FORTY-SEVENTH ANNUAL MATHEMATICS CONTEST
sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

Geometry 2003

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Scoring formula: $4R - W + 40$

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft head (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly 80 minutes to work.

Contributors to TMTA for the Annual Mathematics Contest:

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1. Which of the following is NOT a polygon
   (a) a triangle
   (b) a square
   (c) a trapezoid
   (d) a pentagon
   (e) a circle

2. The number π represents
   (a) the circumference of a circle
   (b) the ratio of the circumference of a circle to its diameter
   (c) the ratio of the circumference of a circle to its radius
   (d) the ratio of the circumference of a circle to its area
   (e) the area of a circle

3. A square is a special kind of rectangle. Which of the following is NOT true.
   (a) A quadrilateral is a special kind of polygon.
   (b) A rhombus is a special kind of parallelogram.
   (c) A trapezoid is a special kind of rectangle.
   (d) An equilateral triangle is a special kind of isosceles triangle.
   (e) A rectangle is a special kind of parallelogram.

4. Which of the following figures has 8 sides?
   (a) a pentagon
   (b) a hexagon
   (c) an octagon
   (d) a decagon
   (e) a parallelogram

5. Two parallel lines are crossed by a transversal. Which of the following pairs of angles are NOT necessarily equal.
   (a) corresponding angles
   (b) alternate interior angles
   (c) alternate exterior angles
   (d) vertical angles
   (e) interior angles on the same side of the transversal

6. What is the perimeter of a square whose side is 7 meters long
   (a) 49 m (b) 49 m^2 (c) 28 m (d) 28 m^2 (e) 48 m^2

7. Which of the following sets of lengths can be the lengths of the sides of a right triangle?
   (a) 20, 22, 28
   (b) 8, 15, 17
   (c) 8, 9, 11
   (d) 4, 5, 6
   (e) 2.5, 3, 5

8. Find the value of X in the diagram:

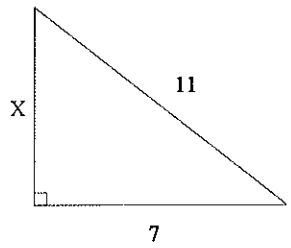
(a) 4

(b) $\sqrt{73}$

(c) $6\sqrt{2}$

(d) 18

(e) $10\sqrt{17}$



9. How many degrees are there in each interior angle of a regular pentagon?

(a) 360 (b) 280 (c) 540 (d) 108 (e) 470

10. The measure of an exterior angle of a regular polygon is 45 degrees. The number of sides the polygon has is:

(a) 8 (b) 9 (c) 5 (d) 15 (e) 6

11. Line l_1 is parallel to line l_2. What is the measure of angle ① ?

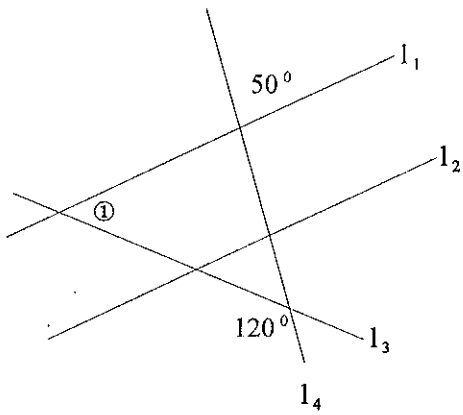
(a) 70°

(b) 50°

(c) 60°

(d) 90°

(e) 30°



12. One side of a right triangle has length 12. What are possible lengths of the other two sides?

(a) only 9 and 15 will work

(b) 9 and 15 will work; 5 and 13 will work

(c) only 5 and 13 will work

(d) 13 and 14 will work

(e) 4 and 5 will work

13. A circle has center O and radius r. The length of chord AB is also r. What is the measure of $\triangle AOB$?

(a) $\pi / 3$ rad. (b) $\pi / 4$ rad. (c) $\pi / 6$ rad. (d) 90° (e) $\pi / 2$ rad.

14. What is the minimum number of points that determine a unique straight line?
(a) 1 (b) 2 (c) 3 (d) 4 (e) 5

15. An equilateral triangle and a square are drawn so that they share a common side. What is the maximum number of additional equilateral triangles that can be drawn so that they share the same vertex with both the square, the original triangle, and each other; and do not overlap or lie inside of the square, the original triangle, or each other?
(a) 2 (b) 3 (c) 4 (d) 5 (e) it depends on the size of the triangles

16. A circle is inscribed in a square. What is the ratio of the area of the circle to the area of the square?
(a) ½ (b) $\pi/2$ (c) $\pi/4$ (d) $2/\pi$ (e) it depends on the radius of the circle

17. Let AD be an altitude of right triangle ABC drawn from the right angle vertex to the hypotenuse. When will triangles ABD and ACD be congruent?
(a) If B is any acute angle.
(b) If B is a right angle.
(c) If B is any obtuse angle.
(d) If triangle ABC is isosceles.
(e) If triangle ABC is scalene.

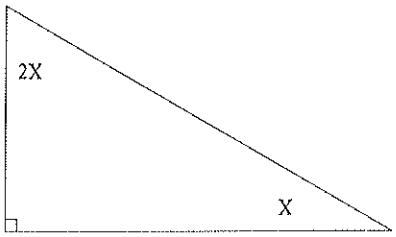
18. Let lines A and B be parallel. Let lines C and D be parallel. If A,B,C and D are in the same plane and line A is not parallel to line C, then the closed figure thus formed will necessarily be a
(a) square
(b) rectangle
(c) rhombus
(d) triangle
(e) quadrilateral

19. Let ABC be an equilateral triangle. Let D and E lie on line segment BC, so that D lies between B and E and so that AD = AE. Give a reason for Step 7 in the following proof:

Statement	Reason
1. AB = AC	1. Definition of equilateral triangle
2. AD = AE	2. given
3. angle ADE = angle AED	3. base angles of an isosceles triangle are equal
4. angle ABC = angle ACB	4. base angles of an isosceles triangle are equal
5. $\triangle$ ABD is similar to $\triangle$ ACE	5. AA
6. angle BAD = angle CAE	6. corresponding angles of similar triangles are equal
7. $\triangle$ ABD $\cong$ $\triangle$ ACE	7. _____

(a) SAS (b) SSA (c) AAA (d) SSS (e) Corresponding parts of congruent triangles

20. Find the degree measure of X in the diagram



(a) 60
(b) 35
(c) 45
(d) 22
(e) 30

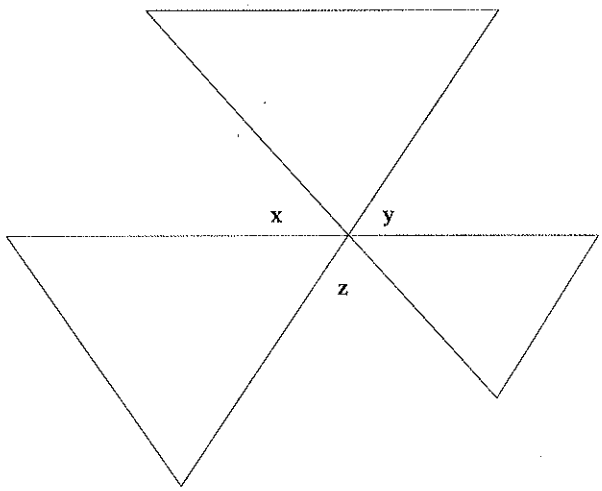
21. The side of a right isosceles triangle with hypotenuse 10 is what length?

(a) 5 (b) 4 (c) $\sqrt{10}$ (d) $5\sqrt{2}$ (e) 5.5

22. What is the distance between points whose spacial coordinates are (-2, 3, 8) and (5, 6, -3)?

(a) 55 (b) $\sqrt{179}$ (c) $\sqrt{181}$ (d) 12.7 (e) $\sqrt{43}$

23. The diagram shows three equilateral triangles with a common vertex. What is the degree measure of x + y + z?



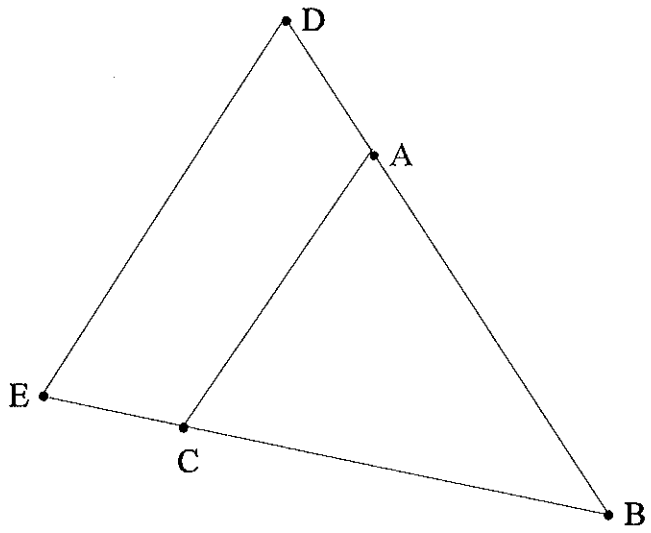
(a) 360
(b) 180
(c) 120
(d) 240
(e) 150

24. Points (1 , 2) and (5 , 2) lie on a circle. What MUST be true about the circle?

(a) The circle has center (3 , 2) and radius 2.
(b) The circle has center (3 , 2).
(c) The circle has center (5 , 2) or (1 , 2) and radius 4.
(d) The center of the circle lies on the line y = 3.
(e) The center of the circle has x-coordinate 3.

25. In the diagram, segment AB has length 5, segment BC has length 7, segment AD has length 4, and AC is parallel to DE. What is the length of segment BE?

(a) 9/5 (b) 11 (c) 12.6 (d) 3 (e) 9.3

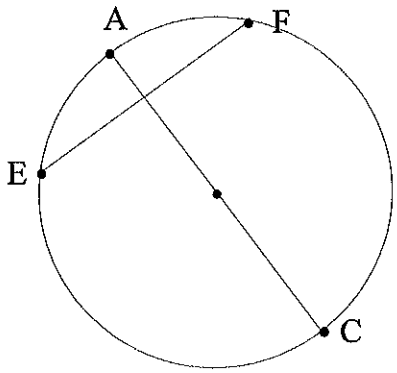


26. Which of the following is not a quadrilateral?

a) square b) rhombus c) prism d) trapezoid e) parallelogram

27. This circle has radius 5. Chord AC is perpendicular to chord EF and bisects EF. What is the length of AC?

(a) 5
(b) 10
(c) 2.5
(d) $\sqrt{34}$
(e) 4



28. For which of the following theorems is the converse NOT a theorem?

(a) If a triangle is isosceles, then the base angles are equal.
(b) If two lines are parallel, then they have a common perpendicular
(c) If alternate interior angles are equal, then lines crossed by a transversal are parallel.
(d) If a figure is both a rectangle and a rhombus, then it is a square.
(e) If a triangle is equilateral, then it is isosceles.

29. If a cube is cut by a plane, which of the following cannot be formed?
(a) a triangle (b) a square (c) a pentagon (d) a hexagon (e) an octagon

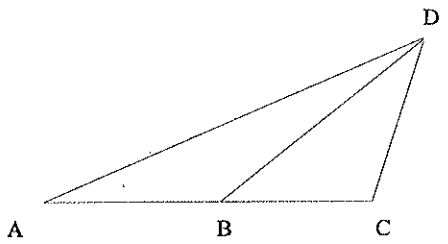
30. Which of the following is NOT necessarily true?
a) A line perpendicular to one plane is necessarily perpendicular to any other plane.
b) If two distinct planes are parallel to a third plane, then they are parallel to each other.
c) If two distinct planes intersect, they intersect in a straight line.
d) If a line has two points in common with a plane, then it lies entirely in that plane.
e) If a line is perpendicular to a plane, then it is perpendicular to every line in that plane that passes through the point where it intersects the plane.

31. How long is the side of the largest equilateral triangle that can be inscribed in a square whose side has length 1.
(a) 1 (b) $.5\sqrt{5}$ (c) $.25\sqrt{5}$ (d) sec 15 degrees (e) sec 25 degrees

32. A circular metal plate of radius 10 cm is heated so that it expands uniformly. After heating, the new radius is 10.5 cm. What is the percent change is the circumference of the plate?
(a) 3.14 cm (b) π cm (c) 0.05 (d) 5% (e) 20%

33. In the diagram, AB = BD, BC = CD, and the angle measure of angle ACD is 130 degrees. What is the measure of angle DAB in degrees?

(a) 25
(b) 30
(c) 12.5
(d) 15
(e) 20



34. A rectangle 40 inches long and 10 inches wide is tiled completely with square tiles ½ inch on each side. How many tiles are needed to cover the rectangle?
(a) 400 (b) 200 (c) 800 (d) 80 (e) 1600

35. Three circles of radii 9, 10, and 11 are tangent to each other externally. What is the perimeter of the triangle formed by joining their centers?
(a) 66 (b) 63 (c) 61 (d) 60 (e) 30

36. If a circle of radius 10 inches has its radius decreased by 4 inches, by what percent is its area decreased?
(a) 40 (b) 36 (c) 64 (d) 44 (e) 62

37. Right triangles ABC and XYZ are similar, with A corresponding to X, B to Y, and C to Z. If BC = 9, AC = 21, and YZ = 24, then the length of XZ is

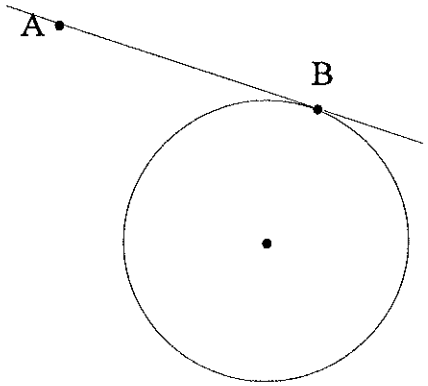
(a) 42 (b) 63 (c) 49 (d) 56 (e) 72

38. Four circles have their centers at the corners of a square 6 units on a side. Each circle is externally tangent to two of the other circles and the circles do not overlap. What is the area that lies inside the square but not inside any of the circles?

(a) $9\pi - 36$
(b) $36\pi - 9$
(c) $9 - 9\pi$
(d) $24 - 9\pi$
(e) $36 - 9\pi$

39. Point A is 10 units away from the center of the circle. The circle has radius 6 units. The given line goes through A and is tangent to the circle at point B. What is the distance from A to B?

(a) 8
(b) 4
(c) $\sqrt{8}$
(d) 2
(e) 16



40. Line ② is parallel to line ③. Line ④ is parallel to line ⑤. How many similar triangles are formed by the given lines?

(a) There are no similar triangles.
(b) There are two similar triangles.
(c) There are three similar triangles.
(d) There are four similar triangles.
(e) There are six similar triangles.