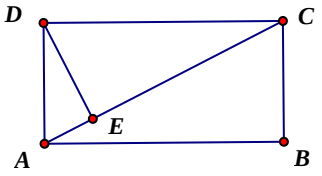
Geometry Contest 2013

1. One pizza has a diameter twice the diameter of a smaller pizza. What is the ratio of the area of the larger pizza to the area of the smaller pizza? A) 4 to 1 B) 2 to 1 C) π to 1 D) 1 to $\sqrt{2}$ E) 2π to 1

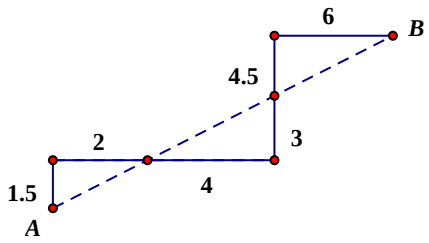
2. In rectangle *ABCD* diagonal *AC* is perpendicular to *DE* at *E*. If *AE = 3* and *DE = 4*, find the area of the rectangle.

A) 50/3
B) 16/3
C) 25
D) 12
E) 100/3



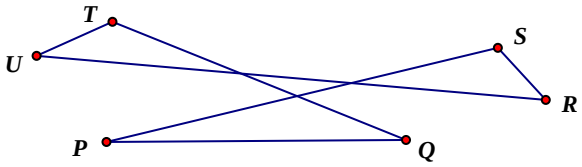
3. In the figure are three similar right triangles with measures of legs as indicated. Find the length of dotted segment *AB*. Assume all measurements are in centimeters (cm).

A) $\sqrt{87.5}$ cm
B) $\sqrt{21}$ cm
C) 21 cm
D) 15 cm
E) 10.5 cm



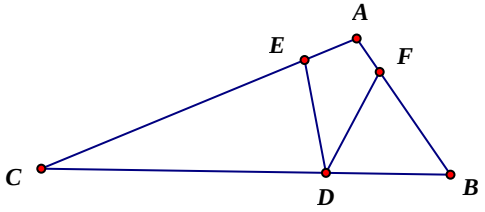
4. The figure is composed of multiple non-congruent and non-parallel line segments which intersect each other with non-congruent angles as shown. Find the sum of the measures of the angles P, Q, R, S, T, and U.

A) 180 degrees
B) 270 degrees
C) 360 degrees
D) 540 degrees
E) 720 degrees



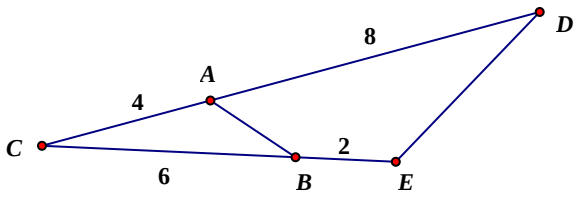
5. If $m\angle A$ = 80 degrees, *CE = CD,* and *BF = BD*, find $m\angle EDF$ in the figure.

A) 40 degrees
B) 50 degrees
C) 100 degrees
D) 130 degrees
E) 140 degrees



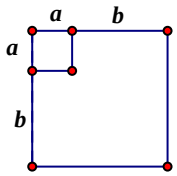
6. If the area of $\triangle ABC$ is *3* square units, and the lengths are given on the figure, then find the area of $\triangle CDE$.

A) 6
B) 8
C) 12
D) 16
E) 20



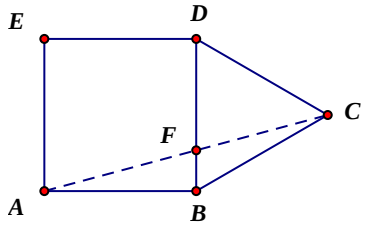
7. A small square with a side of length *a* is placed at the corner of a larger square with a side of length *a+b*. If the area of the small square is 1/9 of the larger square, then find the ratio of *a* to *b*.

A) 1 to 2
B) 1 to 3
C) 1 to 4
D) 1 to 8
E) 1 to 9



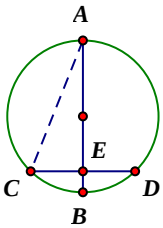
8. Pentagon *ABCDE* is formed by adjoining square *ABDE* and equilateral $\triangle BCD$. Diagonals *AC* and *BD* intersect at *F*. Find $m\angle AFB$.

A) 60 degrees
B) 67.5 degrees
C) 72 degrees
D) 75 degrees
E) 108 degrees



9. Diameter *AB* of length *10* is perpendicular to chord *CD* of length *6* at point *E*. Find the length of the longest chord *AC* joining their endpoints.

A) $\sqrt{10}$ units
B) $\sqrt{34}$ units
C) 8 units
D) $3\sqrt{10}$ units
E) 28 units

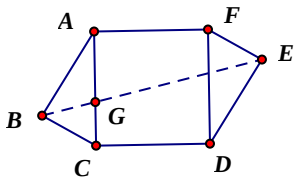


10. A rectangle has sides of length *6* and *8* units. Find the distance from a vertex to the diagonal through two other vertices.

A) 3.6 units B) 4.8 units C) $\sqrt{10}$ units D) 7 units E) $4\sqrt{10}$ units

11. Congruent triangles are constructed on opposite sides of a square as shown below. If $m\angle BAC$ = 25 degrees, $m\angle BCA$ = 65 degrees, and BC is parallel to FE, then find $m\angle AGE$.

A) 20 degrees
B) 45 degrees
C) 70 degrees
D) 110 degrees
E) 160 degrees

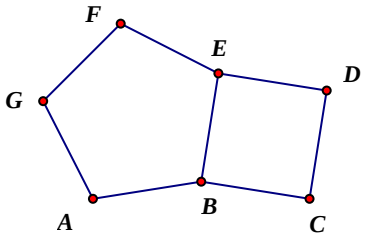


12. Diagonals *AD* and *CE* of regular pentagon *ABCDE* intersect at *F*. Find $m\angle$ AFC.

A) 36 degrees B) 72 degrees C) 90 degrees D) 108 degrees E) 144 degrees

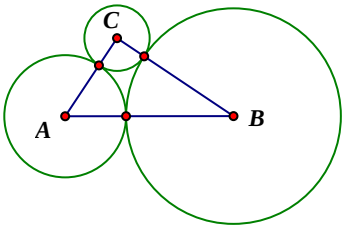
13. A square and a regular pentagon share a side as shown. Find $m\angle ACD$ (not drawn in figure).

A) 90 degrees
B) 99 degrees
C) 100 degrees
D) 108 degrees
E) 162 degrees



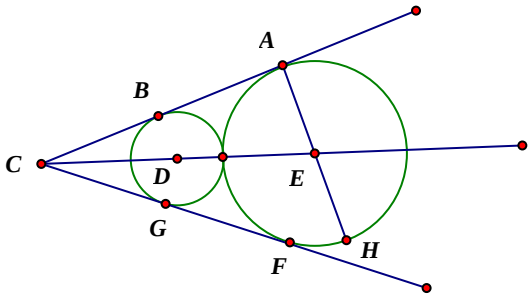
14. Three circles with radii of *3, 5*, and *x* units are mutually tangent in pairs where *x* is the radius of the largest circle. If the lines of centers of these circles are also sides of a right triangle, then find the radius of the largest circle.

A) 4 units
B) 8 units
C) 10 units
D) 12 units
E) 48 units



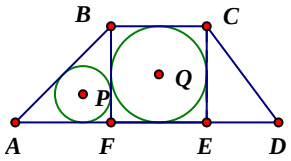
15. Two tangent circles with centers *D* and *E* are also tangent to the sides of a 40 degree angle (i. e., $\angle ACF$) as shown. If segment *AE* is extended to meet the circle again at *H*, find $m\angle FEH$.

A) 20 degrees
B) 40 degrees
C) 70 degrees
D) 100 degrees
E) 140 degrees



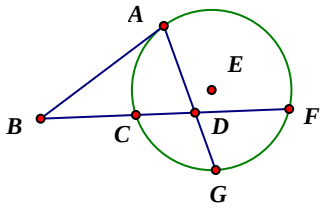
16. Quadrilateral *ABCD* is a trapezoid with circle *P* inscribed in right $\angle$ *ABF* and circle *Q* inscribed in square *BCEF*. If the two isosceles sides of the trapezoid are *5* units each and the area of the square is *16* square units, find the perimeter of the trapezoid.

A) 16 units
B) 18 units
C) 20 units
D) 22 units
E) 24 units



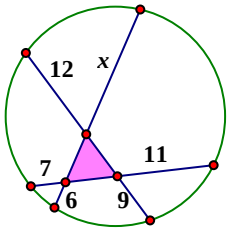
17. Find the length of tangent *AB* for the circle with center *E* if *BC* = 12, *CD* = 8, *AD* =12, and *DG* = 10.

A) $4\sqrt{6}$
B) 20
C) $2\sqrt{105}$
D) $4\sqrt{34}$
E) $\sqrt{673}$



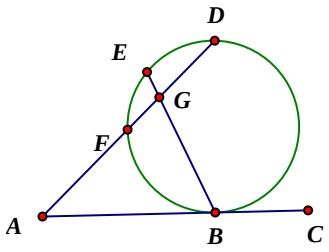
18. The sides of an equilateral triangle *inside* a circle have its sides extended to meet the circle with the extended sides having measurements as shown. Find *x*.

A) 12.5
B) 13
C) 13.5
D) 14
E) 15



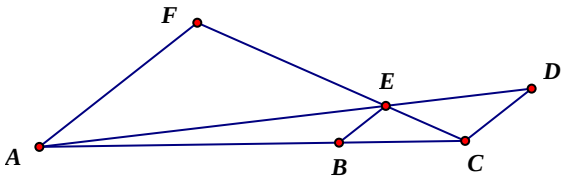
19. In the figure, arc *FE* = 20 degrees, arc *ED* = 50 degrees, arc *DB*= 180 degrees, and segment *AC* is tangent to the circle at B. Segment *AD* intersects the circle at *F* and intersects *BE* at *G*. Find $m\angle A$.

A) 25 degrees
B) 35 degrees
C) 40 degrees
D) 45 degrees
E) 55 degrees



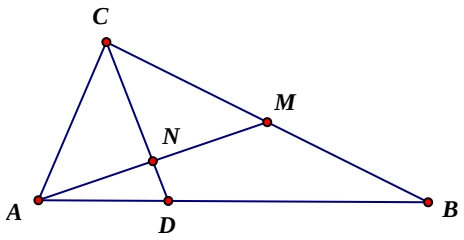
20. Segments *AF*, *BE*, and *CD* are all parallel to each other, segments *AD* and *FC* intersect at *E*, and *B* is a point on *AC*. If *AF = 6* and *CD = 3*, find *BE*.

A) 1
B) $\sqrt{2}$
C) 1.5
D) 2
E) 2.5



21. In scalene $\triangle ABC$, M is the midpoint of BC, N is the midpoint of AM, and D is the intersection of ray CN and AB. Find the ratio of CN to ND.

A) 2 to 1
B) 2.5 to 1
C) 3 to 1
D) 3.5 to 1
E) 4 to 1



22. A point P is 13 units from the center C of a circle. If the diameter of the circle is 10 units, find the length of a tangent segment from P to the circle.

A) $2\sqrt{10}$ units B) $\sqrt{65}$ units C) 12 units D) 18 units E) $\sqrt{194}$ units

23. If two triangles are similar, then what is the maximum number of pairs of parts of the triangles which can be congruent and yet the triangles are not congruent?

A) 2 B) 3 C) 4 D) 5 E) 6

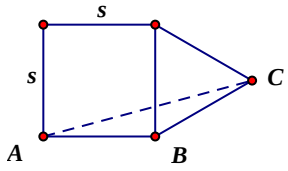
24. Let r be the radius of a circle inscribed in a right triangle with hypotenuse of length c and legs of lengths a and b. Find a formula for r in terms of a, b, and c.

A) $r = \dfrac{a + b - c}{2}$ B) $r = \dfrac{c}{2}$ C) $r = \dfrac{\sqrt{a^2 + b^2}}{2}$ D) $r = \dfrac{\sqrt{a^2 + b^2}}{4}$

E) $r = \dfrac{a^2 + b^2 - c^2}{4}$

25. A pentagon is formed by an adjoining square and an equilateral triangle as shown. If each side of the square has length s, find the area of $\triangle ABC$.

A) $\dfrac{s^2}{2}$ B) $\dfrac{s^2}{4}$ C) $s^2\sqrt{3}$
D) $\dfrac{s^2\sqrt{3}}{2}$ E) $s^2\sqrt{6}$



26. In some quadrilateral $ABCD$, diagonals AC and BD intersect at E and AD is parallel to BC. If $AE = 12$, $BE = 10$, and $CE = 8$, then find DE.

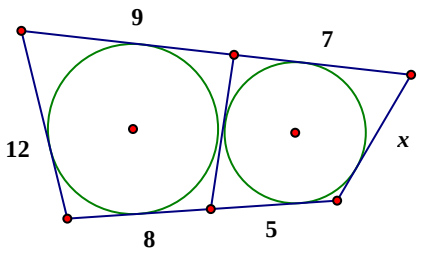
A) 8 B) 9.6 C) 10 D) 15 E) 20

27. If $\triangle$ DEF is formed by extending the sides of $\triangle$ ABC outward such that $AB = BE = 5$, $BC = CF = 3$, and $CA = AD = 4$, then find the ratio of the area of $\triangle$ DEF to the area of $\triangle$ ABC.

A) 3 to 1 B) 4 to 1 C) 5 to 1 D) 6 to 1 E) 7 to 1

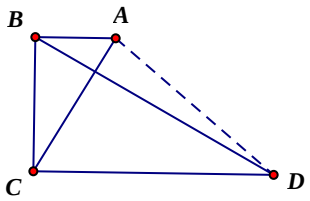
28. In the figure two circles are tangent to each other and are inscribed in quadrilaterals with the measurements as shown. Find the length of tangent segment x. (The figure is not to scale.)

A) 6 units
B) 7 units
C) 8 units
D) 9 units
E) 10 units



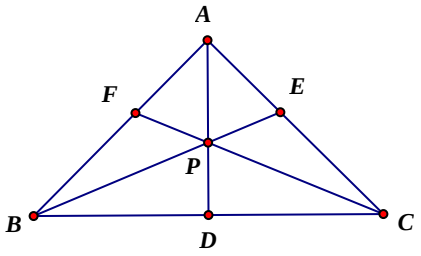
29. Given: $\triangle$ ABC and $\triangle$ BCD are similar with $m\angle ABC = m\angle BCD = 90$ degrees. If $m\angle BAC = 60$ degrees and $AB = 1$ unit, then find AD.

A) 3 units
B) 4 units
C) $2\sqrt{3}$ units
D) $3\sqrt{2}$ units
E) $\sqrt{7}$ units



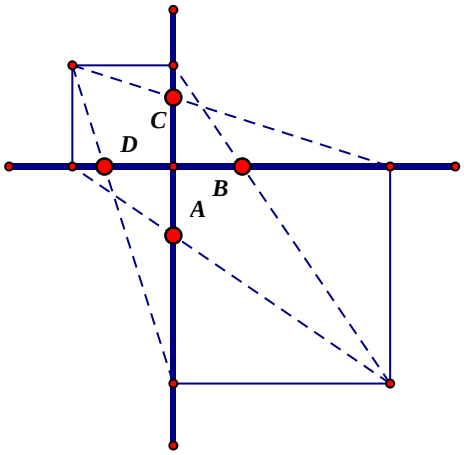
30. Let $\triangle$ ABC be an isosceles right triangle with hypotenuse BC. Let AD be a median and BE be an angle bisector which meet at P. If ray CP intersects AB at F, then find AF/FB.

A) $\frac{\sqrt{3}}{2}$ B) $\frac{1}{2}$ C) 1
D) $\frac{1}{\sqrt{3}}$ E) $\frac{1}{\sqrt{2}}$



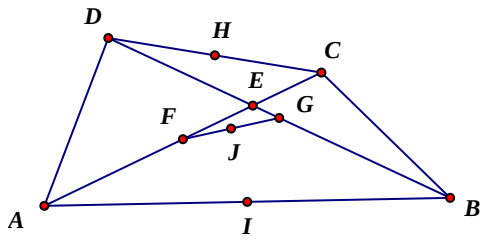
31. Two unequal squares in quadrants II and IV share a common vertex at the origin of a rectangular coordinate system. Segments (dotted) from the remote vertex of one square join with the remaining vertices of the other square and meet the coordinate axes at *A, B, C,* and *D* as shown. What kind of quadrilateral is *ABCD*?

A) a square
B) a rectangle which is not a square
C) a kite which is not a square
D) a rhombus which is not a square
E) an isosceles trapezoid which is not a square



32. In quadrilateral *ABCD* sides *AD* and *BC* are congruent. The diagonals meet at *E*. Points *F, G, H, I,* and *J* are the respective midpoints of *AC*, *BD*, *CD*, *AB*, and *FG*. Which of the following statements are not always true for these conditions? (The figure is not necessarily to scale.)

A) FE = EC
B) H, J, and I are collinear
C) HI is perpendicular to FG
D) FI = HG
E) J is the midpoint of HI

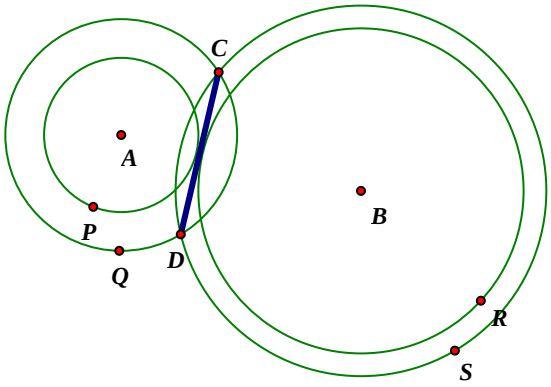


33. Let *P* be a point in the interior of $\triangle ABC$ such that $m\angle ABC = \frac{1}{2}\, m\angle APC$, $m\angle BCA = \frac{1}{2}\, m\angle BPA$, and $m\angle CAB = \frac{1}{2}\, m\angle CPB$, then which statement must follow?

A) Point P is the centroid of the triangle. B) Point P is the circumcenter of the triangle. C) Point P is the incenter of the triangle. D) Point P is the orthocenter of the triangle. E) No such point exists.

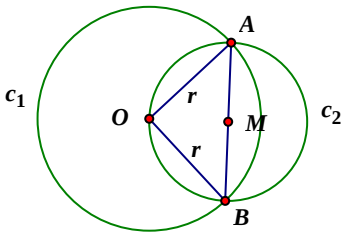
34. In the figure there are two pairs of concentric circles with centers *A* and *B*. The segment *CD* is a common chord for the two larger circles. The segment *CD* is also tangent to the two smaller circles. If radii *BS = 15, BR = 9,* and *AQ = 13,* find *AP*.

A) 5 units
B) 6 units
C) 9 units
D) 12 units
E) 24 units



35. In circle c_1 with radius *r* a smaller circle c_2 is constructed using one of the quadrants of circle c_1 and forming $\sqcup$ *AOB*. Find the square units for the area of the crescent inside the smaller circle but outside the larger circle.

A) $\frac{\pi r^2}{4}$ B) $\frac{\pi r^2\sqrt{2}}{4}$ C) $\frac{r^2}{2}$
D) $\frac{r^2(\pi-2)}{4}$ E) $\frac{\pi r^2(\sqrt{2}-1)}{4}$

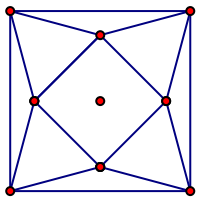


36. Adjacent sides of a parallelogram are one unit and two units. If they meet at 60 degrees, find the difference of the lengths of their diagonals.

A) 1 unit B) 4 units C) $\sqrt{3}$ units D) $\sqrt{7}$ units E) $\sqrt{7}-\sqrt{3}$ units

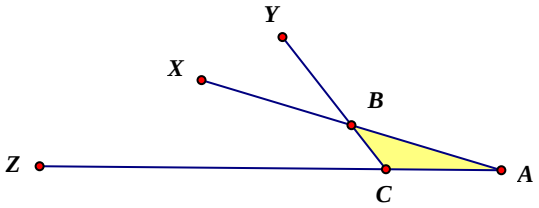
37. An equilateral triangle is constructed inward from each vertex of a square whose sides meet at the vertices of a smaller square. Find the ratio of the area of the original square to the area of the smaller square.

A) $2+\sqrt{3}$ to 1
B) $2-\sqrt{3}$ to 1
C) $\sqrt{3}$ to 1
D) 3 to 1
E) $\sqrt{3}+1$ to 1



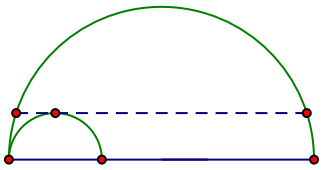
38. In $\triangle ABC$ $AB = 10$, $BC = 8$, $CA = 9$. If B is the midpoint of AX, ZC is 3 times CA, and X, Y, and Z are collinear, find BY.

A) 12
B) 13
C) 14
D) 15
E) 16



39. In the figure the dotted segment is a chord of length 16 which is tangent to the smaller semicircle and also parallel to the diameter of the larger semicircle. Find the area of the region inside the larger semicircle that is outside the smaller semicircle.

A) 4π
B) 8π
C) 16π
D) 20π
E) 32π



40. A rope is stretched around two pulley wheels as shown. If the distance between the centers of the wheels is 10 inches and the radii of the wheels are 2 inches and 7 inches respectively. Find the length of the rope.

A) $9\pi + 10$ inches
B) $9\pi + 20$ inches
C) $32\pi + 10$ inches
D) 10π inches
E) $\frac{32}{3}\pi + 10\sqrt{3}$ inches