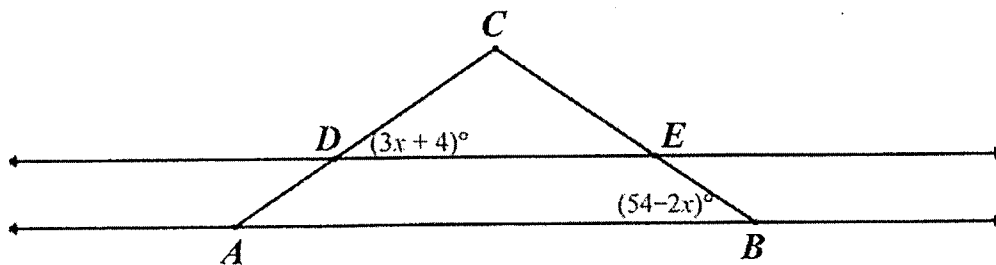


Geometry/Integrated Mathematics II—2019

- If polygon $ABCDE$ is a regular pentagon, what is the measure of $\angle ADB$?
 - 18°
 - 36°
 - 54°
 - 72°
 - 108°
- Diego decides to earn extra money by leading tour groups on exciting explorations. He decides to charge a fee of \$50 per group with an additional charge of \$15 per person in the group. Which of the following functions gives the cost $c(x)$ for a tour group with x members?
 - $c(x) = 50x + 15$
 - $c(x) = 50(x + 15)$
 - $c(x) = 65x$
 - $c(x) = 50 + 15x$
 - $c(x) = (50 + x)15$

- In the given figure, \overline{DE} intersects the congruent sides of isosceles triangle $\triangle ABC$ and is parallel to \overline{AB} . What is the value of x ?

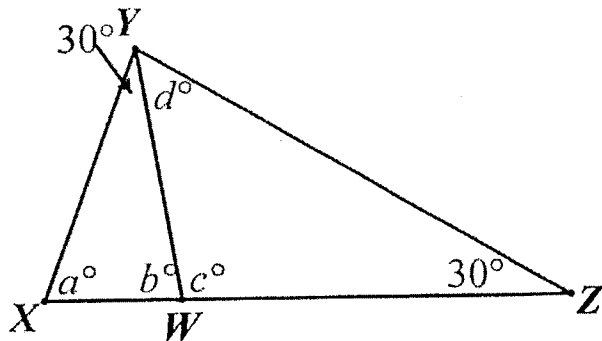


- 10
- $18\frac{2}{3}$
- 34
- 50
- 60

4. What is the value of complex number $\frac{4+2i}{1+i}$?
- (a) 6
 - (b) $3 - i$
 - (c) $6 - 2i$
 - (d) $2 + 6i$
 - (e) $3 + i$
5. A right triangle has one leg measuring 28 cm and hypotenuse measuring 53 cm. What is its perimeter to the nearest centimeter?
- (a) 45
 - (b) 60
 - (c) 81
 - (d) 126
 - (e) 141
6. At noon, a circle whose original diameter was 3 cm begins growing so that its diameter increases by one centimeter every 30 seconds. How much does the circumference grow between 12:02 p.m. and 12:05 p.m. on the same day?
- (a) 2 cm/min
 - (b) 2π cm
 - (c) 6π cm
 - (d) 10 cm
 - (e) 10π cm
7. What is the vertex of the parabola given by the equation $6x^2 - 2y + 48x + 101 = 0$?
- (a) $(4, \frac{5}{2})$
 - (b) $(4, -\frac{5}{2})$
 - (c) $(-4, \frac{5}{2})$
 - (d) $(-\frac{5}{2}, 4)$
 - (e) $(\frac{5}{2}, -4)$

8. Consider the pictured triangle with angles having measures as shown. Which of the following must be true?

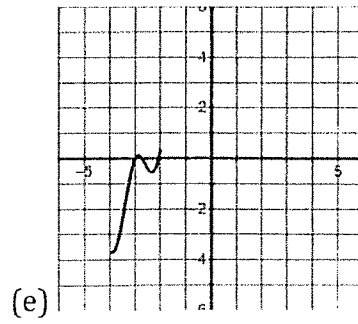
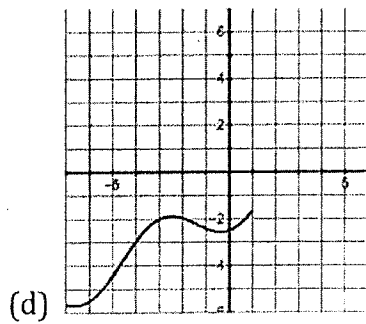
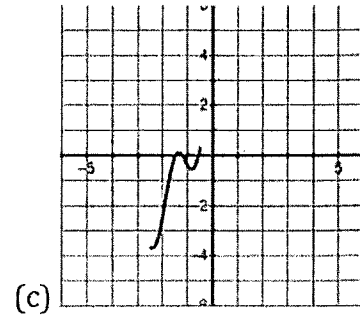
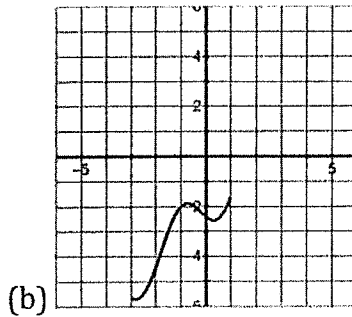
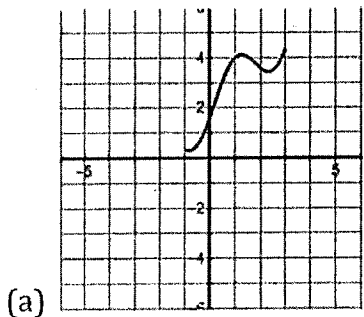
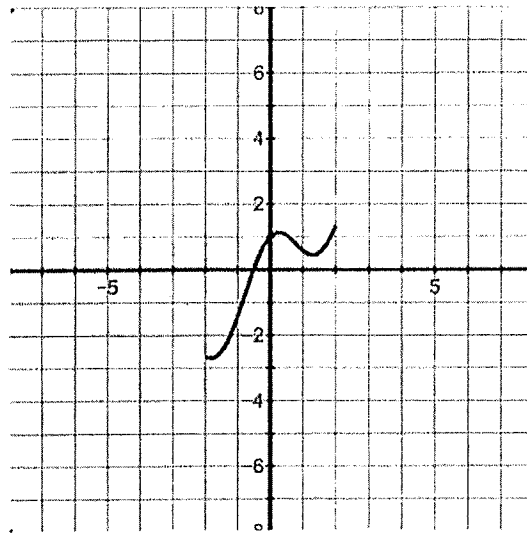
- (a) $a = 60$
- (b) $a = c - 30$
- (c) $a = d$
- (d) $\triangle XYW$ is similar to $\triangle ZWY$
- (e) None of these need be true



9. Consider the plane region R consisting of a rectangle measuring 2π cm by 5π cm together with all of the points inside the rectangle. In square centimeters, what is the area of the plane region consisting of all points whose distance from some point of R is one centimeter or less?

- (a) $10\pi^2 + 7\pi + 1$
- (b) $18\pi^2$
- (c) $10\pi^2 + 15\pi$
- (d) $10\pi^2 + 14\pi$
- (e) $10\pi^2 + 14\pi + 4$

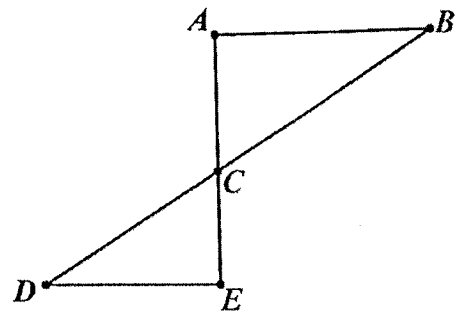
10. Pictured below is the graph of $y = f(x)$ for some function. Which of the choices shows the graph of $y = f(2x + 3) - 1$?



11. If all of the variables in the given expression represent positive real numbers, which of the following is equivalent to

$$(\sqrt{2x})(\sqrt[3]{3y})?$$

- (a) $\sqrt[6]{72x^3y^2}$
 (b) $\sqrt[6]{6xy}$
 (c) $108x^2y^3$
 (d) $xy\sqrt[6]{6}$
 (e) $\sqrt[5]{6xy}$
12. Yanny and Laurel break open their piggy bank and discover that they have 136 coins—all dimes and nickels. If they have a total of \$10, how many nickels do they have?
 (a) 72
 (b) 64
 (c) 36
 (d) 32
 (e) 28
13. How many distinct (non-congruent) isosceles triangles are there with one side 12 cm and one angle 40° ?
 (a) 1
 (b) 2
 (c) 3
 (d) 4
 (e) Infinitely many
14. Consider the figure pictured, in which \overline{BD} intersects \overline{AE} at C . Which of the following sets of hypotheses is NOT sufficient to prove that $\triangle ABC \cong \triangle EDC$?



- (a) C is the midpoint of both \overline{BD} and \overline{AE} .
 (b) C is the midpoint of \overline{AE} , and $\overline{AB} \cong \overline{DE}$.
 (c) $\overline{AE} \perp \overline{AB}$, $\overline{AE} \perp \overline{DE}$, and C is the midpoint of \overline{AE} .
 (d) $\overline{AB} \parallel \overline{DE}$, and $\overline{AB} \cong \overline{DE}$.
 (e) $\angle B \cong \angle D$, and C is the midpoint of \overline{BD} .

15. Which of the following expressions is always equivalent to x ?

(a) $\sqrt{x^2}$

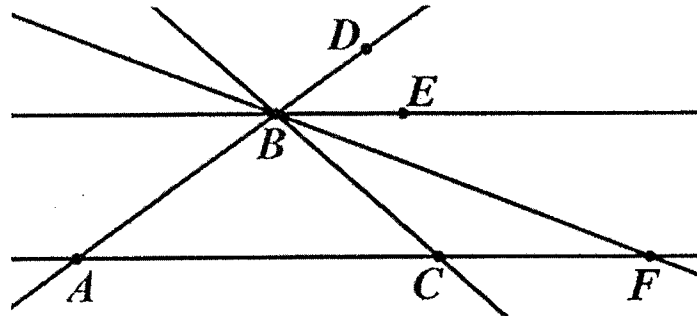
(b) $\frac{x^{9/2}}{x^{7/2}}$

(c) $\frac{x^2 - 2x}{x - 2}$

(d) $\frac{(1+x) - (1-x)}{2}$

(e) These are all equivalent to x .

16. Consider the given diagram, in which $\overline{BE} \parallel \overline{FA}$. If $m\angle DBE = 36^\circ$ and $m\angle BFC = 21^\circ$, what is the measure of $\angle BCA$?



(a) 123°

(b) 57°

(c) 72°

(d) 15°

(e) There is not enough information to answer this question.

17. In the given figure, $AB \perp BC$; $AC \perp CD$; $AD \perp DE$; $AE \perp EF$, and $AB = BC = CD = DE = EF = 1$. What is the length of segment \overline{AF} ?

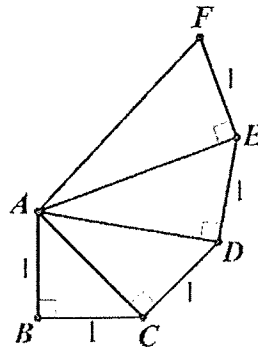
(a) $\sqrt{2}$

(b) 2

(c) $\sqrt{5}$

(d) $2\sqrt{2}$

(e) 3



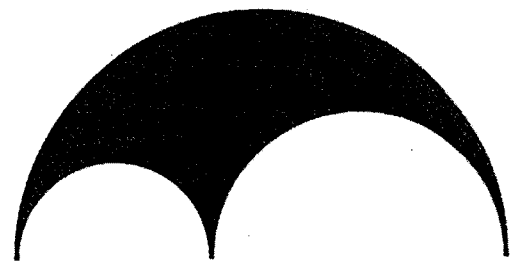
18. Suppose that A is the point $(-1, 2)$ and B is the point $(2, -1)$. What are the coordinates of the point C on line segment \overline{AB} so that $AC = 2CB$?

- (a) $(\frac{1}{2}, \frac{1}{2})$
- (b) $(5, -4)$
- (c) $(1, 0)$
- (d) $(0, 1)$
- (e) $(\frac{2}{3}, -\frac{1}{3})$

19. Laurel gets mad at her phone and throws it from the top of a tall building. Being a stellar mathematics student, she does some calculations, and realizes that at the exact moment that t seconds has elapsed, her phone is $26t$ feet from the building (horizontally) and $-16t^2 + 15t + 34$ feet in the air (vertically). When the phone hits the ground, how many feet is it from the base of the building? (Don't try this at home, kids!)

- (a) 15
- (b) 26
- (c) 39
- (d) 52
- (e) 60

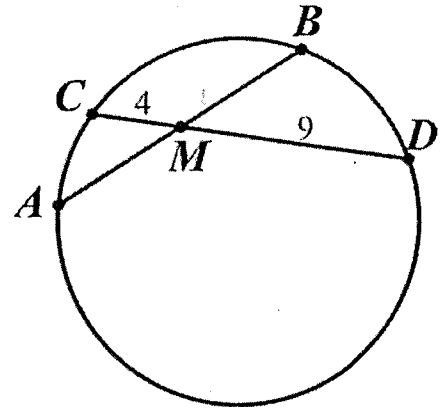
20. The figure shown is known as an *arbelos*, from the Greek for "shoemaker's knife," because it was believed to resemble that tool. It consists of a semicircle with two smaller semicircles removed, designed in such a way that the diameter of each semicircle shares one endpoint with each of the other two. If the large semicircle has diameter 30 mm and the diameters of the two smaller semicircles are in the ratio 3:2, what is the area of the arbelos?



- (a) $432\pi \text{ mm}^2$
- (b) $216\pi \text{ mm}^2$
- (c) $108\pi \text{ mm}^2$
- (d) $54\pi \text{ mm}^2$
- (e) $27\pi \text{ mm}^2$

21. Segments \overline{AB} and \overline{CD} are chords of the same circle, and M is the midpoint of \overline{AB} . If $CM = 4$ inches and $DM = 9$ inches, how long is segment \overline{AB} ?

- (a) 6 inches
 (b) 6.5 inches
 (c) 8 inches
 (d) 12 inches
 (e) 13 inches



22. A triangle inscribed in a circle has sides 10 cm, 24 cm and 26 cm. What is the area of the circle?

- (a) 240 cm^2
 (b) $240\pi \text{ cm}^2$
 (c) $676\pi \text{ cm}^2$
 (d) $169\pi \text{ cm}^2$
 (e) $52\pi \text{ cm}^2$

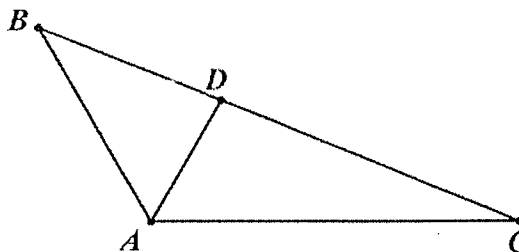
23. A right triangle has area 5 cm^2 and perimeter 12 cm. How long (in centimeters) is its hypotenuse?

- (a) $\sqrt{119}$
 (b) $\frac{41 - \sqrt{241}}{12}$
 (c) $\frac{41 + \sqrt{241}}{12}$
 (d) $\frac{31}{6}$
 (e) $\frac{41}{6}$

24. Two very tall poles stand perpendicular to the ground, one 48 meters tall, the other 80 meters tall. A wire leads from the top of each pole to the bottom of the other; how far above the ground do the wires cross?

- (a) 64 meters
 (b) 48 meters
 (c) 30 meters
 (d) 24 meters
 (e) There is not enough information to solve this problem.

25. In the pictured triangle, $AC = 15$, $AB = 9$, $BC = 21$, and D is a point on \overline{BC} such that \overline{AD} is the angle bisector of $\angle BAC$. What is the length of \overline{AD} ?



- (a) $\frac{45}{8}$
 (b) 8
 (c) $\frac{15}{2}$
 (d) $\frac{9}{2}$
 (e) $3\sqrt{15}$

26. In $\triangle CNU$, $m\angle CNU = 45^\circ$, $m\angle NCU = 30^\circ$, and $CN = 36$ mm. In square millimeters, what is the area of $\triangle CNU$?

- (a) $18\sqrt{3} - 18$
 (b) 324
 (c) 36
 (d) $324\sqrt{3}$
 (e) $324\sqrt{3} - 324$

27. A solid gold sphere with volume 36π cm³ is melted down and re-cast into a right circular cylinder with the same radius as the sphere. What is the height of the cylinder?

- (a) $\frac{4}{\pi}$ cm
 (b) 4 cm
 (c) $\frac{4}{3}$ cm
 (d) 6 cm
 (e) 3 cm

28. A function f whose graph has y -intercept 3 has the interesting property that its average rate of change over the interval $[0, x]$ is simply x . What is the value of $f(7)$?

- (a) 7
 (b) 10
 (c) 21
 (d) 42
 (e) 52

29. Two circular regions with congruent radii of 2 cm are located in the plane so that their centers are 2 cm from each other. In square centimeters, what is the area of their intersection?

(a) $\frac{4}{3}\pi - 4$

(b) $\frac{4}{3}\pi - \sqrt{3}$

(c) $\frac{8}{3}\pi$

(d) $\frac{8}{3}\pi - 2\sqrt{3}$

(e) $\frac{8}{3}\pi - 4$

30. A quadratic function with one root $2 - i$ has a maximum value of -3 . What is the y -intercept of its graph?

(a) 15

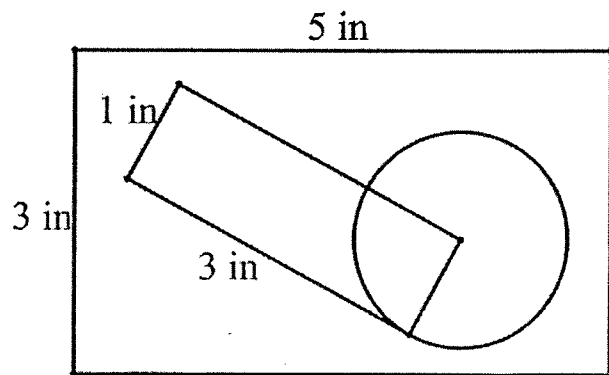
(b) -15

(c) -12

(d) 12

(e) -3

31. The diagram given on the right is a 1×3 rectangle together with a circle with radius 1 inch whose center is at one of the corners of the rectangle, all inside a 3×5 rectangle. Suppose a point is picked at random from the larger rectangle. What is the probability that the point is not in the circle, given that it is not in the smaller rectangle?



(a) $\frac{\pi}{4}$

(b) $1 - \frac{\pi}{4}$

(c) $\frac{3\pi}{4}$

(d) $1 - \frac{\pi}{16}$

(e) $\frac{3\pi}{20}$

32. Suppose that \overline{AC} and \overline{BD} are chords of the same circle intersecting at F . If $m\widehat{AB} = 100^\circ$ and $m\angle AFB = 70^\circ$, what is the measure of \widehat{CD} ?
- (a) 30°
 (b) 40°
 (c) 70°
 (d) 85°
 (e) 170°

33. One morning the Dollywood amusement park opened at 9:00 a.m. Doralee learned from the gate attendants that at 10:00 a.m. there were 300 people in the park. Two hours later, she learned that the count had risen to 1200 people. If Doralee is correct in her assumption that the number y of people in the park after it has been open x hours can be approximately modeled (at least for the first 7 hours) by an exponential function $y = ab^x$, approximately how many people will be in the park at 1:00 p.m.?
- (a) 1500
 (b) 1650
 (c) 2100
 (d) 2400
 (e) 4800

34. The given table depicts some values of a quadratic function $y = f(x)$. What value should replace the question marks?

- (a) 20
 (b) 24
 (c) 28
 (d) 32
 (e) 36

x	y
1	4
2	8
3	16
4	??

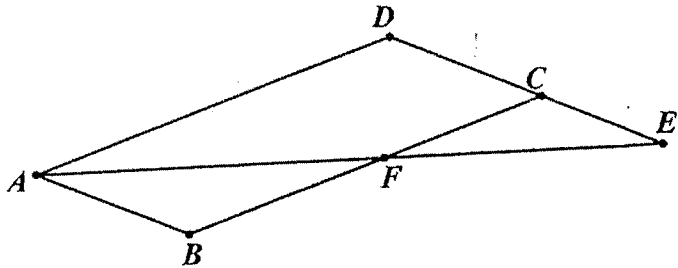
35. If a rectangle has area A and perimeter P , what is the (non-negative) difference between the lengths of its sides?

- (a) $\frac{P}{4} - \sqrt{A}$
 (b) $\frac{P}{2} - \sqrt{A}$
 (c) $\frac{\sqrt{P^2 - 16A}}{2}$
 (d) $\frac{2\sqrt{A - P}}{A + P}$

- (e) There is not enough information to solve this problem.

36. In the given figure, $ABCD$ is a parallelogram, $AB = 16$, $AD = 9$, and $CE = 4$. What is the length of \overline{BF} ?

- (a) 1.8
- (b) 2.75
- (c) 3.2
- (d) 6.25
- (e) 7.2



37. Mrs. Nesbitt needs to pick one more student from either her first or second block class to compete in the TMTA High School Mathematics Competition. Since all of her students are brilliant, she decides to just pick randomly. She knows the following probabilities:

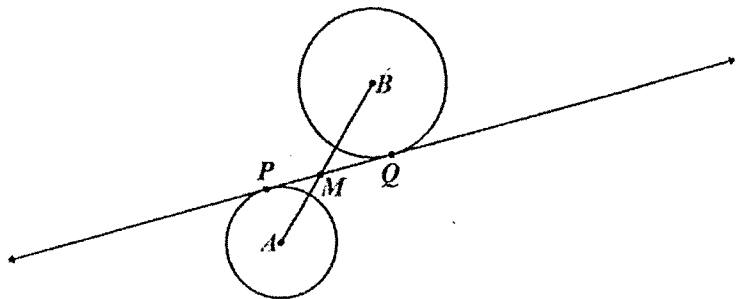
$$\begin{aligned} P(\text{Student is a sophomore}) &= 0.4 \\ P(\text{Student is in her first block class}) &= 0.55 \\ P(\text{Student is a sophomore or in her second block class}) &= 0.66 \end{aligned}$$

Suppose Mrs. Nesbitt picks a student from her first block class. What is the probability that the student is not a sophomore?

- (a) $\frac{34}{55}$
- (b) $\frac{19}{45}$
- (c) $\frac{34}{45}$
- (d) $\frac{19}{55}$
- (e) $\frac{45}{60}$

38. Line \overleftrightarrow{PQ} is tangent to the circles at P and Q , as pictured, and intersects the segment \overline{AB} that joins their centers at M . If $PQ = 9$, the area of $\triangle PAM$ is 6 and the area of $\triangle BQM$ is 24, what is the length of \overline{AB} ?

- (a) 14
- (b) 15
- (c) $9\sqrt{3}$
- (d) 18
- (e) $11\sqrt{3}$



39. The line given by $7x + y = 25$ divides the interior of the circle $x^2 + y^2 = 25$ into two regions; what is the area of the region so formed that does *not* contain the center of the circle?
- (a) $\frac{25\pi}{4}$
- (b) $\frac{25}{4}(\pi - 2)$
- (c) $\frac{25\pi}{2}$
- (d) $\frac{25}{2}(\pi - 2)$
- (e) $25\pi - 50$

40. In the diagram given, \overleftrightarrow{ST} is tangent to the circle with center at P and the circle with center at Q and \overleftrightarrow{PQ} intersects the circle centered at P and the circle centered at Q in points A and B , respectively. If the radius of the circle centered at P is 11, the radius of the circle centered at Q is 3, and $AB = 3$, what is the length of \overleftrightarrow{ST} ?
- (a) 14
- (b) 15
- (c) 17
- (d) 18
- (e) 20

