FORTY-EIGHTH ANNUAL MATHEMATICS CONTEST
sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

Geometry 2004

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Scoring formula: 4R – W + 40

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the <u>best</u> answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft head (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase <u>completely</u>. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly 80 minutes to work.

1. In triangle PQR, the following facts are given:

    i. S lies on $\overline{PQ}$

    ii. $PR = PQ$

    iii. the measure of angle Q is 50°

    iv. $\overline{RS}$ bisects angle PRQ.

    The measure of angle SRQ is equal to

    (a) 10°   (b) 20°   (c) 25°   (d) 30°   (e) 50°

2. If two angles of a quadrilateral are supplementary, the other two angles are

    (a) Acute   (b) Obtuse   (c) Complementary   (d) Supplementary   (e) Equal

3. A circle centered at O has chord $\overline{AB}$ which is 10 inches long and 12 inches from O. What is the length, in inches, of the radius of the circle?

    (a) $\sqrt{244}$ in   (b) 10 in   (c) 12 in   (d) 13 in   (e) $\sqrt{136}$ in

4. Which of the following statements concerning the diagonals of a rectangle is always true?

    (a) The diagonals are of equal length.

    (b) Adjacent angles formed by the diagonals are equal.

    (c) Opposite angles formed by the diagonals are supplementary.

    (d) Adjacent angles formed by the diagonals are complementary.

    (e) The diagonals are perpendicular.

5. In the figure, if $CA = CB$ and $ED = EB$, then which of the following can be concluded?

    (a) $\triangle ABC$ is equilateral

    (b) $\triangle ABC$ is scalene

    (c) $\overline{CA} \parallel \overline{ED}$

    (d) E bisects $\overline{BC}$

    (e) D bisects $\overline{AB}$
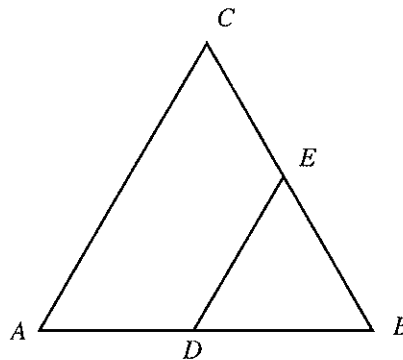
6. Which of the following should be proved equal in order to show that two parallelograms are congruent?

   (a) One pair of corresponding sides

   (b) One pair of corresponding angles

   (c) Two pairs of adjacent sides

   (d) Two pairs of adjacent sides and the included angles

   (e) Two pairs of diagonals

7. If a straight line is drawn from one vertex of an octagon to another vertex, which of the following pairs of polygons could be produced?

   (a) Two quadrilaterals

   (b) A quadrilateral and a heptagon

   (c) A quadrilateral and a hexagon

   (d) A triangle and a hexagon

   (e) A triangle and a pentagon

8. If the hypotenuse of a right triangle is 30 inches long and one angle measures $30°$, then one leg must have a length of

   (a) $\sqrt{30}$ in　　　(b) 10 in　　　(c) 20 in　　　(d) $\sqrt{20}$ in　　　(e) 15 in

9. If triangle ABC is inscribed in a circle of diameter 10 and angle A is acute, then what can be concluded about the length of $\overline{BC}$?

   (a) $BC < \sqrt{10}$　　(b) $BC < 6$　　(c) $BC < 8$　　(d) $BC < 10$　　(e) $BC > 5$

10. In a quadrilateral $ABCD$, $AB = BC$, $CD = AD$, and $AB \neq CD$. Which statement is false?

    (a) $\overline{AC}$ bisects $\overline{BD}$.

    (b) $\overline{BD}$ bisects $\overline{AC}$.

    (c) $\overline{AC}$ and $\overline{BD}$ are perpendicular.

    (d) $\overline{AC}$ and $\overline{BD}$ have different lengths.

    (e) $m\angle BAD = m\angle BCD$

11. A point P is located 9 cm from a plane m. The set of points in m that are exactly 12 cm from P forms a(n)

   (a) hyperbola　　(b) point　　(c) line segment　　(d) parabola　　(e) circle

12. The line segments forming two sides of a regular pentagon are extended to that they meet at point C as shown in the figure. What is the measure of the angle at C?

   (a) 24°
    (b) 36°
    (c) 54°
    (d) 60°
    (e) 72°
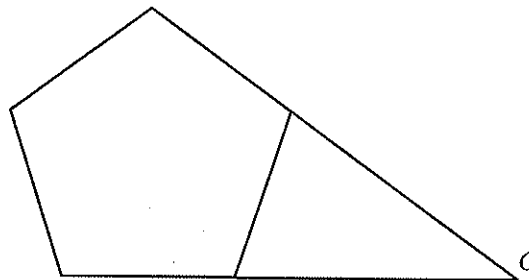
13. Line ℓ intersects a plane m to form an oblique angle. The number of planes that contain ℓ and are perpendicular to m is

    (a) exactly 0
    (b) exactly 1
    (c) exactly 2
    (d) an infinite number
    (e) not determined by this information

14. A circle with radius r is centered at point C, and A and B are two points on the circle. Which of the following statements is *not* true?

    I. If $AB = r\sqrt{2}$ then triangle ACB is a right triangle.

    II. If $AB = r$ then the area of the circle is 6 times the area of the sector ACB.

    III. If $AB = r$ then triangle ACB is an equilateral triangle.

    (a) II only
    (b) III only
    (c) I and III only
    (d) II and III only
    (e) I, II and III

15. Which of the following two-dimensional figures could *not* be folded along the dotted segments so as to form a cube?

(a)

(b)

(c)

(d)

(e)
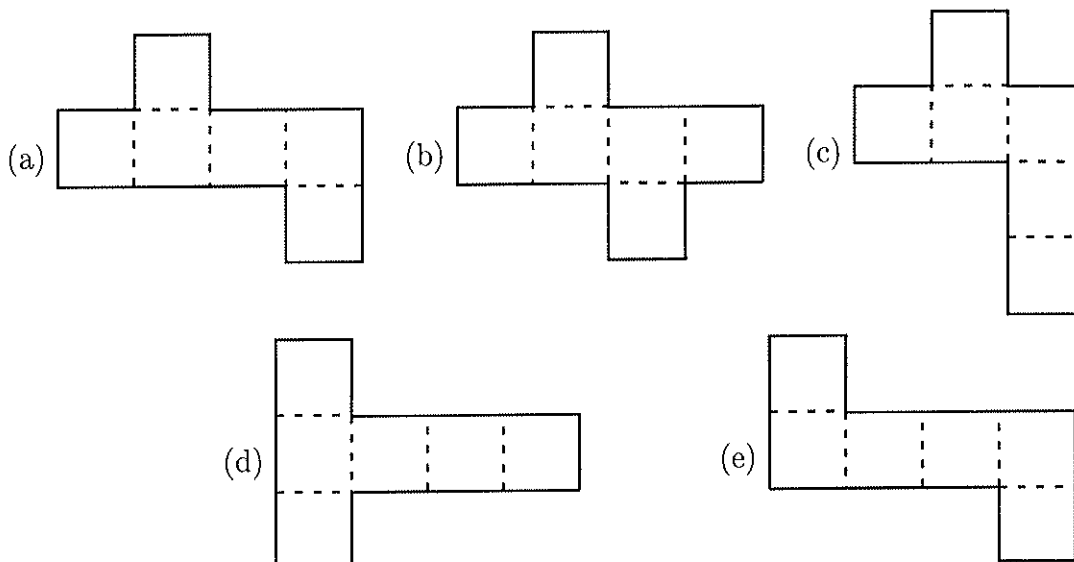
16. Circles C_1 and C_2 are coplanar with radii 3 and 8 respectively, and the distance between their centers is 13 (see figure). Let A be a point on C_1 and B be a point on C_2 such that the line containing A and B is tangent to both circles. What is the length of the segment $\overline{AB}$?
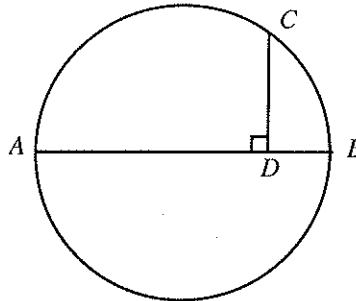
(a) 7

(b) 11

(c) 12

(d) 16

(e) 21

B

A

13

3

8
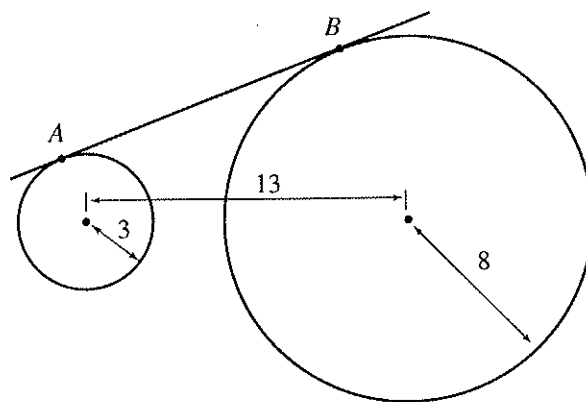
17. A stop sign is to be made by cutting a regular octagon from a square piece of paper. What is the measure of angle $\angle A$?

(a) 30°

(b) 45°

(c) 60°

(d) 120°

(e) 135°

A

STOP
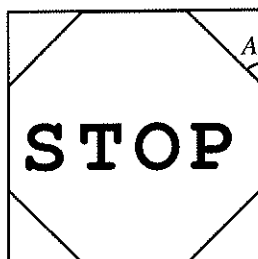
18. In the figure, $\overline{AB}$ is a diameter of the circle and $\overline{CD}$ is perpendicular to $\overline{AB}$. If the length of $\overline{AB}$ is a and the length of $\overline{BD}$ is b then the length of $\overline{CD}$ is

    (a) ab

    (b) $\sqrt{ab}$

    (c) $\frac{1}{ab}$

    (d) $a^2 b^2$

    (e) $\sqrt{ab - b^2}$

19. Given two circles with a common tangent at a point A such that the second circle passes through the center of the first, which of the following is true?

    (a) Every chord of the first circle that passes through A is bisected by the second circle.

    (b) Every chord of the first circle that passes through A is trisected by the second circle.

    (c) Every chord of the first circle that passes through A is tangent to the second circle.

    (d) Every chord of the second circle that passes through A is perpendicular to the line that passes through the center of the first circle and point A.

    (e) None of the above are true.

20. Mary is taller than Ann. Ann is shorter than Sarah. What can be concluded from this information?

    (a) Mary is the tallest of the three.

    (b) Ann is the shortest of the three.

    (c) Sarah is the shortest of the three.

    (d) Ann and Sarah have the same height.

   (e) Mary and Sarah have the same height.

21. Given
    $m\angle AEC = 65°$,
    $m\angle AFC = 75°$, and
    $m\angle A + m\angle C = 60°$.
    Find $m\angle B$.
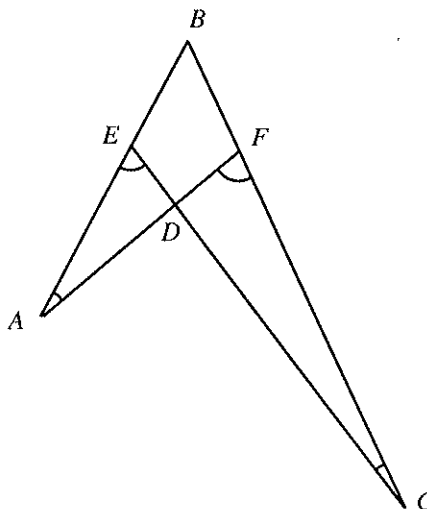
    (a) 20°

    (b) 30°

    (c) 40°

    (d) 50°

    (e) none of these

22. Which of the following is false?

    (a) The area of an isosceles right triangle is equal to one-fourth the square of the length of the hypotenuse.

    (b) If the diagonals of a convex quadrilateral are perpendicular, then the area of the quadrilateral is equal to 1/2 the product of the lengths of the diagonals.

    (c) If D is the midpoint of side $\overline{AC}$ of $\triangle ABC$, then the area of $\triangle ABD$ is equal to the area of $\triangle CBD$.

    (d) If $\triangle ABC \sim \triangle DEF$, $(AB/DE) = 2$, and the perimeter of $\triangle ABC$ is 10, then the perimeter of $\triangle DEF$ is 5.

    (e) If $\triangle ABC \sim \triangle DEF$, $(AB/DE) = 2$, and the area of $\triangle ABC$ is 64, then the area of $\triangle DEF$ is 32.

23. Let x and y be positive real numbers. Then the set of all points at a distance x from one plane, and at a distance y from a second plane which intersects the first plane obliquely is

    (a) four parallel lines

    (b) two parallel lines

    (c) two planes

    (d) four planes

    (e) a line parallel to both planes

24. Points A, B and C are located on a circle. Segment $\overline{AB}$ is congruent to segment $\overline{AC}$. If $AB = 5$ inches and $BC = 8$ inches, what is the diameter of the circle?

    (a) 25/6 in   (b) 25/3 in   (c) $10\sqrt{2}$ in   (d) $5\sqrt{2}$ in   (e) $\sqrt{89}$ in

25. In $\triangle ABC$, $AB = AC$, $BD = DC$ and $AE = ED$.
    Which of the following is incorrect?

    (a) $BE = EC$

    (b) $AE = EC$

    (c) Area of $\triangle BED$ = Area of $\triangle CED$

    (d) Area of $\triangle AEB$ = Area of $\triangle AEC$

    (e) Area of $\triangle ABC = 2 \times$ Area of $\triangle EBC$
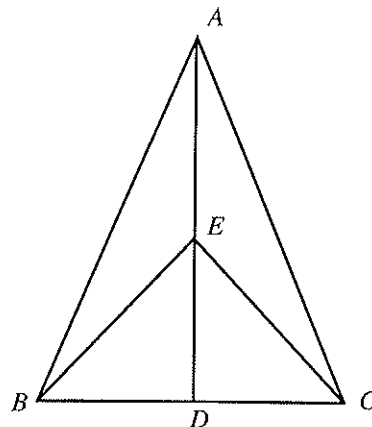
26. Let A be the area of the region bounded by an isosceles triangle. What is the measure of the congruent sides if the length of the third side is L?

    (a) $2A/L$

    (b) $\sqrt{16A^2 + L^4}/2L$

    (c) $\sqrt{A^2 + L^2}/2$

    (d) $(L + A)/\sqrt{3/2}$

    (e) $(AL)/2$

27. The length of any diagonal that is not a diagonal of a face of a rectangular parallelepiped with edges of length a, $2a$, and $3a$, where $a > 0$, is

    (a) $6a$ (b) $\sqrt{14}\,a$ (c) $6a^3$ (d) $\sqrt{14}\,a^3$ (e) none of the above

28. The sum of the measures of the interior angles of a convex n-sided polygon is

    (a) $360°/(n - 2)$

    (b) $180° - 2n$

    (c) $90° \cdot (n - 2)$

    (d) $n\,(180°) - 360°$

    (e) $\frac{n-2}{n} \cdot 180°$

29. The radii of two coplanar circles are 3 and 6 and the distance between their centers is 12. The length of a common internal tangent is

    (a) $3\sqrt{7}$ (b) $2\sqrt{35}$ (c) 8   (d) $\sqrt{135}$ (e) 3

30. Find the distance between the parallel lines $y = -2x + 1$ and $y = -2x + 9$.

    (a) 8   (b) 4   (c) $2\sqrt{2}$ (d) $8/\sqrt{5}$ (e) $4/\sqrt{5}$

31. The locus of the centers of all circles that pass through two given points is a

    (a) circle (b) straight line (c) point (d) parabola (e) hyperbola

32. The diameter $\overline{AB}$ of a circle with center O is a base of the inscribed trapezoid $ABCD$. If $m\angle BAD = 60°$ and the length of the radius $\overline{OA}$ is 8, what is the area of $ABCD$?

    (a) $4\sqrt{3}$ (b) 16 (c) 64π (d) $64\sqrt{3}$ (e) $48\sqrt{3}$

33. Consider the following definition: Two triangles are said to be **similar** if the angle measures of one triangle are the same as the angle measures of the other triangle. With this definition, "is similar to" is a relation on the collection of all triangles that is

    (a) not reflexive, not symmetric, and not transitive.

    (b) reflexive and symmetric but not transitive.

    (c) symmetric and transitive but not reflexive.

    (d) reflexive but not symmetric and not transitive.

    (e) an equivalence relation.

34. The **projection** of a point A in 3-dimensional space onto a plane m is the point P in m such that the line segment $\overline{AP}$ is perpendicular to all line segments in m emanating from P. The projection of a set S of points in space is the collection of all points in m that are projections of points in S. Select the true statement from the following:

    (a) If A and B are points in space with the same projection P, then A and B are the same.

    (b) For a given point P in the plane, the collection of all points in space with projection P is a line.

    (c) Projection is an isometry.

    (d) If two lines in space intersect with angle α, then the projections of these lines onto m intersect at angle α.

    (e) If the projections of two lines into m are parallel, then the lines themselves are parallel.

35. The corners of a pool table are O, A, B, and C, and a Cartesian coordinate system is superimposed as shown in the figure. A pool player's cue ball is at position $(1, 1)$ and she wants to bounce it off the sides OC and CB to strike a ball at position $(7, 4)$. At what point on the side OC should the ball strike?

(a) $(0, \frac{2}{5})$

(b) $(0, \frac{7}{4})$

(c) $(0, \frac{7}{3})$

(d) $(0, \frac{13}{8})$

(e) $(0, \frac{5}{2})$
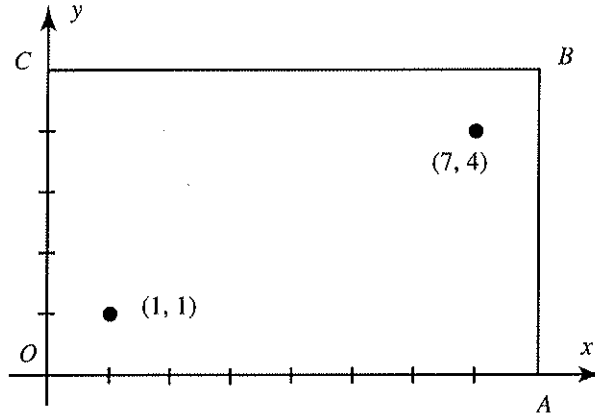
36. The following are known to be true:

- if A is not guilty, then B and C are both guilty.

- either A is not guilty or B is guilty.

- either B is not guilty or C is not guilty.

Who is guilty?

(a) A is the only guilty party.

(b) B is the only guilty party.

(c) C is the only guilty party.

(d) A and B are guilty.

(e) A and C are guilty.

37. Suppose that Alex, who is 2 meters tall, stands at water level on the shore of a calm, wide lake. Assuming that the radius of the earth is 6370 km, approximately how far out into the lake can Alex see?

(a) 1.74 km   (b) 2.82 km   (c) 5.05 km   (d) 14.6 km   (e) 25.5 km

38. Two squares of side length s are set in the plane so that the corner of one is placed at the center of the other as shown here. What is the maximum area of the shaded region in which the squares overlap?
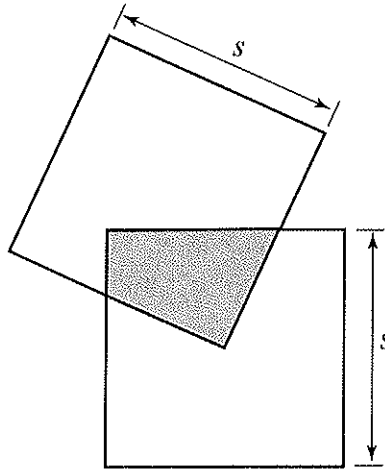
    (a) $\frac{1}{3}s^2$

    (b) $\frac{1}{4}s^2$

    (c) $\frac{2}{\sqrt{3}}s^2$

    (d) $\frac{4}{s^2}$

    (e) $\frac{1}{\pi}s^2$

39. A rhombus of side length s has a diagonal $\overline{AB}$ of length s. if the rhombus is revolved about $\overleftrightarrow{AB}$, what is the volume of the resulting solid?

    (a) $\frac{\pi}{4}s^3$ 　　　 (b) $\frac{\pi}{4}s^3$ 　　　 (c) $\frac{\sqrt{3}\pi}{24}s^3$ 　　　 (d) $\frac{3\pi}{4}s^3$ 　　　 (e) $\frac{4\pi}{3}s^3$

40. Triangle $A'B'C'$ is the reflection of triangle ABC over line l. Triangle $A''B''C''$ is the reflection of triangle $A'B'C'$ over line m. Line l is parallel to line m. Which statement is true?

    (a) The distance between A and A'' is twice the distance between l and m.

    (b) Triangle $A''B''C''$ is the image of triangle ABC under a rotation.

    (c) Triangle $A''B''C''$ is the image of triangle ABC under a translation.

    (d) Triangle $A''B''C''$ is the image of triangle ABC under a glide reflection.

    (e) Triangle $A''B''C''$ is the resulting image if triangle ABC is reflected over line m and that image is reflected over line l.