EIGHTEENTH ANNUAL MATHEMATICS CONTEST
Sponsored by
THE TENNESSEE MATHEMATICS TEACHER'S ASSOCIATION

ALGEBRA II TEST

1974

Scoring Formula: 4R - W

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This test was prepared from a list of Algebra II questions submitted by University of Tennessee at Knoxville.

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school algebra. For each problem there are listed 5 possible answers; one and only one is correct. You are to work each problem, determine the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. You must use a pencil with soft lead (No. 2 lead or softer). A sample problem follows:

1. If $2x = 3$, then x equals

    (a). 2/3.   (b). 3.   (c). 6.

    (d). 3/2.   (e). none of these.

The correct answer for the sample problem is 3/2, which is answer (D); so you would answer this problem by making a heavy black mark under space D as indicated above.

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers so it is advisable not to do much wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet.

The answer sheets will be used for a statewide statistical compilation and will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

1. If $\dfrac{5}{n} - \dfrac{3}{n} = \dfrac{1}{4}$, then n is equal to

    (a). 1/8.
    (b). 1/2.
    (c). 2.
    (d). 4.
    (e). 8.

2. Which of the following, when added to $m + n$, gives $n - m$?

    (a). 2m
    (b). -2m
    (c). 2m - 2n
    (d). m - n
    (e). n - m

3. Phil has 15 coins; some are quarters and the rest are nickels. If he has $2.95 in coins, how many nickels does he have?

    (a). 4
    (b). 5
    (c). 6
    (d). 9
    (e). 11

4. If $x = 3/4$, then x^{-2} is equal to

    (a). -16/9.
    (b). -6/8.
    (c). -9/16.
    (d). 8/6.
    (e). 16/9.

5. If $\log_{10} x = 2$, then x is equal to

    (a).   2.
    (b).   4.
    (c). 12.
    (d). 20.
    (e). 100.

6. If $x^2 - y^2 = 5$ and $x^2 + y^2 = 13$, then $x^4 - y^4$ is equal to
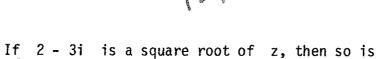
    (a).   1.
    (b). 18.
    (c). 25.
    (d). 64.
    (e). 65.

7. $(-2i)(-2i)$ is equal to

    (a). -4i.
    (b).   4i.
    (c). -4.
    (d).   4.
    (e).   8.

8. If an integer x is divided by another integer y, the quotient is 24. If the sum of the two integers is 75, then x is equal to

    (a).   3.
    (b).   8.
    (c). 25.
    (d). 48.
    (e). 72.

9. For what value of k will the roots of the equation $x^2 + kx + 9 = 0$ be equal?

   (a). 0

   (b). 3

   (c). 6

   (d). 9

   (e). 18

10. The two solutions of the equation $2x^2 - x - 4 = 0$ are

   (a). opposite in sign.

   (b). both positive and equal.

   (c). both negative and equal.

   (d). both positive but unequal.

   (e). both negative but unequal.

11. Evaluate $\sqrt{3^6}$.

   (a). 6

   (b). 9

   (c). 27

   (d). $9\sqrt{3}$

   (e). 1/27

12. If $2 - 3i$ is a square root of z, then so is

   (a). $2 - 3i$.

   (b). $-2 + 3i$.

   (c). $-2 - 3i$.

   (d). $2 + 3i$.

   (e). $\sqrt{13}$.

13. Factor $16x^4 - 1$.

    (a). $(4x^2 + 1)^2$

    (b). $(4x^2 - 1)^2$

    (c). $(4x^2 + 1)(2x + 1)(2x - 1)$

    (d). $(2x - 1)^4$

    (e). $(4x^2 - 1)(2x + 1)(2x + 1)$

14. The expression $\log(a^2 b) + \log\left(\dfrac{b}{a}\right) + \log\left(\dfrac{a}{b}\right) - \log(b)$ reduces to

    (a). $\log(b^2)$.

    (b). $\log(a/b)$.

    (c). $2 \log(a)$.

    (d). $\log(a)$.

    (e). $2 \log(b)$.

15. $[\sqrt{75} - \sqrt{12} + \sqrt{27}]$ simplifies to

    (a). $6\sqrt{3}$.

    (b). $4\sqrt{3}$.

    (c). $\sqrt{5} + \sqrt{3}$.

    (d). $3\sqrt{6}$.

    (e). $\sqrt{90}$.

16. The roots of $3x^2 - 2x - 1 = 0$ are

    (a). $1/3, -1$.

    (b). $-1/3, 1$.

    (c). $-1/3 \pm (4/3)\sqrt{2}\, i$.

    (d). $-1/3, -1$.

    (e). $1/3, 1$.

17. The roots of $x^3 + 6x^2 + 11x + 6 = 0$ are

    (a). 1, 2, -3.
    (b). 1, 2, 3.
    (c). 1, -2, -3.
    (d). -1, -2, -3.
    (e). -1, 2, 3.

18. Suppose 5 different prizes are given randomly to 8 people, so that no one of the 8 people receives more than one prize. In how many ways may the 5 prizes be distributed?

    (a). 5!

    (b). 8!

    (c). $\dfrac{8!}{5!\ 3!}$

    (d). 8!/3!

    (e). 5^8.

19. Let $n(\overline{X})$ denote the number of elements in a set $\overline{X}$. If $n(A) = 17$, $n(B) = 11$ and $n(A \cap B) = 4$, then $n(A \cup B)$ is equal to which one of the following?

    (a). 24
    (b). 28
    (c). 21
    (d). 15
    (e). Not enough information is given to determine $n(A \cup B)$.

20. Find a value for k such that when $x^3 - 7x^2 + 3x + k$ is divided by $(x - 2)$ the remainder is -8.

    (a). 0
    (b). 2
    (c). 14
    (d). 8
    (e). 6

21. If $b = \log_e x$, then e^{-2b} is equal to which one of the following?

    (a).   -2

    (b). $1/x$

    (c). $-x^2$

    (d). $1/x^2$

    (e). x^2

22. Evaluate $\sqrt{2^8 + 2^8}$. The result is equal to which one of the following?

    (a). 2^4

    (b). 2^3

    (c). 4^8

    (d).   256

    (e).   None of these.

23. Solve for x in the following equation: $\dfrac{5}{2x} - \dfrac{7}{3x} = 1 - \dfrac{1}{6x}$.

    (a). $x = 1/2$

    (b). $x = 1/3$

    (c). $x = 3$

    (d). $x = 0$

    (e). $x = 1/5$

24. Which one of the following facts proves that the function f defined by $f(x) = |x|$ is <u>not</u> a one-to-one mapping?
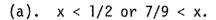
    (a). $f(0) = 0$

    (b). $f(x^2) = x^2$

    (c). $f(1) = 1$

    (d). $f(1) = f(-1)$

    (e). $f(x) \geq 0$

25. $\dfrac{7a - 28b}{a - 4b} = 7$ for all a, b provided that which one of the following holds true?

    (a). $a \neq 0$

    (b). a and b can be any two numbers

    (c). $a \neq -4b$

    (d). $a \neq 0$ and $b \neq 0$

    (e). $a \neq 4b$

26. Which one of the following equations defines a function f having the property that $f(a + b) = f(a) + f(b)$ for every a and b?

    (a). $f(x) = x^2$

    (b). $f(x) = -2x$

    (c). $f(x) = x^3$

    (d). $f(x) = |x|$

    (e). $f(x) = e^x$

27. Solve the system of equations $\begin{cases} 2x + y - z = 2 \\ x - y + z = 7 \\ x + 2y + z = 4. \end{cases}$ The solution is:

    (a). $x = 0, y = 0, z = 0$

    (b). $x = 3, y = -1, z = 3$

    (c). $x = 3, y = 1, z = 3$

    (d). $x = -3, y = 1, z = -3$

    (e). No unique solution exists.

28. A person selects the 13 hearts from a standard deck of playing cards. He then shuffles these 13 hearts and randomly draws 3 cards without replacement. The probability that the ace of hearts is one of the three cards drawn is

    (a). 1/13.

    (b). 1/3.

    (c). 3/13.

    (d). 3/52.

    (e). 1/4.

29. Solve $3 + \sqrt{3x + 1} = x$. The solution set for x is

    (a). $\{1, 8\}$.

    (b). $\{8\}$.

    (c). $\{1\}$.

    (d). $\{-2\}$.

    (e). $\{-1, 8\}$.

30. If $\dfrac{x + 2}{2x - 3} \div \dfrac{x^2 - 4}{2x^2 - 3x}$ exists, then it can be reduced to which one of the following?

    (a). $\dfrac{2x - 3}{x + 2}$

    (b). $\dfrac{x}{x - 2}$

    (c). $\dfrac{x - 2}{x}$

    (d). $\dfrac{x + 2}{x}$

    (e). $\dfrac{(x + 2)^2(x - 2)}{x(2x - 3)^2}$

31. Solve the inequality $|x| > 5$. The solution for x is

    (a). $x > 5$.

    (b). $-5 < x < 5$.

    (c). $x > 5$ and $x < -5$.

    (d). $x > 5$ or $x < -5$.

    (e). $x = \pm 5$.

32. The roots of $x^2 + x + 1 = 0$ are

   (a). ± 1.
   (b). $\pm\sqrt{3}$.
   (c). $-1/2 \pm \sqrt{3}/2$.
   (d). 1, 1.
   (e). none of these.

33. Find all values of λ such that the system $\begin{cases}(-5 - \lambda)x + 2y = 0 \\ 2x - (2 + \lambda)y = 0\end{cases}$ has a solution <u>other</u> than the trivial solution $x = y = 0$. The solution for λ is

   (a). $-1, -6$.
   (b). 0.
   (c). -3.
   (d). -1.
   (e). $0, -3$.

34. Consider the distinguishable permutations of the letters of the word, TENNESSEE. How many of these permutations begin with "N", end with "N", and have T as the middle letter?

   (a). $\dfrac{9!}{2!2!4!}$

   (b). $\dfrac{9!}{2!4!}$

   (c). $6!$

   (d). 15

   (e). 30

35. Solve $x^{\frac{2}{3}} + x^{\frac{1}{3}} - 6 = 0$. The solution is

   (a). $x = 27$ or $x = 8$.
   (b). $x = -3$ or $x = 2$.
   (c). $x = -27$ or $x = 8$.
   (d). $x = 8$.
   (e). $x = \sqrt[3]{-3}$ or $x = \sqrt[3]{2}$.

36. Solve the inequality $\frac{x + 2}{2x - 1} < 5$. The solution for x is

   (a). $x < 1/2$ or $7/9 < x$.

   (b). $x < 2$.

   (c). $1/2 < x < 2$.

   (d). $-1/2 < x < 2$.

   (e). $1/2 < x < 7/9$.

37. Let x denote a real number. Which one of the following statements is true for $-x$?

   (a). It is negative.

   (b). It is positive.

   (c). It is positive when x is negative.

   (d). It is $= |-x|$.

   (e). None of these.

38. A train whose total length is x miles is running at the rate of 60 miles per hour. It overtakes another train, whose total length is 1/18 of a mile, running at the rate of 40 miles per hour in the same direction on a parallel track, and completely passes it in half a minute. In what time would the first train completely pass another of its own length going in the opposite direction at 30 miles per hour?

   (a). 1 minute

   (b). 8 seconds

   (c). 8 8/9 seconds

   (d). 1/9 second

   (e). Answer cannot be determined from the given information.

39. A three-digit number is equal to 25 times the sum of its digits. If the three digits are reversed, the resulting number exceeds the given number by 198. The tens digit is one less than the sum of the hundreds and units digits. The tens digit of the given number is which one of the following?

(a). 2

(b). 3

(c). 6

(d). 7

(e). 5

40. If $3^x + 3^{x-1} = 36$, then $(x)^{2x}$ is equal to which one of the following?

(a). 37/2

(b). 27

(c). 3

(d). 12

(e). None of these.