TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

FIFTY-NINTH ANNUAL MATHEMATICS CONTEST

2015

Geometry

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Scoring formula: 4 x (Number Right) – (Number Wrong) + 40

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

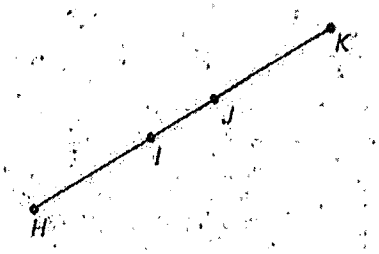
This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you; if you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly eighty minutes to work.

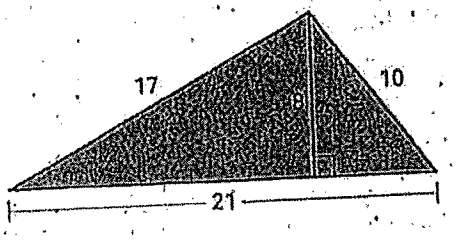
1. In the diagram of the collinear points, HK = 9, IJ = 1, and HI = JK. Find the length of HJ.

a. 5
b. 8
c. 3
d. 10
e. 4



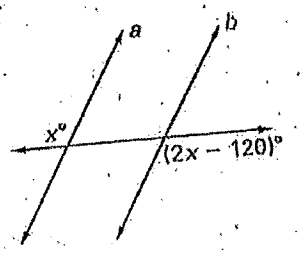
2. Find the perimeter of the right triangle in the diagram below with the smallest area.

a. 31
b. 40
c. 48
d. 24
e. 60



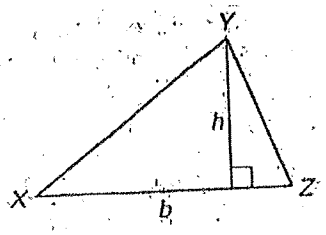
3. Find the value of x if line a ∥ b.

a. 92°
b. 95°
c. 70°
d. 100°
e. 120°

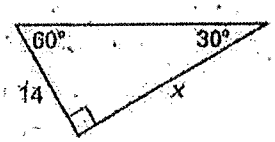


4. Triangle XYZ has an area of 25 square inches. The ratio of the base of ΔXYZ to the height of ΔXYZ is 2:1. Find the base and height of ΔXYZ.

a. base = 5 inches; height = 10 inches
b. base = 10 inches; height = 5 inches
c. base = 12.5 inches; height = 25 inches
d. base = 6 inches; height = 12 inches
e. base = 12 inches; height = 6 inches

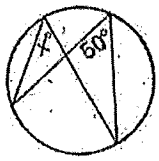


5. Find the value of x.



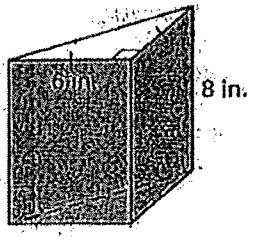
a. 14
b. $\frac{14\sqrt{3}}{3}$
c. 28
d. $14\sqrt{3}$
e. $14\sqrt{2}$

6. Find the value of x.



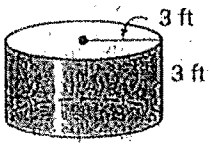
a. 40°
b. 130°
c. 50°
d. 80°
e. 30°

7. Find the surface area, rounded to the nearest whole number, of the right prism.



a. 198 square inches
b. 18 square inches
c. 144 square inches
d. 200 square inches
e. 100 square inches

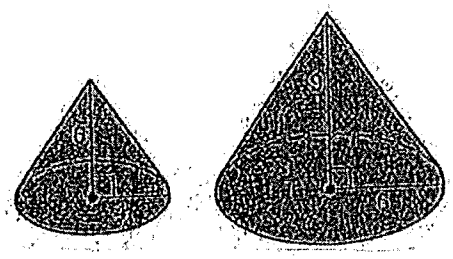
8. Find the volume of the figure below.



a. $54\pi\ \text{ft}^2$
b. $54\pi\ \text{ft}^3$
c. $18\pi\ \text{ft}^2$
d. $18\pi\ \text{ft}^3$
e. $27\pi\ \text{ft}^3$

9. Decide whether the two solids below are similar. If so, find the scale factor.

a. Similar; 2:9
b. Similar; 1:3
c. Similar; 2:3
d. Similar; 8:27
e. Not Similar



10. Which statement is a corollary of the theorem, "The sum of the three interior angles of a triangle is 180°."

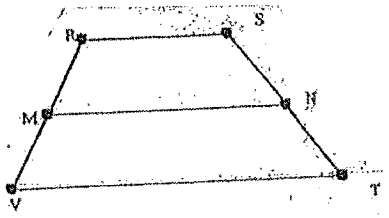
a. Each interior angle of a square is a right angle.
b. Each interior angle of an equiangular triangle measures 60°.
c. The diagonals of a rectangle are congruent.
d. All of the above.
e. None of the above.

11. In a proof, what reason can be cited that justifies the statement $\overline{PQ} \cong \overline{PQ}$?

a. Symmetric Property of Congruence
b. Substitution Property of Equality
c. Transitive Property of Congruence
d. Identity
e. Given/Assumption

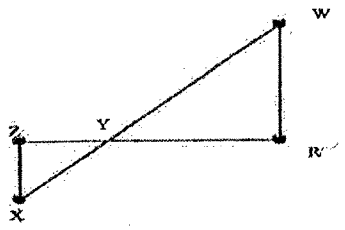
12. For trapezoid RSTV, $\overline{RS} \parallel \overline{VT}$. If M is the midpoint of $\overline{RV}$ and N is the midpoint of $\overline{ST}$, it follows that:

a. $\overline{MN} \parallel \overline{RS}$, $\overline{MN} \parallel \overline{VT}$, and MN = RS + VT
b. $\overline{MN} \parallel \overline{RS}$, $\overline{MN} \parallel \overline{VT}$, and MN = 0.5(RV + ST)
c. $\overline{MN} \parallel \overline{VT}$ and MN = RS + VT
d. $\overline{MN} \parallel \overline{RS}$, $\overline{MN} \parallel \overline{VT}$, and MN = 0.5(RS + VT)
e. None of the above



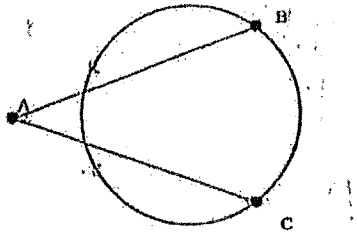
13. If $\overline{XZ} \perp \overline{ZR}$ and $\overline{WR} \perp \overline{ZR}$, which method is used to prove that $\triangle XYZ \sim \triangle WYR$?

a. AA
b. SAS~
c. SAS
d. SSS~
e. SSS



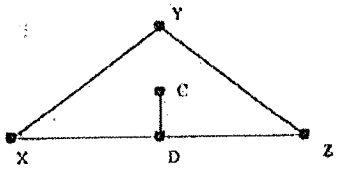
14. If $m\overset{\frown}{BC} = 105°$ and $m\overset{\frown}{RS} = 27°$, find $m\angle BAC$.

a. 39°
b. 44°
c. 78°
d. 88°
e. 66°



15. In isosceles triangle $\triangle XYZ$, XY = YZ = 10, and XZ = 16. Where C is the centroid of $\triangle XYZ$, find the distance from C to side $\overline{XZ}$ of the triangle.

a. 2
b. 2.5
c. 3
d. 4
e. 8



16. In $\odot P$, a sector is bounded by radii $\overline{PX}$ and $\overline{PY}$ and arc $\overset{\frown}{XY}$. If $m\overset{\frown}{XY} = 90°$ and PX = 6 m, find the exact area of the sector.

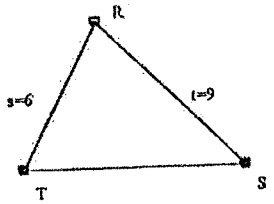
a. $9\pi\ m^2$
b. $12\pi\ m^2$
c. $3\pi\ m^2$
d. $36\pi\ m^2$
e. $18\pi\ m^2$

17. A regular dodecahedron is manufactured as a desk calendar. What is the likelihood that the month facing upward is a month that begins with the letter "J?"

a. $\frac{1}{12}$
b. $\frac{1}{6}$
c. $\frac{1}{4}$
d. $\frac{1}{3}$
e. $\frac{1}{10}$

18. In $\triangle RST$, s = 6, t = 9, and $m\angle S = 38^\circ$. Find $m\angle T$ to the nearest degree.

a. 65°
b. 69°
c. 73°
d. 60°
e. 67°

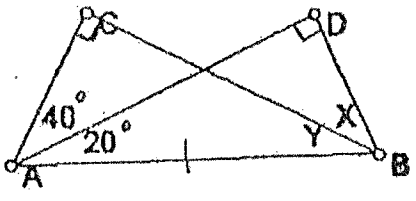


19. If a triangle is scalene, then it cannot be:

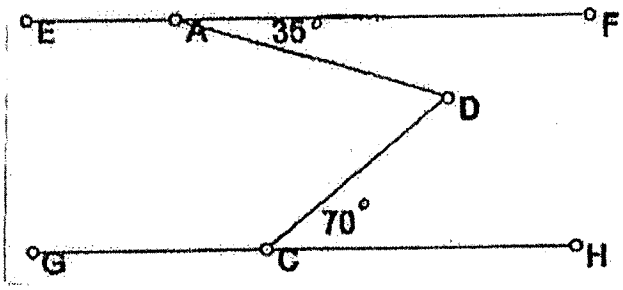
a. Right
b. Obtuse
c. Isosceles
d. Acute
e. Congruent to a Right Triangle

20. Find the value of x and y.

a. X = 20, Y = 60
b. X = 40, Y = 20
c. X = 30, Y = 40
d. X = 40, Y = 30
e. X = 35, Y = 35



21. If $\overline{EF} \parallel \overline{GH}$, then find the m∠D.

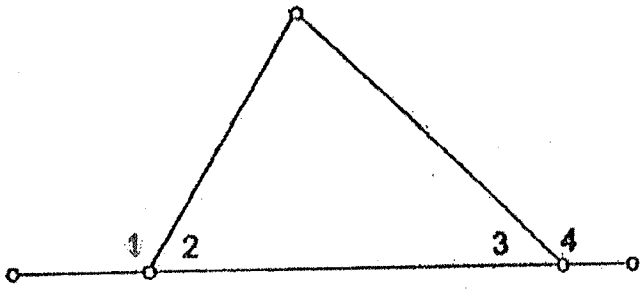


a. 35°
b. 105°
c. 70°
d. 140°
e. 75°

22. What is the measure of each interior angle of a regular decagon?

a. 144°
b. 72°
c. 100°
d. 140°
e. 36°

23. Find the m∠4 if the m∠1 = 3x + 66, the m∠2 = m∠3, and m∠3 = 4x – 40.



a. 154°
b. 22°
c. 124°
d. 48°
e. 132°

24. On a number line point R has coordinate 2 and point S has coordinate 6. A is the midpoint of RS, B is the midpoint of RA, C is the midpoint of RB, and D is the midpoint of RC. Find the coordinate of point D.

a. 2.5
b. 4
c. 2.25
d. 4.25
e. 4.5

25. For any angle D, the difference of the measure of a supplement of angle D and a complement of angle D is a certain number. Find that number.

a. 0
b. 90
c. 180
d. 270
e. 360

26. The coordinates of rectangle RECT are R(-3, -1), E(-3, 2), C(5, 2), and T(5, -1). If a point is chosen at random from inside the rectangle, what is the probability that the point lies in the first quadrant?

a. $\frac{7}{11}$
b. $\frac{1}{4}$
c. $\frac{1}{2}$
d. $\frac{5}{24}$
e. $\frac{5}{12}$

27. What is the converse of the inverse of the contrapositive of $p \rightarrow q$ called?

a. Conditional
b. Converse
c. Inverse
d. Contrapositive
e. Biconditional

28. Solve for x if $\overline{AB}$ is perpendicular to $\overline{BC}$, $\overline{BD}$ bisects $\angle ABC$, $m\angle ABC = x + 5y$, and $m\angle DBC = 2x + 2y + 3$.

a. 3.75
b. 18
c. 7
d. 15
e. 6

29. Your friend's house is located at (-1, 8) and your house is located at (7, 2). Find the shortest distance from your house to your friend's house.

a. $6\sqrt{2}$
b. 6
c. 8
d. 14
e. 10

30. A six-pointed regular star consists of two interlocking equilateral triangles. What is the ratio of the area of the entire star to the area of one of the equilateral triangles?

a. 1
b. 2
c. $\frac{4}{3}$
d. 1.3
e. 5

31. If two rectangles have perimeters of 100cm, the ratio of the length of the longer sides of the rectangles is 4:3, and the ratio of the shorter sides of the rectangles is 1:2, then the ratio of the areas of the rectangles is

a. 1:1
b. 1:2
c. 2:3
d. 3:4
e. 4:6

32. The ratio of two complementary angles is 7:2. What are the angle measures?

a. 10° and 80°
b. 20° and 70°
c. 40° and 140°
d. 10° and 170°
e. 12° and 78°

33. Tom is standing in a hole that is 4 feet deep. Bill asks him how much deeper he is going to dig the hole. Tom replies that he will dig 4ft 2in deeper and that the top of his head will then be the same distance below ground level that it is now above ground level. How tall is Tom?

a. 7 feet, 2 inches
b. 6 feet, 1 inch
c. 5 feet, 8 inches
d. 6 feet
e. 5 feet

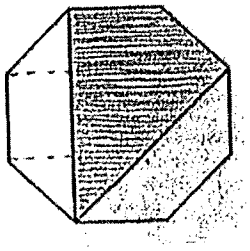
34. Let PQRS be a square piece of paper. P is folded onto R, and then Q is folded onto S. The area of the resulting figure is 9 square inches. Find the perimeter of square PQRS.

a. 6 inches
b. 36 inches
c. 24 inches
d. 12 inches
e. $12 + 6\sqrt{2}$

35. A particular bike wheel makes 1056 revolutions in one mile. What is the radius of the wheel in feet?

a. $\frac{2}{\pi}$ feet
b. $\frac{5}{\pi}$ feet
c. $\frac{5}{4\pi}$ feet
d. $\frac{5}{2\pi}$ feet
e. π feet

36. Suppose one vertex angle of a regular octagon is trisected (as shown in the figure below). Each side of the octagon measures 4 units. Find the area of the shaded region.



a. 4
b. $4(4 + 4\sqrt{2})$
c. 16
d. $12\sqrt{2}$
e. $32\sqrt{2}$

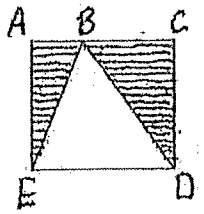
37. If a rectangular field is 100 meters long, what is the smallest size paper you need to draw a map of the field on a scale of 1:1000?

a. 0.001 meters
b. 0.01 meters
c. 0.1 meters
d. 1 meter
e. 10 meters

38. A stone is thrown into a cylindrical water tank that is 2 feet in diameter, causing the water to rise 1.5 inches. What is the volume of the stone?

a. 864π in^3
b. π in^3
c. 4π in^3
d. 216π in^3
e. The volume can't be determined

39. If ABCDE is a square whose edge measures 12 cm, what is the area of the shaded region?



a. 144 cm^2
b. 217 cm^2
c. 108 cm^2
d. 0.5 cm^2
e. 72 cm^2

40. If a jar of peanut butter that is 3 inches in diameter and 4 inches high sells for 60 cents, what is a fair price for a jar that is 6 inches in in diameter and 6 inches high?

a. \$0.02
b. \$0.10
c. \$0.30
d. \$3.00
e. \$3.60