FORTY-SIXTH ANNUAL MATHEMATICS CONTEST
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Geometry · 2002

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Scoring formula: $4R - W + 40$

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft head (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly 80 minutes to work.

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1. A straight angle is divided into three angles whose measures are in the ratio 9:5:2. What is the measure of the angle formed when the largest of these three angles is bisected?

   a. 54°   b. 50°30'   c. 101°15'   d. 50°7'30"   e. 50°37'30"

2. In the diagram, $\overline{XY}$ is tangent to circle O at Y. $\overline{YO} = 8$, $\overline{XY} = 15$. Find the length of the secant XZ.
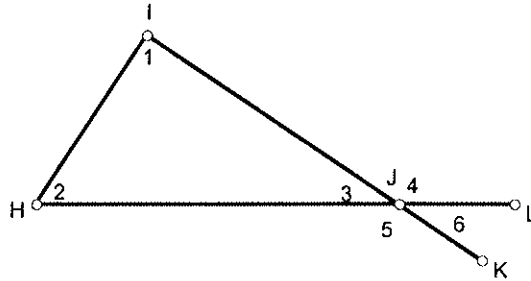
   a. 25

   b. 12

   c. 17

   d. 24

   e. 9
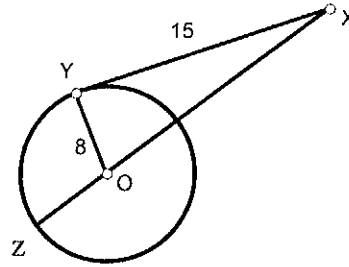
3. The diameter of a bicycle tire is 24 inches. How many revolutions will the tire make in a one mile race?

   a. $\dfrac{7920}{\pi}$ b. $\dfrac{2640}{\pi}$ c. $\dfrac{3960}{\pi}$ d. 220π e. $\dfrac{3520}{\pi}$

4. In HIJ, $HJ > IJ > HI$. Which of the following statements is true?

   a. $m\angle 6 > m\angle 1$

   b. $m\angle 3 > m\angle 2$

   c. $m\angle 6 > m\angle 2$

   d. $m\angle 1 > m\angle 3$

   e. $m\angle 2 > m\angle 4$

5. Which of the following polygons listed below can be drawn so that it has no axes of symmetry?

   a. isosceles triangle   b. parallelogram   c. kite

        d. isosceles trapezoid   e. rhombus

6. In the figure shown, lines m and n lie on plane p and intersect at point A. Lines l and k intersect plane p at points A and B, respectively; $l \perp n$ at A. Which of the following statements must be true?

    a. $k \perp m$

    b. $k \perp p$

    c. $m \perp n$

    d. $l \perp m$

    e. $k \parallel l$
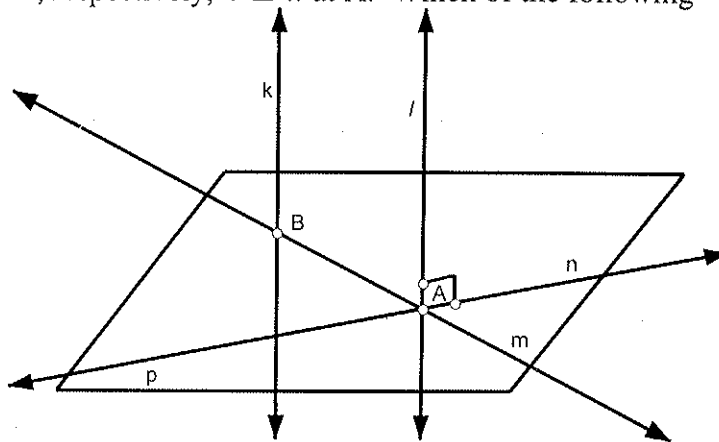
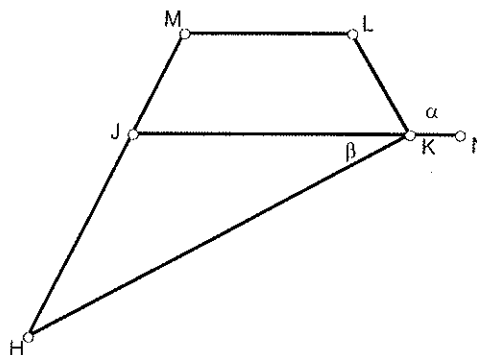7. The measure of an exterior angle of a regular polygon is 24°. How many diagonals can be drawn in the polygon?

    a. 90   b. 252   c. 54   d. 64   e. 102

8. Which of the following statements about similar polygons is not true?

    a. Corresponding sides are in proportion.
    b. Corresponding angles are congruent.
    c. Ratio of perimeters is equal to the ratio of the lengths of corresponding sides.
    d. Ratio of areas is equal to the ratio of the lengths of corresponding sides.
    e. Ratio of the lengths of corresponding diagonals is equal the ratio of corresponding perimeters.
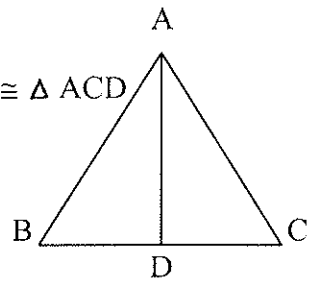
9. In the diagram, trapezoid JKLM is isosceles. Triangle JHK is isosceles with base $\overline{HK}$. Find the measure of β in terms of α.

    a. $\beta = \alpha$

    b. $\beta = \dfrac{\alpha}{2}$

    c. $\beta = 180° - \dfrac{\alpha}{2}$

    d. $\beta = 90° - \alpha$

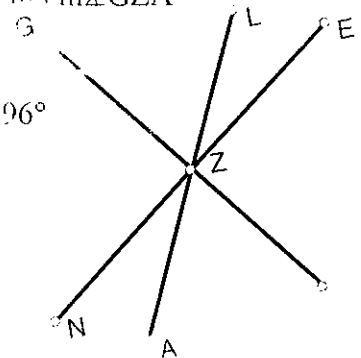   e. $\beta = 90° - \dfrac{\alpha}{2}$

10. In the figure shown, $\overline{AD}$ is a median and an altitude in $\triangle ABC$. Which of the following statements cannot be proved?

a. $\triangle ADB$ is a right triangle.   b. $\triangle ABD \sim \triangle ACD$ c. $\triangle ABD \cong \triangle ACD$

d. $\triangle ABC$ is equilateral   e. $\triangle ABC$ is isosceles

11. In the figure shown, $\overrightarrow{GZ} \perp \overrightarrow{NE}$, m$\angle$GZL = 24x, m$\angle$LZE = 16x. Find m$\angle$GZA

a. 136°   b. 126°   c. 116°   d. 106°   e. 96°

12. A triangle has sides of lengths 14, 11, and 7. Find the length of the altitude of the triangle drawn to the longest side.

a. $\dfrac{6\sqrt{10}}{7}$ b. $\dfrac{12\sqrt{10}}{7}$ c. $\dfrac{24\sqrt{10}}{7}$ d. $\dfrac{12\sqrt{5}}{7}$ e. $\dfrac{10\sqrt{10}}{7}$
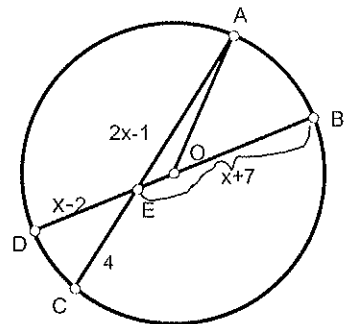
13. If the name of a polygon is chosen at random from the list of names below, what is the probability that the diagonals of the named polygon are congruent or perpendicular?

parallelogram   rectangle   rhombus   square   kite   trapezoid

a. $\dfrac{1}{6}$ b. $\dfrac{1}{3}$ c. $\dfrac{1}{2}$ d. $\dfrac{2}{3}$ e. $\dfrac{5}{6}$
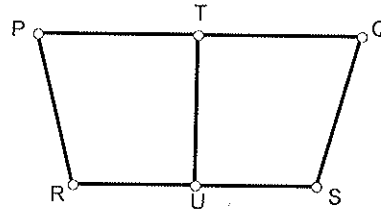
14. In $\odot O$, m$\angle$AOD = 140°, $AE = 2x - 1$, $CE = 4$, $BE = x + 7$, $DE = x - 2$. Find the length of arc AB.

a. $\dfrac{1}{6}\pi$ b. $\dfrac{8}{3}\pi$ c. $\dfrac{5}{3}\pi$ d. $\dfrac{4}{3}\pi$ e. $\dfrac{7}{3}\pi$

15. In the diagram, $\angle R \cong \angle S$, $\overline{PT} \cong \overline{QT}$, $\overline{PR} \cong \overline{QS}$, $\overline{RU} \cong \overline{SU}$. Which of the following cannot be proven?

a. $\overline{PT} \cong \overline{TU}$

b. $\overline{PQ} \| \overline{RS}$

c. $PTUR \sim QTUS$

d. $\overline{RS} \perp \overline{TU}$

e. $\angle P$ is supplementary to $\angle R$

16. When using a compass and straightedge to bisect an angle, what is the minimum number of arcs necessary to complete the construction?

a. one   b. two   c. three   d. four   e. five

17. The endpoints of the diameter of a circle are (4,-5) and (-2,3). Find the center and area of the circle.

a. (-1,1), 100π b. (-1,1), 25π c. (1,1), 25π

d. (1,-1), 100π e. (1,-1), 25π

18. Find the area of a regular hexagon with side of measure 10 units.

a. 60 units^2 b. $150\sqrt{3}$ units^2 c. 150 units^2 d. $25\sqrt{3}$ units^2 e. $150\sqrt{2}$ units^2

19. Which of the following statements is false in Euclidean geometry?

a. Through a point on a plane, there is only one line perpendicular to the given plane.
b. Through a point on a line, there is only one plane perpendicular to the given line.
c. Through a point on a line, there is only one line perpendicular to the given line.
d. Through a point not on a plane, there is only one line perpendicular to the given plane.
e. Through a point not on a line, there is only one line perpendicular to the given line.

20. In the figure shown, $\overline{HO}$ and $\overline{DR}$ are parallel chords in the circle. $\overrightarrow{CH}$ and $\overrightarrow{CR}$ are tangent to the circle at points H and R. The measure of $\angle ORD$ is 100°. Find the measures of $\angle C$ and $\angle T$.

a. $\angle C = 20°, \angle T = 20°$

b. $\angle C = 20°, \angle T = 40°$

c. $\angle C = 40°, \angle T = 20°$

d. $\angle C = 40°, \angle T = 40°$

e. $\angle C = 50°, \angle T = 50°$
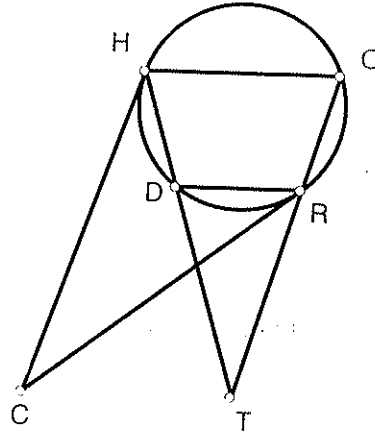
21. Twice the supplement of an angle is added to three times the complement of an angle. The sum is the measure of an interior angle of a regular nonagon. What is the measure of the supplement of the angle?
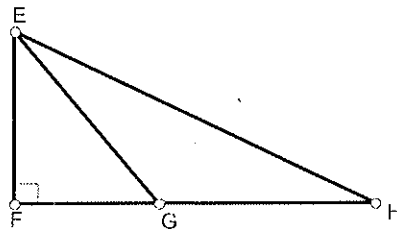
a. 98°   b. 94°   c. 90°   d. 86°   e. 82°

22. Which of the following combinations of regular polygons will not tessellate?

a. 3 triangles, 2 squares   b. 2 octagons, 1 square   c. 2 triangles, 2 hexagons

d. 2 dodecagons, 1 triangle   e. 3 triangles, 1 hexagon

23. In right triangle EFH, $\overline{EF} = 1$, $\overline{FG} = 1$, and $\overline{EG} \cong \overline{GH}$. Find the length of $\overline{EH}$.
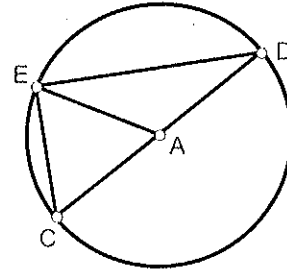
a. 4

b. $\sqrt{5}$

c. $\sqrt{4 + 2\sqrt{2}}$

d. $2 + \sqrt{2}$

e. $4 + 2\sqrt{2}$

24. What is the name of the locus in space of all points 6 units away from a given point?

    a. circle   b. segment   c. sphere   d. cylinder   e. cone

25. $\odot A$ has a diameter of 12 mm. Chord $\overline{EC}$ has length 6 mm. Find the area of $\triangle EAD$.

    a. 18mm^2

    b. 36 mm^2

    c. $6\sqrt{3} \text{ mm}^2$

    d. $9\sqrt{3} \text{ mm}^2$

    e. $18\sqrt{3} \text{ mm}^2$

26. In Euclidean geometry, which of the following statements describing a polygon is false?

    a. No two segments with a common endpoint are collinear.
    b. For n at least 4, a convex polygon of n sides has $n - 2$ diagonals.
    c. No two segments intersect, except for common endpoints.
    d. All points are coplanar.
    e. No more than two segments are perpendicular to the same segment.

27. $\odot C$ and $\odot D$ are tangent at the origin and are tangent to 3 sides of the rectangle as shown. The coordinates of D are (a,b). The coordinates of C are (-a,-b). Find the area of the rectangle.

    a. $8a + 8b$

    b. $8(a^2 + b^2)$

    c. $8\sqrt{a^2 + b^2}$

    d. $6(a^2 + b^2)$

    e. $6\sqrt{a^2 + b^2}$
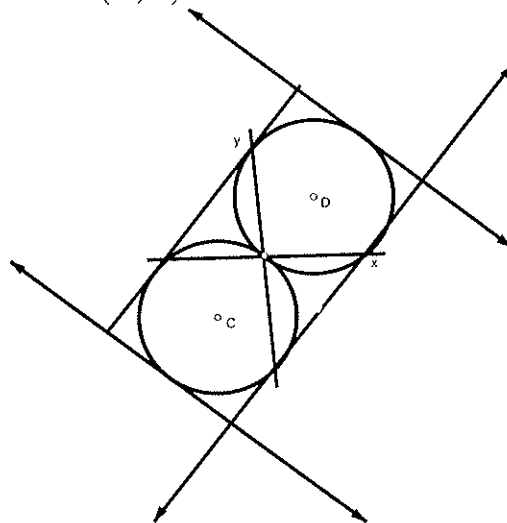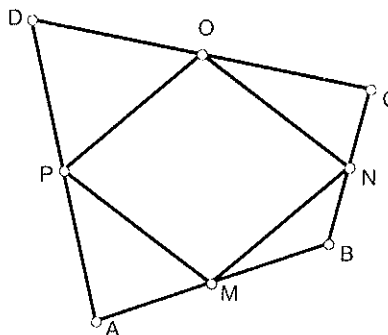
28. Find the distance between the parallel lines $y = 3x - 4$ and $y = 3x + 1$.

    a. 5   b. $\sqrt{10}$ c. $\dfrac{5}{2}$ d. $\dfrac{\sqrt{10}}{2}$ e. 2
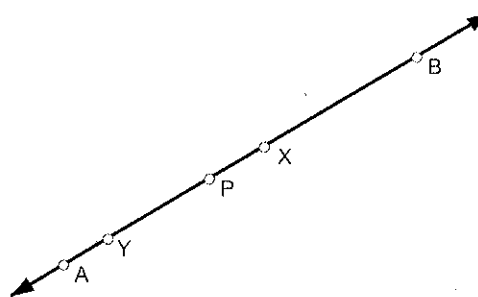
29. In the diagram, quadrilateral ABCD has midpoints M, N, O, P of sides $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$, respectively. Which name best describes figure MNOP?

    a. quadrilateral

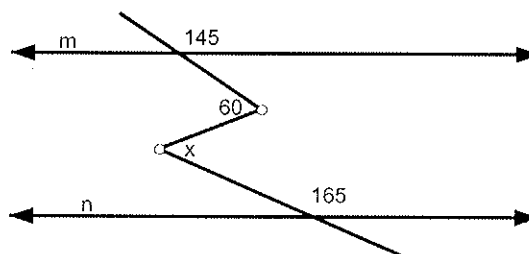   b. parallelogram

    c. rhombus

    d. trapezoid

    e. kite

30. $\overrightarrow{PB}$ and $\overrightarrow{PA}$ are opposite rays on $\overleftrightarrow{AB}$. X is a point on $\overrightarrow{PB}$. Y is a point on $\overrightarrow{PA}$, as shown in the figure. Which of the following must be false?

    a. $\overrightarrow{XB} \subset \overrightarrow{YB}$

    b. $\overrightarrow{XB} \cap \overrightarrow{XY} = \overline{XY}$

    c. $\overrightarrow{PX} \cup \overrightarrow{PY} = \overleftrightarrow{XY}$

    d. $\overleftrightarrow{AB} \subset \overrightarrow{YP}$

    e. $\overrightarrow{AY} \cap \overrightarrow{BX} = \overline{AB}$
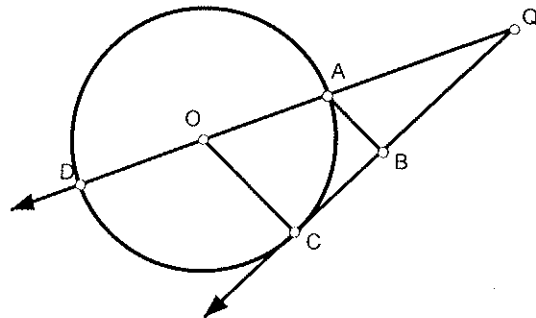
31. Find the measure of angle x in the diagram shown where m ∥ n.

    a. 15°

    b. 35°

    c. 40°

    d. 45°

    e. 60°

32. In the figure, $\overline{QC}$ is tangent to $\odot O$ at C. $\overrightarrow{QO}$ intersects $\odot O$ at A and D. $\overline{AB} \parallel \overline{OC}$. $\overline{CB} = 12$, $\overline{AB} = 8$. Find the length of secant $\overrightarrow{QD}$.
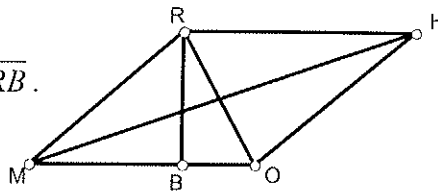
a. $\overline{QD} = 20.8$

b. $\overline{QD} = 46.8$

c. $\overline{QD} = 26$

d. $\overline{QD} = 39$

e. $\overline{QD} = 43$

33. The equation of $\odot Q$ is $(x - 3)^2 + (y + 5)^2 = 17$. If $\odot Q$ is reflected over the x axis and translated four units to the left, which of the points below is not located on the new circle, $\odot Q'$?

a. (0,1)   b. (3,4)   c. (-2,9)   d. (-5,6)   e. (-3,8)

34. In rhombus RHOM , diagonal MH is 24 units in length, diagonal RO is 18 units, $\overline{RB} \perp \overline{MO}$. Find the length of $\overline{RB}$.

a. 14.4   b. 28.8   c. 30.8   d. 19.2   e. 15.4

35. A right cylindrical can of height 15 cm. contains 360 cm^3 of soda. Three-fifths of the soda is poured into a conical paper cup with the same radius as the can. The soda fills the cup completely. What is the height of the paper cup?

a. 24 cm.   b. $\dfrac{24}{\pi}$ cm.   c. 14.4 cm.   d. 9 cm.   e. 27 cm.

36. $\triangle$ LMN is equilateral with perimeter 15. $\overline{LF}$ and $\overline{MG}$ are medians which intersect at X. $\overline{XZ} \parallel \overline{LN}$. Find the length of $\overline{FZ}$.
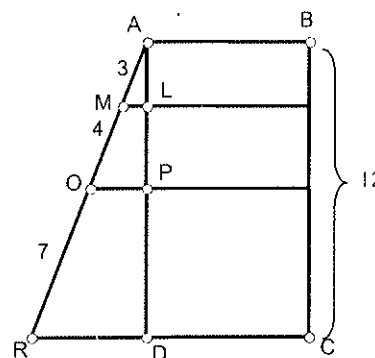
a. $\dfrac{2}{5}$ b. $\dfrac{6}{5}$ c. $\dfrac{5}{6}$ d. 2   e. 1
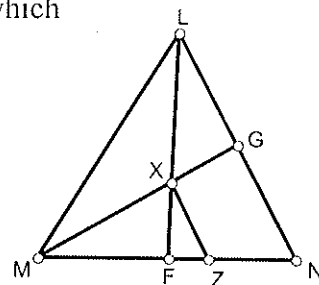
37. An orange is $8\frac{1}{2}$ cm. in diameter. When its peel is removed, the peeled orange is 8 cm. in diameter. What is the ratio of the volume of the peel to the volume of the peeled orange?
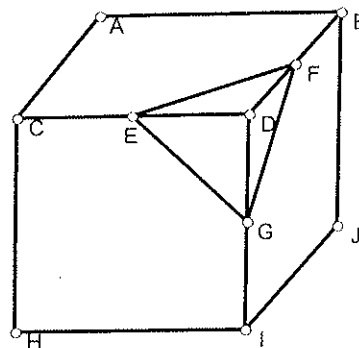
a. $17^3 - 1$

b. $\dfrac{17^3 - 4^3}{4^3}$

c. $\dfrac{4\pi(17^3 - 4^3)}{3}$

d. $\dfrac{17^3 - 4^6}{4^6}$

e. $\dfrac{4\pi(17^3 - 4^6)}{3}$

38. In the diagram, ABCR is a trapezoid with bases $\overline{AB}$ and $\overline{RC}$. ABCD is a rectangle. $\overline{AM} = 3$, $\overline{MO} = 4$, $\overline{OR} = 7$, $\overline{BC} = 12$. Find the perimeter of trapezoid DROP.

a. $4\sqrt{13}$

b. 26

c. $13 + \sqrt{13}$

d. 24

e. $13 + 3\sqrt{13}$

39. In the diagram, each edge of the cube has length 24. E, F, and G are the midpoints of $\overline{CD}$, $\overline{BD}$, and $\overline{DI}$. Find the volume of pyramid DEFG.

a. $72\sqrt{3}$ units^3

b. 144 units^3

c. $144\sqrt{3}$ units^3

d. 288 units^3

e. 432 units^3

40. $\overline{MK}$ and $\overline{LJ}$ are the hypotenuses of overlapping right triangles KLM and JKL. $\overline{MK} \perp \overline{LJ}$, the length of $\overline{MK}$ is $6\sqrt{5}$, the length of $\overline{LK}$ is $6\sqrt{3}$. Find the length of $\overline{JK}$.

a. $9\sqrt{2}$

b. $\dfrac{18}{\sqrt{5}}$

c. $\dfrac{12}{\sqrt{5}}$

d. $6\sqrt{3}$

e. $\dfrac{6\sqrt{6}}{\sqrt{5}}$