

$x^3 - x = 0$

$x(x+1)(x-1) = 0$

$x = 0, -1, 1$

19. A committee contains 10 men and 12 women. A subcommittee of 2 men and 3 women is to be selected from the committee. In how many different ways can this be done?

(a) 1100

(b) 1410

(c) 9900

(d) 26,334

(e) none of these

$\binom{10}{2} \cdot \binom{12}{3}$

$10 \quad 2$
$12 \quad 3$

$= \frac{10!}{2!\,8!} \cdot \frac{12!}{3!\,9!}$

$= \frac{10 \cdot 9}{1 \cdot 2} \cdot \frac{12 \cdot 11 \cdot 10}{1 \cdot 2 \cdot 3} = 99 \cdot 100 = 9900$

20. Which of these functions (defined for positive real numbers) has the property $f(x + y) = f(x) + f(y)$?

(a) $f(x) = 2x + 3$ $\quad f(x+y) = 2(x+y)+3 = 2x+2y+3$; $f(x)+f(y) = (2x+3)+(2y+3)$

(b) $f(x) = \frac{x}{x+1}$ $\quad f(x+y) = \frac{x+y}{x+y+1}$; $f(x)+f(y) = \frac{x}{x+1} + \frac{y}{y+1}$

(c) $f(x) = \sin x$

(d) $f(x) = \log x$

(e) none of these ✓

21. Sin $(\tan^{-1} \frac{\sqrt{2}}{4})$ is:

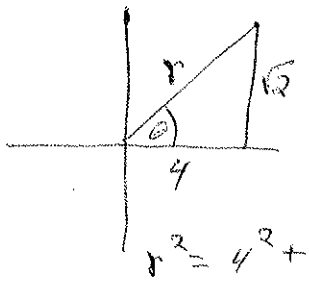
(a) 1/2

(b) 1/3

(c) 1/4

(d) $\sqrt{2}/2$

(e) $\sqrt{3}/3$



$r^2 = 4^2 + (\sqrt{2})^2$

$r^2 = 16 + 2$

$r^2 = 18$

$r = \sqrt{18} = 3\sqrt{2}$

$\sin\theta = \frac{\sqrt{2}}{3\sqrt{2}} = \frac{1}{3}$

22. If an eight-inch (diameter) pizza serves two, how many should two twelve-inch pizzas serve?

(a) 5

(b) 6

(c) 8

(d) 9

(e) 10

23. What is the sum of the x and y coordinates of the simultaneous solution of

$x^2 + y^2 = 25$

$4x + 3y = 25$?

(a) 7

(b) $7\frac{1}{2}$

(c) 8

(d) $8\frac{1}{2}$

(e) none of these

24. For real x, what is the smallest value of $x^2 + 6x + 11$?

(a) -3

(b) 0

(c) 2

(d) 11

(e) there is no smallest value

25. If $a > 0$, what is the y-intercept of the perpendicular bisector of the segment from the origin to the point (a, a^2) ?

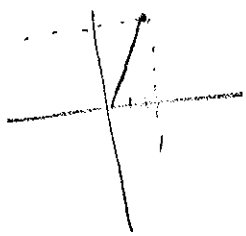
(a) $a^2 + 1$

(b) $a^2 + \frac{1}{2}$

(c) $\frac{a^3 + a}{2}$

(d) $\frac{a^2 + 1}{2}$

(e) $\frac{a^2 - 1}{2}$



26. If the equation $x^2 + bx + c = 0$ has exactly one real solution r, then what is b? (Assume b and c are real.)

(a) $b = r/2$

(b) $b = -r/2$

(c) $b = 2r$

(d) $b = -2r$

(e) $b = r^2$

27. How many points do the graphs of $16x^2 - 96x + 9y^2 = 0$ and $x^2 - 2x - y^2 = 0$ have in common?

(a) 0

(b) 1

(c) 2

(d) 3

(e) 4

28. For a certain year a man receives \$265 in interest from \$3000 he has invested, part at 8% interest and part at 9% interest. How much did he invest at 8%?

(a) \$500

(b) \$750

(c) \$1000

(d) \$2500

(e) none of these

29. The solution set of $10^{\log_{10} x^2} = x$ is:

(a) {0}

(b) {1}

(c) {0,1}

(d) {10}

(e) the empty set

30. What is the solution set for $|x| - 1 \leq 2x$?

(a) $\{x| -1 \leq x\}$

(b) $\{x| -1/3 \leq x < 0\}$

(c) $\{x| -1 \leq x < 0\}$

(d) $\{x| 0 \leq x\}$

(e) $\{x| -1/3 \leq x\}$

31. In the system of common logarithms, log 2 = .3010. What is the value of x if log x = -1.6990 ?

(a) 1/4

(b) 1/2

(c) -20

(d) -200

(e) none of these

32. The set of all x such that $\left|\frac{3x + 1}{x - 6}\right| < 2$ is:

(a) $\{x| x > 11/5\}$

(b) $\{x| x < -13\}$

(c) $\{x| x > 0\}$

(d) $\{x| -13 < x < 11/5\}$

(e) $\{x| x > 11/5 \text{ or } x < -13\}$

33. A teacher uses a linear function to rescale the scores on a set of test papers so that the maximum possible score is still 100, and so that a grade of 56 becomes an 80 on the new scale. Using the teacher's formula, what would be the new score corresponding to an old score of 67?

(a) 67

(b) 85

(c) 91

(d) 94

(e) none of these

34. A musician wishes to write a musical composition that contains a theme which occurs 10 different times, with other themes interposed. The first time the theme appears, it is to last for 20 seconds. On each succeeding occurrence, the theme is to last some fixed length of time longer than in its previous occurrence. If the total amount of time devoted to this theme in the composition is to be 400 seconds, then how long is the second occurrence of the theme?

(a) $21\frac{2}{3}$ seconds

(b) $22\frac{5}{8}$ seconds

(c) $23\frac{3}{7}$ seconds

(d) $24\frac{4}{9}$ seconds

(e) none of these

35. What is the number of distinct sets which can be obtained from the sets $\{1,2\}$, $\{2,3,4\}$, and $\{3,4,5\}$ by repeated applications of the operations $\cup$ and $\cap$?

(a) 8

(b) 9

(c) 11

(d) 16

(e) unlimited

36. What is the remainder when 2^{360} is divided by 5 ?

(a) 0

(b) 1

(c) 2

(d) 3

(e) 4

37. If $f(x) = |x - 1|$ and $g(x) = x^2$, each with domain the set of all real numbers, what is the solution set for the equation $f(g(x)) = g(f(x))$?

(a) $\{0\}$

(b) $\{1\}$

(c) $\{0,1\}$

(d) $\{x|x \leq 0\} \cup \{x|x \geq 1\}$

(e) the set of all real numbers

38. In Δ ABC, angle BAC is a right angle, $AD \perp BC$, $\overline{BD} = 10$, $\overline{DC} = 5$. Then, to the nearest tenth, what is $\overline{AD}$?

(a) 6.8

(b) 6.9

(c) 7.0

(d) 7.1

(e) 7.5

39. The operation * is defined for all real numbers x and y by $x * y = x+y - xy$. This operation is:

(a) commutative but not associative

(b) associative but not commutative

(c) associative and commutative

(d) neither associative nor commutative

(e) not well-defined

40. Two vertical poles, one twelve feet tall and one eight feet tall, are fifteen feet apart. A wire is to run from the top of one pole to a stake on the ground between the poles and from the stake to the top of the second pole. Where should the stake be placed so that the total length of wire is as small as possible?

(a) 6 ft. from the 12 ft. pole

(b) 7 ½ ft. from the 12 ft. pole

(c) 8 ft. from the 12 ft. pole

(d) 9 ft. from the 12 ft. pole

(e) none of these