Key

SEVENTEENTH ANNUAL MATHEMATICS CONTEST

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THE TENNESSEE MATHEMATICS TEACHER'S ASSOCIATION

ALGEBRA II TEST

1973

Scoring Formula: 4R - W

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This test was prepared from a list of Algebra II questions submitted by Martin Junior College and other mathematics teachers across Tennessee.

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school algebra. For each problem there are listed 5 possible answers; one and only one is correct. You are to work each problem, determine the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. You must use a pencil with soft lead (No. 2 lead or softer). A sample problem follows:

1. If $2x = 3$, then x equals

   (a). 2/3. (b). 3. (c). 6.

   (d). 3/2. (e). none of these.

	a	b	c	d	e
1.	☐	☐	☐	■	☐

The correct answer for the sample problem is 3/2, which is answer (d); so you would answer this problem by making a heavy black mark under space d as indicated above.

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers so it is advisable not to do much wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet.

The answer sheets will be used for a statewide statistical compilation and will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

1. The factors of $x^3 - 2x + 1$ are:

   (a). $(x - 1)(x^2 + 1)$

   (b). $(x + 1)(x^2 - 1)$

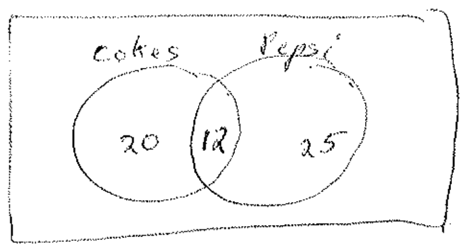
   (c). $(x + 1)(x^2 + 1)$

   (d). $x(x - 1)(x - 1)$

   (e) none of these

2. There are 70 people at a party. Thirty-two of the people like Cokes and 37 of the people like Pepsis. If 12 people like Pepsi and Coke, how many people do not like either one?

   (a). 1
   (b). 13
   (c). 39
   (d). 44
   (e). 56

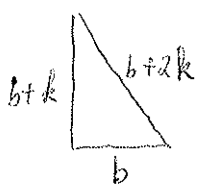


20
12
25
57

70
-57
13

3. The sides of a right triangle are in arithmetic progression, and the shortest is of length b. Find the hypotenuse of the triangle.

   (a). $b + 2$
   (b). $b - 3$
   (c). $\frac{3b}{5}$
   (d). $\frac{5b}{3}$
   (e). $b/3$



$(b+2k)^2 = (b+k)^2 + b^2$

$\cancel{b^2} + 4bk + 4k^2 = \cancel{b^2} + 2bk + k^2 + b^2$

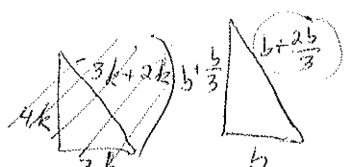
$2bk + 3k^2 = b^2$

$0 = b^2 - 2bk - 3k^2$

$0 = (b-3k)(b+k)$

$b = 3k$ or $\cancel{b = -k}$

$k = \frac{b}{3}$



4. The solution set of the inequality $|6 - 2x| \geq 2$ is

   (a). $\{x \mid x \geq 4 \text{ or } x \leq 2\}$.
   (b). $\{x \mid x \leq 2\}$.
   (c). $\{x \mid x \geq 2\}$.
   (d). $\{x \mid x \leq 2 \text{ or } x \geq -2\}$.
   (e). $\{x \mid 2 \leq x \leq 4\}$.

$-(6-2x) \geq 2$ or $6 - 2x \geq 2$

$6 - 2x \leq -2$ $\qquad -2x \geq -4$

$-2x \leq -8$ $\qquad x \leq 2$

$x \geq 4$

5. If the two solutions of $x^2 + 4x + c = 0$ are real and unequal, which one of the following is true for all possible values of the constant c?

$b^2 - 4ac = 16 - 4(1)c > 0$

$16 > 4c$

$4 > c$

(a). $c \neq 0$
(b). $c = 0$
(c). $c > 1$
(d). $c < 4$
(e). $c > 4$

6. If the graph of the equation $cy = dx^2 - 4$ passes through the points (2,0) and (-4,3), then c is equal to

$c \cdot 0 = d \cdot 4 - 4$ $\quad$ $c \cdot 3 = 1 \cdot 16 - 4$

$d = 1$ $\quad$ $3c = 12$

$c = 4$

(a). 4.
(b). 0.
(c). -9/2.
(d). -20/3.
(e). -20.

7. The number of possible distinct arrangements without repetition of the four letters A, N, P, S which do <u>not</u> spell the word SNAP is

$4! - 1 = 1 \cdot 2 \cdot 3 \cdot 4 - 1$

$= 24 - 1$

$= 23$

(a). 4.
(b). 23.
(c). 24.
(d). 255.
(e). 256.

8. If $a_n = (1/10)^{n-1}$, where n is a positive integer, then which one of the following statements is true about $\sum_{n=1}^{\infty} a_n$? $= \sum_{n=1}^{\infty} \frac{1}{10^{n-1}} = \frac{1}{10^0} + \frac{1}{10} + \frac{1}{10^2} + \frac{1}{10^3} + \cdots$

$=$

(a). It is smaller than 1.
(b). It represents a rational number.
(c). It represents an irrational number.
(d). It is larger than 2.
(e). It does not exist.

9. The system of linear equations $\begin{cases} 3x - 2y = 5 \\ kx + 3y = 7 \end{cases}$ will have a unique solution pair (x,y) <u>except</u> when k is

$D = \begin{vmatrix} 3 & -2 \\ k & 3 \end{vmatrix} = 9 - -2k = 9 + 2k = 0$

$k = -\frac{9}{2}$

(a). -3.
(b). -2.
(c). 0.
(d). 9/2.
(e). -9/2.

10. If $\log_{20} 10 = a$, then $\log_{20} 2$ is equal to

(a). a/5.
(b). a/2.
(c). a/10.
(d). 1 - a.
(e). a - 2.

$a = \log_{20} 10 = \log_{20} 2 \cdot 5$

$= \log_{20} 2 + \log_{20} 5$

$\Rightarrow \log_{20} 2 = a - \log_{20} 5$

11. Given $(\log_k x)(\log_5 k) = 3$. Solving for x gives:

Recall $\log_b n = \frac{\log_a n}{\log_a b}$

OR $(\log_b n)(\log_a b) = \log_a n$

$\Rightarrow \log_5 X = 3$

$5^3 = X$

$125 = X$

(a). $x = 5k^3$
(b). $x = 2k/125$
(c). $x = k^2$
(d). $x = 125$
(e). cannot be determined from the given information

12. A chemist has m ounces of salt water that is m% salt. How many ounces of salt must be added to make a solution that is 2m% salt?

(a). $m/(100 + m)$
(b). $2m/(100 - 2m)$
(c). $m^2/(100 - 2m)$
(d). $m^2(100 + 2m)$
(e). $2m/(100 + m)$

m% salt	100% salt	2m% salt
m oz.	X oz.	m+X oz.
m^2	100X	2m(m+x)

$m^2 + 100X = 2m(m+x)$

$m^2 + 100X = 2m^2 + 2mX$

$0 = m^2 + 2mX - 100X$

$X(100 - 2m) = m^2$

$X = \frac{m^2}{100 - 2m}$

13. For one root of $ax^2 + bx + c = 0$ to be double the other, the coefficients a, b, and c must be related as follows:

(a). $4b^2 = 9c$
(b). $2b^2 = 9ac$
(c). $2b^2 = 9a$
(d). $b^2 - 8ac = 0$
(e). $9b^2 = 2ac$

$r + 2r = -\frac{b}{a}$ $\quad r \cdot 2r = \frac{c}{a}$

$3r = -\frac{b}{a}$ $\quad 2r^2 = \frac{c}{a}$

$r = -\frac{b}{3a}$ $\quad r^2 = \frac{c}{2a}$

$\left(-\frac{b}{3a}\right)^2 = \frac{c}{2a}$

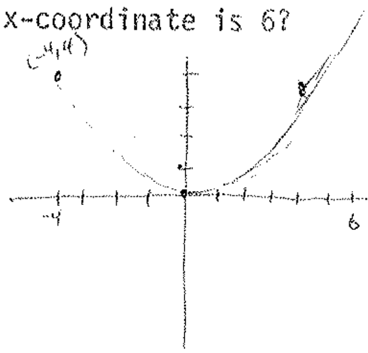
$\frac{b^2}{9a^2} = \frac{c}{2a}$

$2ab^2 = 9a^2c$

$2b^2 = 9ac$

14. A parabola with vertex at the origin and whose axis is the y-axis, passes through the point (-4,4). What is the y-coordinate of the point on this parabola whose x-coordinate is 6?

(a). 24
(b). 9
(c). $\sqrt{6}/2$
(d). 6
(e). $2\sqrt{6}$



$y = \frac{1}{4a} x^2$

$4 = \frac{1}{4a}(-4)^2$

$4 = \frac{16}{4a}$

$16a = 16$

$a = 1$

$y = \frac{1}{4(1)} x^2$

$y = \frac{1}{4}(6)^2$

$y = \frac{1}{4} \cdot 36$

$y = 9$

15. A train running between two towns arrives at its destination 10 minutes late when it goes 40 mph and 16 minutes late when it goes 30 mph. The distance between the towns in miles is

(a). 720.
(b). 12.
(c). 8 6/7.
(d). 33 1/3.
(e). 5 1/3.

40 mph
$t + \frac{10}{60}$

30 mph
$t + \frac{16}{60}$

$40\left(t + \frac{1}{6}\right) = 30\left(t + \frac{4}{15}\right)$

$40t + 6\frac{2}{3} = 30t + 8$

$10t = \frac{4}{3}$

$t = \frac{4}{30} = \frac{2}{15}$ hr.

then $t + \frac{1}{6} = \frac{4}{30} + \frac{1}{6} = \frac{9}{30} = \frac{3}{10}$

$d = 40(t + \frac{1}{6}) = 40 \cdot \frac{3}{10} = 12$ mi

16. There are three discs denoted by A,B,C. One disc is red, one is blue, and one is white. Deduce the colors of A, B, and C respectively from the fact that one, and only one, of the following statements is true:
(1) A is red; (2) B is not red; (3) C is not blue.

X

(a). A - red, B - white, C - blue
(b). A - white, B - red, C - blue
(c). A - blue, B - red, C - white
(d). A - red, B - blue, C - white
(e). cannot be determined

~~Red white Blue~~
~~(1) TRUE NOX~~
(2)

	A	B	C	
(1) TRUE	Red	Red	Blue	X
(2) TRUE	not Red	not Red	Blue	X
(3) TRUE	not Red, ~~white~~ blue	Red	~~Blue~~ white	

21. Express $(1 - i)^7(1 + i)^8$ in the form $a + bi$:

(a). 128 + 128i

(b). 2 + 2i

(c). 9728 + 0i

(d). 64 + 64i

(e). none of these

$(1-i)^2 = 1-2i+i^2 = -2i$

$(1+i)^2 = 1+2i+i^2 = 2i$

$(1-i)^7(1+i)^8$

$= (1-i)(-2i)^3 \ (2i)^4$

$= (1-i)(-8i^3)(16i^4)$

$= (1-i)(8i)(16)$

$= 128i - 128i^2$

$= 128i + 128$

22. Set A = {1, -1, i, -i}, where the elements of A are from the complex number field. Consider the statements:

I. A is closed with respect to addition. false, $1+1 = 2 \notin A$

II. A is closed with respect to multiplication. true

III. If $a, b \in A$, then $a + b = b + a$. true

IV. If $a, b \in A$, then $a \div b = b \div a$. false, $\frac{1}{i} \neq \frac{i}{1}$

V. A is closed with respect to division.

Which one of the statements below is true?

(a). Only I and II are true.

(b). Only II, III, and IV are true.

(c). Only III, IV, and V are true.

(d). Only II, III, and V are true.

(e). All are true.

23. Consider the functions below:

I. $F(x) = b^x$, $b \neq 1$ and $b > 0$

II. $F(x) = \log_b x$, $b \neq 1$ and $b > 0$

III. $F(x) = ax$, $a \neq 0$

IV. $F(x) = ax^2$, $a \neq 0$

Which of the functions above has the property that $F(u + v) = F(u) + F(v)$?

(a). I

(b). II

(c). III

(d). IV

(e). All of them

I) $F(u+v) = b^{u+v}$, but $F(u)+F(v) = b^u + b^v$, no.

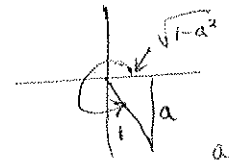
II) $F(u+v) = \log_b(u+v)$, but $F(u)+F(v) = \log u + \log v = \log uv$, no.

III) $F(u+v) = a(u+v)$, and $F(u)+F(v) = au + av$. YES

24. If $\sin(t) = a$ and $+3\pi/2 < t < 2\pi$, then $\sin(2t)$ is

(a). $2a$.
(b). $-2a$.
(c). $2a\sqrt{1 - a^2}$.
(d). $-2a\sqrt{1 - a^2}$.
(e). $\sqrt{a^2 - 1}$.

$\sin 2t = 2 \sin t \cos t$
$= 2 \cdot a \cdot \sqrt{1-a^2}$



$a < 0$

25. The graph of the set of ordered pairs (x,y) defined by $\{(x,y) \mid 9x^2 + 4y^2 - 18x - 16y - 24 = 0\}$ represents

(a). a circle.
(b). an ellipse.
(c). a parabola.
(d). a hyperbola.
(e). a pair of intersecting straight lines.

$9x^2 - 18x + \underline{9} + 4y^2 - 16y + \underline{16} = 24 + \underline{9+16}$
$9(x^2 - 2x + \underline{1}) + 4(y^2 - 4y + \underline{4}) = 24 + \underline{25}$
$9(x-1)^2 + 4(y-2)^2 = 49$

26. Evaluate the determinant: $\begin{vmatrix} 1 & 0 & 3 & 2 \\ \cancel{-2} & \cancel{3} & \cancel{4} & \cancel{0} \\ -3 & 0 & 3 & 5 \\ 7 & 0 & 3 & 6 \end{vmatrix} = 3\begin{vmatrix} 1 & 3 & 2 \\ -3 & 3 & 5 \\ 7 & 3 & 6 \end{vmatrix} = 3(1)\begin{vmatrix} 3 & 5 \\ 3 & 6 \end{vmatrix} - (3)(3)\begin{vmatrix} -3 & 5 \\ 7 & 6 \end{vmatrix} + 3(2)\begin{vmatrix} -3 & 3 \\ 7 & 3 \end{vmatrix}$

$= 3[18 - 15] - 9[-18 - 35] + 6[-9 - 21]$
$= 3(3) - 9(-53) + 6(-30)$
$= 9 + 477 - 180$
$= 306$

(a). 3
(b). -18
(c). 102
(d). 306
(e). -6

27. Two evenly matched baseball teams play a series of 6 games. The probability that team A will win exactly 4 games is $p = q = 1$

(a). 2/3.
(b). 1/120.
(c). 15/64.
(d). 1/64.
(e). 1/16.

$P(\text{4 wins}) = \binom{6}{4}\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^2 = \frac{6!}{4!\,2!} \cdot \frac{1}{2^6}$
$= \frac{\overset{3}{\cancel{6}} \cdot 5}{1 \cdot \cancel{2}} \cdot \frac{1}{64}$
$= \frac{15}{64}$

28. If $x^3 - 2x^2 - 5x + 9$ is divided by $x - a$ and the remainder is 9, then a is equal to $f(a) = 9 \Rightarrow a^3 - 2a^2 - 5a + 9 = 9$

(a). 1.
(b). 3.
(c). 10.
(d). - 1.
(e). none of these.

$a(a^2 - 2a - 5) = 0$

$a = 0 \quad a = \frac{2 \pm \sqrt{4 - 4(-5)}}{2}$

$= \frac{2 \pm \sqrt{24}}{2} = 1 \pm \sqrt{6}$

29. $\left[\frac{81^{-3/4} - 3^{-2}}{(9/25)^{-1/2} + (1/3)x^0}\right]^{-1/3}$, $x \neq 0$, is equal to

$= \left[\frac{\frac{1}{27} - \frac{1}{9}}{\frac{5}{3} + \frac{1}{3}}\right]^{-\frac{1}{3}} = \left[\frac{1 - 3}{54}\right]^{-\frac{1}{3}} = \left[-\frac{1}{27}\right] = \left[-27\right]^{\frac{1}{3}} = -3$

(a). 1.
(b). -3.
(c). 5.
(d). 1/3.
(e). none of these.

30. If $y = (x - 1)/(1 - 2x)$ and $(1 - 2x) \neq 0$, then $(y - 1)/(1 - 2y)$ is equal to

(a). $(3x - 2)/(3 - 4x)$.
(b). x.
(c). $(2 - 3x)/(3 + 4x)$.
(d). $(3x + 2)/(3 + 4x)$.
(e). $(3x + 2)/(3 - 4x)$.

$\frac{y-1}{1-2y} = \frac{\frac{x-1}{1-2x} - 1}{1 - \frac{2(x-1)}{1-2x}} = \frac{(x-1) - (1-2x)}{(1-2x) - 2(x-1)} = \frac{x - 1 - 1 + 2x}{1 - 2x - 2x + 2}$

$= \frac{3x - 2}{3 - 4x}$

31. Reduce $\frac{4^{2x+1} \cdot 8^3}{16^x \cdot 32^2 - 2^{4x+7} \cdot 64}$ to lowest terms:

(a). - 16/7
(b). 1
(c). 5/16
(d). 2^{2x+3}
(e). - 2/7

$= \frac{2^{4x+2} \cdot 2^9}{2^{4x} \cdot 2^{10} - 2^{4x+7} \cdot 2^6} = \frac{\cancel{2^{4x}} \cdot 2^2 \cdot \cancel{2^9}^{\,2^3}}{\cancel{2^{4x}} \cdot \cancel{2^6}(2^4 - 2^7)}$

$= \frac{2^5}{2^4(1 - 2^3)} = \frac{2}{-7}$

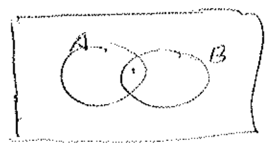
32. If $\log_8 M + \log_8 (1/6) = 2/3$, then M is equal to

(a). 1/2.
(b). 4.
(c). 24.
(d). 2/3.
(e). 48.

$$\log_8 \frac{M}{6} = \frac{2}{3}$$
$$8^{\frac{2}{3}} = \frac{M}{6}$$
$$4 = \frac{M}{6}$$
$$24 = M$$

33. Consider the four statements below, where A and B are sets and A' and B' are the respective complements of the sets:

I. If $A \subseteq B$, then $A \cap B = A$. true
II. If $A \subseteq B$, then $A' \subseteq B'$. false
III. $(A \cap B)' = A' \cup B'$. true
IV. $A \cup (A \cap B) = A$. true
$\cancel{A\cup(A\cap B) = (A\cup A)\cap(A\cup B)}$



Determine the one answer below which is correct:

(a). I, II, III, and IV are true
(b). Only II, III, and IV are true
(c). Only I, and IV are true
(d). Only I, III, and IV are true.
(e). Only I and III are true

1. The line joining the point (4,3) to the origin intersects the line $2x + 2y - 7 = 0$ at the point: $y = \frac{3}{4}x$

(a). (3/2, 2)
(b). (35/16, 21/16)
(c). (2, 3/2)
(d). (-1, 2)
(e). (0, 7)

$$2x + 2\left(\tfrac{3}{4}x\right) - 7 = 0$$
$$2x + \tfrac{3}{2}x - 7 = 0$$
$$4x + 3x - 14 = 0$$
$$7x = 14$$
$$x = 2$$
$$y = \tfrac{3}{4}(2) = \tfrac{3}{2}$$

35. If (x - y)/(x + y) = 5/2, then x/y is equal to:

(a). 7/3
(b). -7/3
(c). 3
(d). -7/5
(e). -5/7

$\frac{x-y}{x+y} = \frac{5}{2}$

$5x+5y = 2x-2y$

$3x = -7y$

$\frac{x}{y} = \frac{-7}{3}$

36. The maximum value of $6 - 3x - 3x^2$ is

(a). 1/2.
(b). -6/35.
(c). 4/27.
(d). 27/4.
(e). -1/2.

$y = -3(x^2 + x + \frac{1}{4}) + 6 + \frac{3}{4}$

$y = -3(x + \frac{1}{2})^2 + 6\frac{3}{4}$

max $y = 0 + 6\frac{3}{4}$

37. If a man drives M miles at an average speed of 30 mph and another M miles at 50 mph, then his average speed for the 2M miles is

(a). 40 mph.
(b). 45 mph.
(c). 35 mph.
(d). 37 1/2 mph.
(e). Not enough is given to determine the average speed.

$\frac{M}{50} + \frac{M}{30} = \frac{2M}{x}$

$\frac{1}{50} + \frac{1}{30} = \frac{2}{x}$

$30x + 50x = 2(50)(30)$

$80x = (100)(30)$

$x = \frac{(100)(30)}{80}$

$x = \frac{300}{8} = 37\frac{1}{2}$

38. Given the equation $x^3 - x^2 + 17x + 87 \overset{= f(x)}{=} 0$. Determine the one answer below which is correct. The equation has

Factors of 87: 1 3
87 29

(a). all roots equal. no
(b). one real root and two imaginary roots.
(c). three imaginary roots. no
(d). two real roots and one imaginary root. no
(e). three real roots.

$f(3) = 3^3 - 3^2 + 17(3) + 87$
$= 27 - 9 + 51 + 87 > 0$

$f(-3) = -27 - 9 - 51 + 87 = 0 \therefore -3$ is root

$$\begin{array}{r} x^2 - 4x + 29 \\ x+3 \overline{) x^3 - x^2 + 17x + 87} \\ x^3 + 3x^2 \\ \hline -4x^2 + 17x \\ -4x^2 - 12x \\ \hline 29x + 87 \\ 29x + 87 \\ \hline 0 \end{array}$$

$b^2 - 4ac$
$= 16 - 4(1)(29)$
$= 16 - 4(29) < 0$
$\Rightarrow$ 2 imag. rts.

39. Four positive integers are given. Select any three of these integers, find their arithmetic average, and add this result to the fourth integer. Do this for all of the possible distinct triplets of these numbers. The results obtained are 29, 23, 21, and 17. One of the original integers is:

(a). 19
(b). 21
(c). 23
(d). 29
(e). 17

$\frac{a+b+c}{3}+d=29$ $\quad a+b+c+3d=87$

$\frac{a+b+d}{3}+c=23$ $\quad a+b+3c+d=69$

$\frac{a+c+d}{3}+b=21$ $\quad a+3b+c+d=63$

$\frac{b+c+d}{3}+a=17$ $\quad 3a+b+c+d=51$

$\therefore 2a+2b+2c+2d=90$

$a+b+c+d=45$

$a+b+c+3d=87$

$-2d=-42$

$d=21$

40. A ball is rolling in such a way that its rate of decrease in distance travelled every second is 1/4 ft/sec/sec. If it rolls 16 feet before coming to rest, how far did it roll during the first second?

(a). 8 feet
(b). 12 feet
(c). 8/3 feet
(d). 2 feet
(e). none of these

$\frac{k}{4}+\cdots+\frac{3}{4}+\frac{2}{4}+\frac{1}{4}=16$

$\frac{k(k+1)}{2}=4\cdot 16$

$k^2+k=128$

$k^2+k-128=0$

$k=\frac{-1\pm\sqrt{1-4(-128)}}{2}=\frac{-1\pm\sqrt{513}}{2}$