TENTH ANNUAL MATHEMATICS CONTEST

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ALGEBRA II TEST

1966

Scoring Formula: $4R - W$.

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DIRECTIONS:

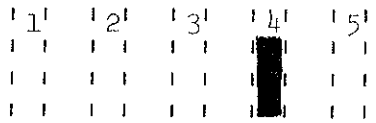
Do not open this booklet until you are told to do so.

This is a test of your competence in high school algebra. For each problem there are listed 5 possible answers. You are to work the problems, determine the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. A sample follows:

1. If $2x = 3$, then x equals:

(1) 2/3; (2) 3; (3) 6;

(4) 3/2; (5) none of these.



The correct answer for the sample problem is 3/2, which is answer (4); so you would answer this problem by making a heavy black mark under space 4 as indicated above.

If you should change your mind about an answer, be sure to erase completely. Avoid wild guessing, as wrong answers count against you. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

1. If x and y are unequal rational numbers, then $x^2 - 3y^2$

(1) is always less than zero
(2) is always greater than zero
(3) can equal zero
(4) cannot equal zero
(5) is an irrational number.

2. $\sqrt{125\ a^2}$ equals

(1) $5\sqrt{5}\ a$
(2) $5|a|$
(3) $5\sqrt{5}|a|$
(4) $-5\sqrt{5}|a|$
(5) $|5a|$.

3. In changing $4b\sqrt[3]{7a}$ to a radical whose coefficient is 1, you get as a correct answer:

(1) $\sqrt[3]{28\ ab^3}$
(2) $\sqrt[3]{16\ a^3b}$
(3) $\sqrt[3]{64b^3 + 7a}$
(4) $\sqrt[3]{448\ a^3b^3}$
(5) $\sqrt[3]{448\ ab^3}$

4. Solve for x:

$$(a^3)^x = \frac{1}{a} .$$

(1) 1/3
(2) 0
(3) - 1/3
(4) - 4
(5) none of these.

5. Solve:

$$\begin{vmatrix} 2 & -x & 5 \\ 1 & -3 & 4 \\ -1 & 3 & x \end{vmatrix} = 0.$$ One of the values of x is:

(1) 6
(2) 4
(3) 9
(4) 3
(5) You cannot tell.

6. The area A of a circular sector varies jointly as the central angle θ and the square of the radius r. If θ is held constant, what effec does doubling the radius have on the area?

(1) Divides it by 4
(2) Doubles it
(3) Multiplies it by 4
(4) Multiplies it by 2θ
(5) None of these.

7. The equation $\sqrt{x + 7} + x = 13$

(1) has two roots
(2) has no root
(3) has three roots
(4) has one root
(5) none of the above is correct.

8. The parabola $4y = 4x^2 - 4x - 7$ has a horizontal tangent at the point

(1) $(1/2, -2)$
(2) $(0, -7/4)$
(3) $(5/2, 2)$
(4) $(5, 73/4)$
(5) none of these.

9. The equation of the line that has a y-intercept of -3 and forms a right angle with the line $-2x - 3y = 5$ is

(1) $3y + 2x = -6$
(2) $2y - 3x = -6$
(3) $3y + 2x = -9$
(4) $2y - 3x = -9$
(5) $2y - 3x = -3$.

10. To rationalize the denominator of the fraction $\dfrac{1}{(5 + \sqrt[3]{9})}$ one may multiply the numerator and the denominator by:

(1) $25 - 5\sqrt[3]{9} + 3\sqrt[3]{3}$
(2) $25 + 5\sqrt[3]{9} + 3\sqrt[3]{3}$
(3) $5 - \sqrt[3]{9}$
(4) $25 - 3\sqrt[3]{3}$
(5) $25 + \sqrt[3]{9}$

11. A solution (x, y, z) of the system of linear equations

$$\begin{aligned} -x + y + 4z &= 0 \\ x + 3y + 8z &= 0 \\ x + 2y + 5z &= 0 \end{aligned}$$

is

(1) $(2, -2, 1)$
(2) $(1, -3, 1)$
(3) $(-1, -2, 1)$
(4) $(1, 1, 0)$
(5) none of these.

12. Let $A = \{x: x \text{ is a whole number and } 2 < x < 3\}$ and let $B = \{0\}$. Then set A

(1) is equal to set B
(2) includes set B
(3) is equivalent to set B
(4) is a subset of set B
(5) none of these

25. Another way to write $(a^{-1} + b^{-1})^{-1}$ is

(1) $\frac{a + b}{ab}$

(2) $\frac{1}{a} + \frac{1}{b}$

(3) $\frac{ab}{a + b}$

(4) $a + b$

(5) ab.

26. The sum of the digits of a two-place number is 12. If the order of the digits were reversed, the new number would be twice the original number less the sum of the digits. What is the original number?

(1) 75

(2) 66

(3) 57

(4) 48

(5) 42.

27. If the sides of a square are each increased by 12 inches, the area is increased by 200 square inches. The length of a side of the original square is

(1) 2 inches

(2) 2 1/3 inches

(3) 10 1/2 inches

(4) 3 2/3 inches

(5) 2 1/4 inches.

28. Let y, z, w be real numbers.

(a) $(yz)w = y(zw)$

(b) $y(zw) = y(wz)$

What law of multiplication in the set of real numbers was used in each step? Check the one answer that answers both (a) and (b)

(1) (a) Distributive Law; (b) Commutative Law

(2) (a) Associative Law; (b) Commutative Law

(3) (a) Commutative Law; (b) Associative Law

(4) (a) Associative Law; (b) Distributive Law

(5) (a) Commutative Law; (b) Distributive Law.

29. The reciprocal of $3i - 2$ in the form of $a + bi$ is

(1) $2 - 3i$

(2) $-2 - 3i$

(3) $\frac{2}{5} + \frac{3}{5}i$

(4) $\frac{-2}{5} + \frac{3}{5}i$

(5) $\frac{-2}{13} + \frac{-3}{13}i$.

30. A can do a piece of work in 8 days and B can do the same work in 5 days. How many days will it take them working together to do this work?

(1) 5 days
(2) 6 1/2 days
(3) 2 days
(4) 3 1/3 days
(5) none of these.

31. If $x - 5$ divides $2x^3 - 3x^2 - kx + 20$ with a remainder of zero, then k equals:

(1) 31
(2) 69
(3) 5
(4) 39
(5) 0

32. In solving the equation $\frac{6}{z^2 - 1} = \frac{1}{2} + \frac{1}{1 - z}$ we find that one value of z is:

(1) 1
(2) -5
(3) $-3 - 2\sqrt{7}$
(4) -3
(5) $\frac{(1 + 2\sqrt{19})}{4}$

33. If one root of the equation $2x^2 - 5x + c = 0$ is 4, then the other root is:

(1) - 3/2
(2) 13/2
(3) 4 - c
(4) - 12
(5) 25/8.

34. One man walks 4 miles per hour and a second man walks 6 miles per hour. If they leave at the same time and walk in opposite directions on a road from the same point of departure, how long will it take them to be 25 miles apart?

(1) 2.5 hours
(2) 6.5 hours
(3) 10 hours
(4) 1.4 hours
(5) none of these.

35. The 50^{th} term of the progression $1 + x, 1 + 4x, 1 + 7x, \ldots$ is

(1) 1 + 150x
(2) 148x + 1
(3) 50 + 3725x
(4) 50 + 148x
(5) x + 148.

36. Determine k so that the roots of $x^2 + 2kx + 1 = 3k$ will be equal.

(1) 1
(2) -1
(3) i
(4) -i
(5) $1 \pm \sqrt{2}$

37. Which one of the following is equal to $\dfrac{\frac{2}{a} - \frac{3}{b}}{\frac{4}{a^2} - \frac{9}{b^2}}$?

(1) $3a + 2b$

(2) $\dfrac{ab}{3a + 2b}$

(3) $\dfrac{3a + 2b}{ab}$

(4) $\dfrac{ab}{3a - 2b}$

(5) $\dfrac{2b - 3a}{ab}$

38. If the function $x^2 - kx + k - 1$ has one of its zeros twice the other, then one of the numbers for k is:

(1) 1
(2) -1
(3) 3
(4) -3
(5) - 3/2.

39. The coefficient of the term involving x^8 in the expansion of $(x^2 + 3y)^{10}$ is:

(1) $(3^7)(70)$
(2) $(3)(70)$
(3) $(3^7)(5)$
(4) $(3^6)(5)$
(5) none of these.

40. If a polynomial $P(x)$ is divided by $(x - 2)$ the remainder is 1 and when divided by $(x - 1)$ the remainder is 2. If $P(x)$ is divided by $(x - 1)(x - 2)$, the remainder is:

(1) 2
(2) 3
(3) $- x + 3$
(4) - 2
(5) $- 2x + 10$.