FIFTIETH ANNUAL MATHEMATICS CONTEST
sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

Geometry 2006

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Scoring formula: 4R - W + 40

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft head (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly 80 minutes to work.

Contributors to TMTA for the Annual Mathematics Contest:

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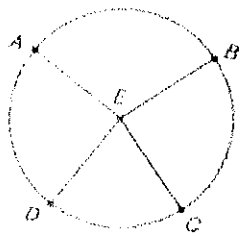
1. What is the measure of the angle formed by the hands of the clock when it is 4:30 p.m.?

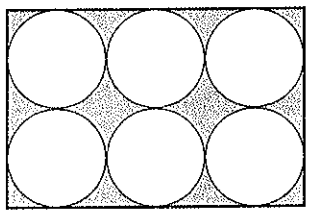
A. 15° B. 30° C. 45° D. 60° E. 42.5°

2. In the given circle with center E, $m\angle AEB = (4x+18)^\circ$, $m\angle BEC = (5x+4)^\circ$, $m\angle CED = (3x+4)^\circ$, and $m\angle AED = (5x-6.)^\circ$ Find x.



A. 10 B. 20 C. 30 D. 5 E. 17

3. Consider six circles which are internally tangent to a rectangle and pairwise tangent to each other as shown below. The dimensions of the rectangle are 6" by 4". How many square inches is the area of the shaded region (outside the circles and inside the rectangle)?



A. $24-24\pi$ B. $24-6\pi$ C. $24-3\pi$ D. $12-6\pi$ E. $24-12\pi$

4. The distance between the centers of two tangent circles with radii r_1 and r_2 is always: (Let $r_1 > r_2$)

A. $r_1 + r_2$ B. $r_1 - r_2$ C. $|r_1 - r_2|$ D. $|r_1| - |r_2|$ E. None of these

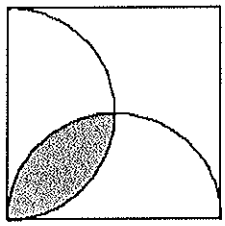
5. Write the equation of the line which is the perpendicular bisector of the segment whose end points are (4,-6) and (8,0).

A. $x+3y-3=0$
B. $2x+6y-6=0$
C. $4x-6y-6=0$
D. $2x+3y-3=0$
E. $2x-6y-6=0$

6. The measures of one interior angle of a regular polygon is 135°. The polygon is a(n):

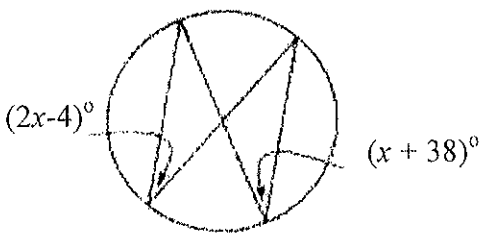
A. pentagon C. heptagon E. decagon
B. hexagon D. octagon

7. Consider the square below. Two semicircles are drawn each passing through 2 vertices and the center of the square whose sides have length 4 units. How many square units is the area of the shaded region?



A. $\pi + 2$ B. $\pi + 4$ C. $2\pi - 4$ D. $2\pi - 2$ E. $2\pi + 2$

8. Find the value of x.

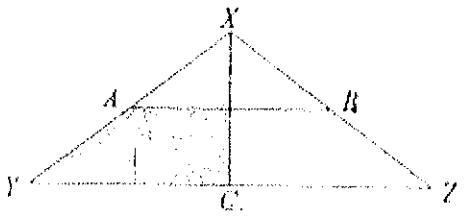


A. 42 B. 37 C. 47 D. 52 E. 45

9. The diagonal of a rectangle has length 10 inches and the perimeter is 28 inches. How many square inches is the area of the rectangle?

A. 12 B. 16 C. 24 D. 36 E. 48

10. The area of triangle XYZ is 8 square inches. Points A and B are midpoints of congruent segments $\overline{XY}$ and $\overline{XZ}$. The altitude is $\overline{XC}$. The area (in square inches) of the shaded region is:



A. $1\frac{1}{2}$ B. 2 C. $2\frac{1}{2}$ D. 3 E. $3\frac{1}{2}$

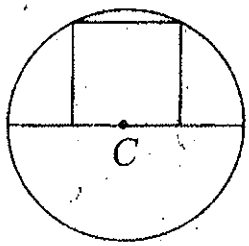
11. A square and an equilateral triangle each have a perimeter of 1 meter. Let A be the area of the circle circumscribed about the square and let B be the area of the circle circumscribed about the triangle. Find the ratio A/B.

A. $\frac{9}{16}$ B. $\frac{3}{4}$ C. $\frac{27}{32}$ D. $\frac{3\sqrt{6}}{8}$ E. 1

12. A triangle and a square have the same base and equal areas. If the length of the common base is x and the height of the triangle is h, what is the height of the triangle in terms of x?

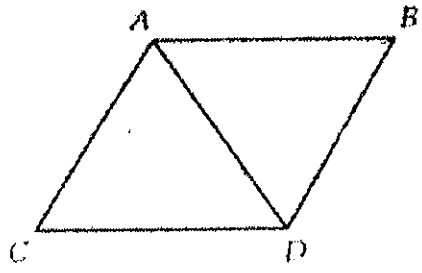
A. $\frac{x}{4}$ B. $\frac{2x}{3}$ C. $2x$ D. $\frac{3x}{2}$ E. $4x$

13. A square of maximum area is inscribed in a semicircle of radius R as shown. What is the area of the square?



A. $\frac{3}{5}R^2$ B. $\frac{2\sqrt{5}}{5}R$ C. $\frac{4}{5}R^2$ D. $\frac{16}{25}R^4$ E. $\frac{\sqrt{5}}{5}R$

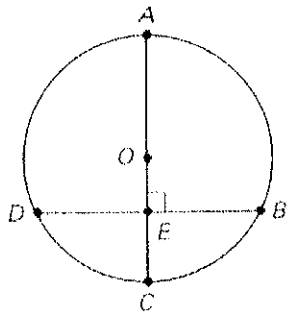
14. If $ABDC$ is a rhombus and ABD is an equilateral triangle, and AD = 4 inches, what is the area of the rhombus?



Note: Figure may not be drawn to scale.

A. $4\sqrt{2}$ in^2 B. $4\sqrt{3}$ in^2 C. $\frac{16}{\sqrt{3}}$ in^2 D. $8\sqrt{3}$ in^2 E. 16 in^2

15. A round table top with center O has these dimensions . $\overline{OE}$ is 7 in., $\overline{DB}$ is 48 in., and $\overline{DB} \perp \overline{AC}$. Find the length of $\overline{AC}$.



A. 14 inches B. 25 inches C. 50 inches D. 28 inches E. 38 inches

16. An isosceles trapezoid has vertices at (-2, 1), (2, 1), (5, -1), and (-5, -1). Find the measure of each diagonal.

A. $\sqrt{53}$ B. 10 C. 4 D. $\sqrt{13}$ E. 20

17. The diagonals of a quadrilateral *MNOP* intersect at *X*. Which statement guarantees that *MNOP* is a rectangle?

A. $MO = NP$
B. $\angle PMN \cong \angle MNO \cong \angle NOP$
C. $\underline{MX} = \underline{NX} = OX = PX$
D. $MO \perp NP$
E. Each pair of consecutive angles is supplementary

18. In ΔRST, $\overrightarrow{SU}$ is the perpendicular bisector of $\overline{RT}$ and *U* lies on $\overline{RT}$. Which statement(s) must be true?

I. ΔRST is equilateral.
II. $\Delta RSU \cong \Delta TSU$
III. The ray $\overrightarrow{SU}$ is the bisector of $\angle RST$

A. I only B. II only C. III only D. II and III only E. I, II and III

19. In ΔABC, the measures of $\angle A$, $\angle B$ and $\angle C$ are in the ratio 2: 5: 5 respectively.
$m\angle B =$ _____.

A. 75° B. 60° C. 30° D. 40° E. 100°

20. Find the geometric mean of $3x$ and $3y$.

A. $3\sqrt{xy}$ B. $\sqrt{3xy}$ C. $3\sqrt{x+y}$ D. $\sqrt{3(x+y)}$ E. $9xy$

21. $\overline{AB}$ and $\overline{AC}$ are tangent to circle O at B and C. If the measure of minor arc BC is x, then $m\angle BAC =$_____.

A. x B. $180 - x$ C. $360 - x$ D. $180 + x$ E. $\frac{1}{2}x$

22. The locus of the centers of all 8cm chords in a circle of radius 5 cm is a ___.

A. point B. segment C. line D. ray E. circle

23. The diagonal of a rectangle is equal to the sum of the length of the shorter side and one-half the length of the longer side. Find the ratio of the length of the shorter side to the length of the longer side.

A. 2:3 B. 3:4 C. 1:4 D. 1:2 E. 2:5

24. Which of the following is not a Platonic Solid?

I. hexahedron
II. icosahedron
III. octahedron
IV. dodecahedron

A. I and III B. only I C. only V D. III & V E. None of these

25. In the right triangle ABC, D is the mid point of AB and $\overline{CE} \perp \overline{AB}$ with point E on AB. The hypotenuse is AB. If $CD = 4$ inches and $BE = 5$ inches, then CE is:

A. $\sqrt{41}$ inches B. 3 inches C. $2\sqrt{10}$ inches D. $\sqrt{15}$ inches E. None of these