



TENNESSEE MATHEMATICS TEACHERS ASSOCIATION

SIXTY-FIFTH ANNUAL MATHEMATICS CONTEST

2023

Geometry/Integrated Math II

Scoring Formula: $4 \times (\text{Number Right}) - (\text{Number Wrong}) + 40$

Directions:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

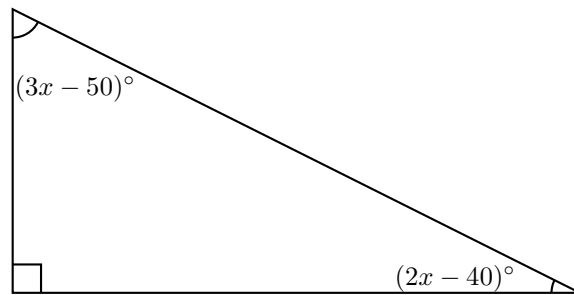
This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you; if you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly eighty minutes to work.

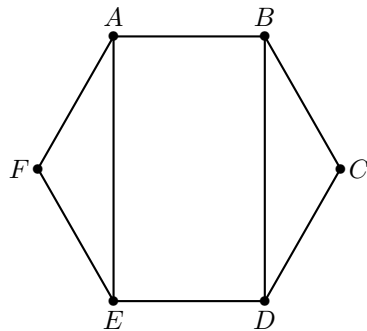
1. The sides of a triangles are length 7 cm, 23 cm, and 25 cm. Which of the following best describes the triangle?
- (a) No such triangle is possible.
 - (b) It is an obtuse triangle.
 - (c) It is a right triangle.
 - (d) It is an acute triangle.
 - (e) It is an isosceles triangle.

2. Consider the right triangle below. What is the value of x ?



- (a) $x = 54$ (b) $x = 10$ (c) $x = 36$ (d) $x = 42$ (e) $x = 38$

3. Hexagon $ABCDEF$ is regular with side length 4 cm. What is the area of Rectangle $ABDE$?



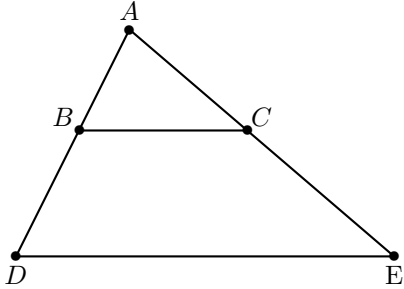
- (a) $8 + 8\sqrt{3}$ cm²
- (b) 32 cm²
- (c) 24 cm²
- (d) $16\sqrt{3}$ cm²
- (e) $8\sqrt{3}$ cm²

4. What is the equation of a circle with radius 5 centered at the point $(1, -2)$?
- (a) $(x + 1)^2 + (y - 2)^2 = 5$
 - (b) $(x - 1)^2 + (y + 2)^2 = 25$
 - (c) $(x - 1)^2 + (y + 2)^2 = 5$
 - (d) $(x + 1)^2 + (y - 2)^2 = 25$
 - (e) $(x - 1) + (y + 2) = 5$
5. The measure of each interior angle of a regular polygon is 160° . How many sides does the polygon have?
- (a) 22
 - (b) 20
 - (c) 18
 - (d) 16
 - (e) 24
6. A cube has surface area of 36 square inches. What is the volume of a sphere inscribed in the cube?
- (a) 36π cubic inches
 - (b) 8π cubic inches
 - (c) $\pi\sqrt{6}$ cubic inches
 - (d) $8\pi\sqrt{6}$ cubic inches
 - (e) π cubic inches
7. A rhombus has perimeter 100 cm and its shorter diagonal is 30 cm. What is the area of the rhombus?
- (a) 300 cm^2
 - (b) 400 cm^2
 - (c) 450 cm^2
 - (d) 600 cm^2
 - (e) 500 cm^2
8. What is the maximum value of the function $f(x) = -9 + 3x - \frac{x^2}{6}$?
- (a) 3
 - (b) 6
 - (c) 4.5
 - (d) 9
 - (e) -9
9. A regular tetrahedron has a volume of 9 cubic inches. What is the surface area of the tetrahedron?
- (a) $18\sqrt[3]{3}$ cubic inches
 - (b) $24\sqrt[3]{3}$ cubic inches
 - (c) $18\sqrt{3}$ cubic inches
 - (d) $12\sqrt[3]{3}$ cubic inches
 - (e) $12\sqrt{3}$ cubic inches

10. Which of the following describe shape of the faces of a regular icosahedron?
- (a) All equilateral triangles
 - (b) Some regular hexagons and some regular pentagons
 - (c) All regular pentagons
 - (d) All regular hexagons
 - (e) All squares
11. A non-convex polygon is drawn. What is the minimum number of sides that the polygon can have?
- (a) 4
 - (b) 8
 - (c) 6
 - (d) 5
 - (e) 3
12. Which of the following points is always equidistant from the vertices of a triangle?
- (a) the orthocenter
 - (b) the midpoint of the longest side
 - (c) the centroid
 - (d) the incenter
 - (e) the circumcenter
13. If $\angle A$ is complementary to $\angle B$ and $\angle A$ is supplementary to $\angle C$, what is the value of $m\angle C - m\angle B$?
- (a) 60°
 - (b) 0°
 - (c) 180°
 - (d) 90°
 - (e) 45°
14. Which of the following cannot be used to prove that two triangles are congruent?
- (a) Side-Angle-Side
 - (b) Angle-Angle-Side
 - (c) Angle-Side-Angle
 - (d) Side-Side-Side
 - (e) Side-Side-Angle
15. Which of the following are always congruent?
- (a) corresponding angles
 - (b) vertical angles
 - (c) alternate exterior angles
 - (d) alternate interior angles
 - (e) all of the above

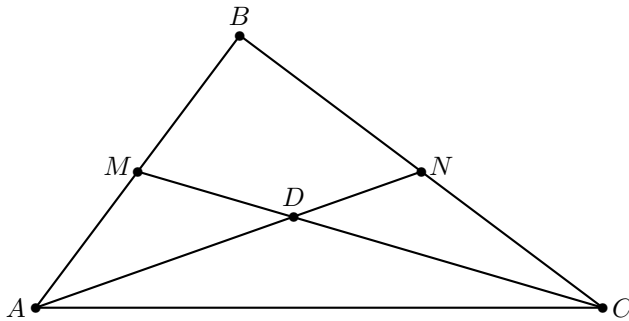
16. Two cubes have faces whose areas have the ratio 2:1. What is the ratio of their volumes?
 (a) $2\sqrt{2} : 1$ (b) $2 : 1$ (c) $\sqrt{2} : 1$ (d) $4 : 1$ (e) $\sqrt[3]{2} : 1$

17. In the following diagram $\overleftrightarrow{BC} \parallel \overleftrightarrow{DE}$. If $m\overline{DE} = 15.0$ cm, $m\overline{AB} = 4.4$ cm, $m\overline{BD} = 5.6$ cm, what is $m\overline{BC}$ rounded to the nearest tenth of a centimeter?



- (a) 6.4 cm (b) 7.0 cm (c) 11.8 cm (d) 7.5 cm (e) 6.6 cm

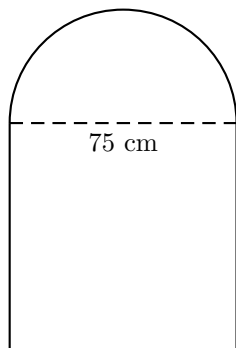
18. In the diagram below, point M is the midpoint of \overline{AB} and point N is the midpoint of \overline{BC} . If the area of $\triangle ABC = 120$ cm², what is the area of $\triangle ADC$?



- (a) 60 cm² (b) 30 cm² (c) 40 cm² (d) $20\sqrt{3}$ cm² (e) $30\sqrt{2}$ cm²

19. A trapezoid has 3 sides that are length x and one side that is length $2x$. What is the measure of the smallest interior angle of the trapezoid?
 (a) 30° (b) 45° (c) 9° (d) 20° (e) 60°

20. A decorative window is made from a semicircle and a square as shown below. What is the area of the window to the nearest square centimeter?



- (a) 7834 cm² (b) 11148 cm² (c) 10043 cm² (d) 5876 cm² (e) 14461 cm²

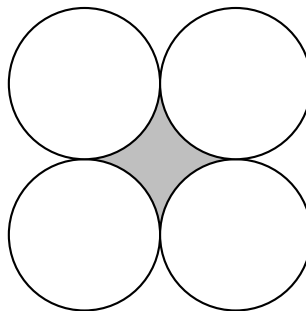
21. A regular n -gon is inscribed in a circle of radius 1 cm. Which of the following values for n gives a regular polygon whose perimeter is an integer number of centimeters?

- (a) $n = 3$ (b) $n = 6$ (c) $n = 8$ (d) $n = 4$ (e) $n = 5$

22. A high definition TV screen is a rectangle whose height and width have a ratio of 9:16. The size of the screen is measured diagonally. How wide is the screen on a 90-inch TV? Round to the nearest inch.

- (a) 90 inches (b) 80 inches (c) 78 inches (d) 82 inches (e) 44 inches

23. In the diagram below, all circles have diameter 1 cm and intersecting circles are tangent to one another. What is the area of the shaded region?



- (a) $4 - \pi$ cm² (b) $1 - \frac{\pi}{4}$ cm² (c) $2\sqrt{2} - \frac{\pi}{2}$ cm² (d) $\sqrt{2} - \frac{\pi}{4}$ cm² (e) $2 - \frac{\pi}{2}$ cm²

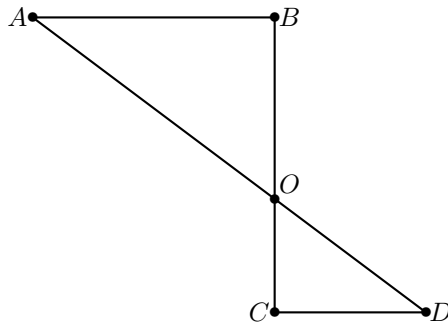
24. How many edges are in a square pyramid?

- (a) 6 (b) 12 (c) 8 (d) 16 (e) 4

25. Which of the following is NOT true about the diagonals of any rectangle?

- (a) They are congruent.
(b) Together with the sides, they form 4 triangles with equal area.
(c) They are perpendicular.
(d) They bisect each other.
(e) All of the above are always true about a rectangle.

26. In the diagram below, $\overleftrightarrow{AB} \perp \overleftrightarrow{BC}$ and $\overleftrightarrow{CD} \perp \overleftrightarrow{BC}$, if $m\overline{BO} = 6$ cm, $m\overline{AO} = 10$ cm, $m\overline{CD} = 5$, what is $m\overline{DO}$?



- (a) 6.25 cm (b) 9.67 cm (c) 8.33 cm (d) 7.25 cm (e) 7.33 cm

27. A triangle is circumscribed by a circle where the longest side of the triangle is a diameter of the circle. What can be said about the triangle?

- (a) It is an isosceles triangle.
(b) It is an obtuse triangle.
(c) It is a scalene triangle.
(d) It is a right triangle.
(e) It is an acute triangle.

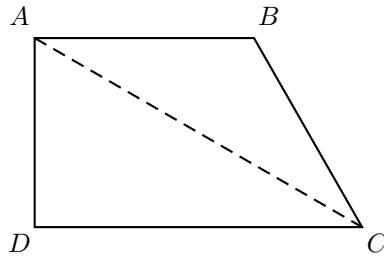
28. If point A is located at (3,1) and point B is located at (7, -3), which of the following for point C would make $\triangle ABC$ isosceles?

- (a) (0, -1) (b) (6, 3) (c) (-1, -1) (d) (3, -6) (e) (-1, -3)

29. A regular n -polygon has apothem of length a , radius of length r , and side of length s . Which of the following gives the area of the polygon?

- (a) $2nrs$ (b) $2ans$ (c) $\frac{1}{2}ans$ (d) $\frac{1}{2}anrs$ (e) $\frac{1}{2}nrs$

30. In the figure below, $m\overline{AB} = m\overline{BC}$, $\angle BAD$ and $\angle D$ are right angles, $m\overline{AC} = 10$ cm, and $m\angle B = 120^\circ$. Find the perimeter of quadrilateral $ABCD$.



- (a) $\frac{15 + 35\sqrt{3}}{6}$ cm
 (b) $\frac{25 + 25\sqrt{3}}{3}$ cm
 (c) $\frac{15 + 35\sqrt{3}}{3}$ cm
 (d) $10 + 10\sqrt{3}$ cm
 (e) $15\sqrt{3}$ cm

31. A rhombus has diagonals of length 8 cm and 4 cm. What is its perimeter? Round to the nearest tenth of a centimeter.

- (a) 16.0 cm (b) 21.2 cm (c) 17.9 cm (d) 20.0 cm (e) 22.4 cm

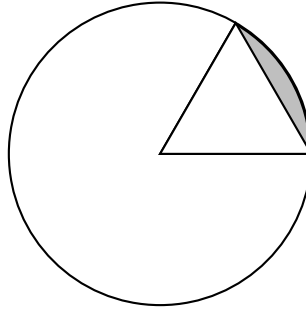
32. A traditional soccer ball is a polyhedron called a truncated icosahedron. A truncated icosahedron has 12 regular pentagon and 20 regular hexagon faces. How many vertices does a truncated icosahedron have?

- (a) 90 (b) 45 (c) 60 (d) 180 (e) 120

33. A sugar ice cream cone has height of 4.6 inches and a diameter of 1.9 inches. What is the lateral surface area of the ice cream cone to the nearest tenth of a square inch?

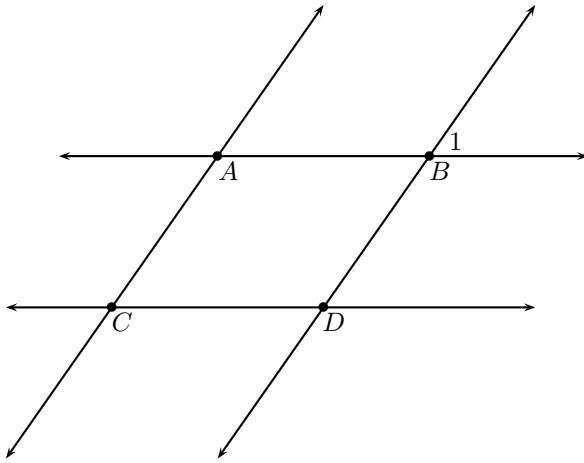
- (a) 16.9 in.² (b) 14.0 in.² (c) 4.3 in.² (d) 29.4 in.² (e) 17.2 in.²

34. The triangle in the picture below has a vertex in the center of the circle and is equilateral with side length 4 cm. What is the area of the shaded region rounded to the nearest hundredth of a square centimeter?



- (a) 0.38 cm^2 (b) 8.38 cm^2 (c) 4.91 cm^2 (d) 2.45 cm^2 (e) 1.45 cm^2

35. In the following picture, $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ and $\overleftrightarrow{AC} \parallel \overleftrightarrow{BD}$. How many angles are corresponding angles with $\angle 1$?



- (a) 1 (b) 5 (c) 3 (d) 2 (e) 7

36. A circle is inscribed in a regular hexagon. Another hexagon is then inscribed in the circle. If the area of the smaller hexagon is 1 m^2 , what is the area of the larger hexagon?

- (a) $\sqrt{3}$ (b) $\frac{4}{3}$ (c) $\sqrt{2}$ (d) $\frac{\sqrt{6}}{2}$ (e) $\frac{5}{4}$

37. A sphere and a circular cone both have radius r and equal volume. What is the height h of the cone in terms of r ?

- (a) $h = 4r$ (b) $h = r$ (c) $h = 3r$ (d) $h = 6r$ (e) $h = 2r$

38. How high does a cylinder with diameter 2 cm have to be so the lateral surface area is equal to the area of the base? Round to the nearest hundredth of a centimeter.

- (a) 0.67 cm (b) 0.75 cm (c) 1 cm (d) 0.33 cm (e) 0.5 cm

39. Find the area of the quadrilateral with vertices $(0, 0)$, $(1, 3)$, $(6, 1)$, and $(4, 5)$.

- (a) 16.5 sq. units (b) 20.5 sq. units (c) 22.5 sq. units (d) 14.5 sq. units (e) 18.5 sq. units

40. A cylinder and sphere both have radius r . What must the height h be in terms of r for the surface area of the sphere to equal the lateral surface area of the cylinder?

- (a) $h = 3r$ (b) $h = r$ (c) $h = 6r$ (d) $h = 2r$ (e) $h = 4r$

