FORTY-THIRD ANNUAL MATHEMATICS CONTEST
sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

Geometry 1999

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Scoring formula: 4R - W + 40

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft head (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly 80 minutes to work.

Contributors to TMTA for the Annual Mathematics Contest:

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1. **A four-foot statue casts a shadow of six feet. At the same instant a tree standing nearby casts an eight-foot shadow. The height of the tree to the nearest tenth of a foot is**

   (a) 5.3 (b) 6.0 (c) 10.0 (d) 10.7 (e) 12.0

2. **A clear plastic rectangular box is tightly sealed and contains a blue liquid. The box is 2 inches by 3 inches by 6 inches in dimension. When the box is sitting on its smallest face the liquid is 1 inch from the top. When the box is sitting on its largest face, the distance (in inches) the liquid will be from top is**

   (a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) 1 (d) $\frac{4}{3}$ (e) $\frac{5}{3}$

3. **Which of the following sets of numbers cannot be the lengths of the sides of a right triangle?**

   (a) $\{1, \frac{4}{3}, \frac{5}{3}\}$ (b) $\{12, 22.5, 25.5\}$ (c) $\{\sqrt{3}, \sqrt{4}, \sqrt{5}\}$

   (d) $\{3.5, 12, 12.5\}$ (e) $\{\sqrt{3}, \sqrt{12}, 3\}$

4. **In the figure below, $\overline{ST} \parallel \overline{PQ}$, the length of $\overline{BS}$ is a, the length of $\overline{SP}$ is b, the length of $\overline{BT}$ is c, and the length of $\overline{TQ}$ is d. Which of the following is not correct?**

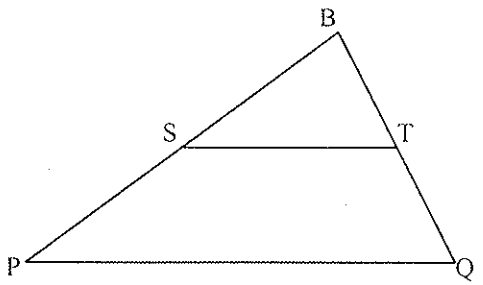
   (a) $\frac{b}{a} = \frac{d}{c}$

   (b) $\frac{a+b}{a} = \frac{c+d}{c}$

   (c) $\frac{a+b}{c+d} = \frac{a}{c}$

   (d) $ab = cd$

   (e) $\frac{cb}{2} = \frac{ad}{2}$



5. **Two cubes have sides in the ratio 2 : 3. The ratio of the diagonal of a face of the smaller cube to the diagonal of a face in the larger cube is**

(a) $4:9$ (b) $8:27$ (c) $2:3$ (d) $\sqrt{2}:\sqrt{3}$ (e) $\sqrt[3]{2}:\sqrt[3]{3}$

6. **A diameter and a chord of a circle intersect in such a way that the diameter splits the chord into two segments of lengths 8 and 9 centimeters and the chord splits the diameter into two segments whose lengths are in the ratio 2 : 1. The diameter of the circle measured in centimeters is**

(a) 6 (b) $\frac{34}{3}$ (c) 12 (d) 17 (e) 18

7. **Using twelve points in a plane, no three collinear, how many distinct triangles can be formed with vertices selected from these twelve points?**

(a) 4 (b) 36 (c) 120 (d) 220 (e) 1320

8. **A rectangular barn is 40 feet wide and 50 feet long. A horse is tied to a corner of the barn by a 50-foot long rope. How many square feet of ground outside of the barn can the horse graze?**

(a) 1600π (b) 1650π (c) 1875π (d) 1900π (e) 2500π

9. **The slopes of two lines are 2 and –3. Find the slope of the line which bisects the acute angle between them.**

(a) $5\sqrt{2}+7$ (b) $\frac{-1}{2}$ (c) $\frac{5}{2}$ (d) $\frac{\sqrt{6}}{3}$ (e) $\sqrt{2}-1$

10. **A chord in a circle of diameter 50 inches has length 48 inches. How far in inches is the chord from the center of the circle?**

(a) 1 (b) 2 (c) $\sqrt{5}$ (d) 7 (e) 14

11. **Three mutually tangent circles, each with radius 3 centimeters are placed inside a larger circle such that each of the smaller circles is tangent to the larger circle. What is the radius in centimeters of the larger circle?**

(a) 6 (b) $2\sqrt{3}+3$ (c) $3\sqrt{3}+3$ (d) $\frac{3\sqrt{3}}{2}+3$ (e) $\frac{9\sqrt{3}}{4}+3$

12. **Determine the volume of the solid formed by revolving the region in the xy-plane bounded by the triangle with vertices (2, 0), (10, 0), and (5, 4) about the x-axis.**

(a) $5\sqrt{41}$ (b) $\frac{128}{3}\pi$ (c) 16π (d) $\frac{256}{3}\pi$ (e) 32

13. **A square is inscribed in a circle. The region inside the square is painted blue and the remaining regions are painted red. If this is the layout for a dart board, what is the probability that a randomly thrown dart will land in one of the red areas?**

(a) $\frac{1}{2\pi}$ (b) $\frac{1}{5}$ (c) $\frac{4}{5}$ (d) $\frac{2}{\pi}$ (e) $\frac{\pi-2}{\pi}$

14. **Define a procedure P as follows:**

P: Subdivide a square into 4 congruent squares and shade 3 of these congruent squares.

Start with a square with side 256 inches and perform P on this square and then perform P on the new unshaded square. If we repeat this process on each new unshaded square until we have performed P on a total of 10 squares, what is the total area of the shaded regions in square inches?

(a) $\frac{262143}{4}$ (b) $\frac{1048575}{16}$ (c) $\frac{4184303}{64}$ (d) 65536 (e) $\frac{131067}{2}$

15. **A regular hexagon is inscribed in a circle. Another circle is inscribed inside the hexagon. What is the ratio of the area of the smaller circle to the area of the larger circle?**

(a) $\frac{3}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{4}$ (e) $\frac{1}{6}$

16. Find the center of the circle in the xy-plane that passes through the points (4, -5), (6, 7), and (-8, -3).

(a) (0, 0) (b) (-1, 5) (c) (-1, 2) (d) (-2, -1) (e) (-2, 2)

17. How many diagonals does a convex polygon with n sides have?

(a) $n-3$ (b) $\frac{n(n-1)}{2}$ (c) n (d) $\frac{n(n-3)}{2}$ (e) $n-2$

18. In ΔABC, $\angle A$ and $\angle B$ are congruent. If side BC has length 14 centimeters and side AC is twice as long as side AB, what is the perimeter in centimeters of the triangle?

(a) 20 (b) 28 (c) 35 (d) 56 (e) 70

19. If the statements, "If X, then Y" and "If not Z, then not Y" are true, which of the following statements must also be true?

(a) If not Y, then not Z.
(b) If Y, then not X.
(c) If not Z, then not X.
(d) If not X, then not Z.
(e) If Z, then Y.

20. Consider a cube with sides of length 4 centimeters. Connect the centers of each face to the center of each adjoining face to form the edges of a new solid. The surface area of this new solid in square centimeters will be

(a) $12\sqrt{3}$ (b) $16\sqrt{3}$ (c) $8+12\sqrt{3}$ (d) 48 (e) $32\sqrt{3}$

21. In the figure, C is the center of the circle, points T, A, and D are on the circle, BT is tangent to the circle, C is on $\overline{AT}$, D is on $\overline{AB}$, and $\overline{CD}$ is perpendicular to $\overline{AT}$. If the length of segment $\overline{CD}$ is r and the length of segment $\overline{DB}$ is x, which of the following represents the length of segment $\overline{BT}$?

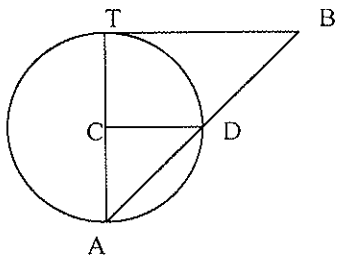
(a) $r + \sqrt{x^2 - r^2}$

(b) $\sqrt{x^2 - r^2}$

(c) $\sqrt{(r+x)^2 - (2r)^2}$

(d) $x - \sqrt{2}r$

(e) $r + x$



22. A trapezoid has parallel sides of lengths 13 inches and 21 inches. The longer of the two nonparallel sides is 17 inches and the shorter of the two nonparallel sides is perpendicular to a parallel side. What is the area in square inches of the trapezoid?

(a) 221 (b) 225 (c) 247 (d) 255 (e) 289

23. Two parallel lines are cut by a transversal forming the eight numbered angles shown below. If $\angle 1$ is an acute angle, how many pairs of congruent angles can be formed from these eight numbered angles?

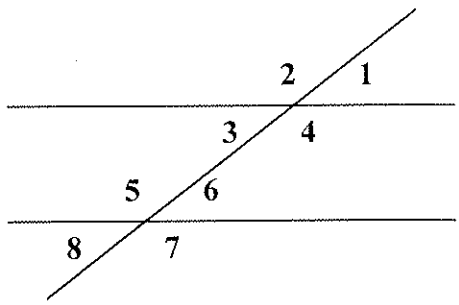
(a) 4

(b) 8

(c) 12

(d) 16

(e) 28



24. ΔABC is a right triangle with hypotenuse $\overline{AC}$. A point D is located on side $\overline{BC}$ in such a way that ΔABD and ΔADC are isosceles triangles. If segment $\overline{AB}$ has length 1 centimeter, what is the length in centimeters of $\overline{AC}$?

(a) 2 (b) $2\sqrt{2}+4$ (c) $\sqrt{2\sqrt{2}+4}$ (d) $2+\sqrt{2}$ (e) $\sqrt{2+\sqrt{2}}$

25. If the diameter of a bicycle wheel is 28 inches, how many complete revolutions does the wheel make in traveling 10 miles?

(a) 600 (b) 3601 (c) 6157 (d) 7202 (e) 24630

26. Euclid's "Elements of Geometry" containing thirteen books covering geometry, number theory, and geometric algebra dates from

(a) 300 BC (b) 820 AD c) 1570 AD (d) 1637 AD (e) 1829 AD

27. A square is inscribed in a circle with radius 30 centimeters. The perimeter of the square in centimeters will be

(a) $30\sqrt{2}$ (b) 60 (c) $60\sqrt{2}$ (d) 120 (e) $120\sqrt{2}$

28. The sum of the measures of the interior angles of a convex polygon is 3780°. How many sides does the polygon have?

(a) 19 (b) 20 (c) 21 (d) 22 (e) 23

29. The ends of a glass tube are circles with radii 7 centimeters and 5.25 centimeters. The segment joining the centers of the circles has length 6 centimeters and is perpendicular to each of the planes containing the circles. The outside surface area of the tube in square centimeters, not including the ends, is

(a) 55.125π (b) 73.5π (c) 75π (d) 76.5625π (e) 84π

30. **A window is in the shape of a rectangle capped with a semicircle as shown below. If the diameter of the circle is 4 feet and the perimeter of the window is 40 feet, then the area of the window in square feet is**

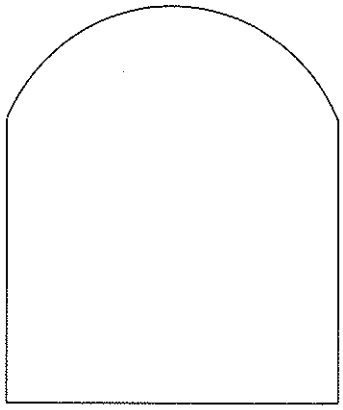
(a) $18 - \pi$

(b) 64

(c) $72 - 2\pi$

(d) 72

(e) $72 + 8\pi$



31. **Assume the dots in the picture below are 1 centimeter apart vertically and horizontally. The perimeter in centimeters is best approximated by**

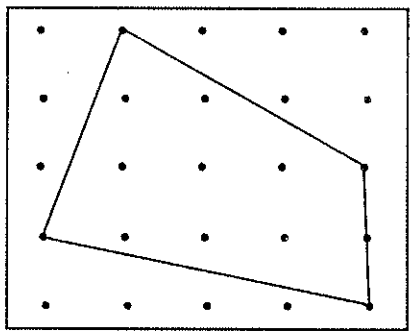
(a) 11.6

(b) 12.9

(c) 13.8

(d) 15.9

(e) 16.3



32. **A baseball field is a square with sides of length 90 feet. The pitcher's mound is located 60 feet from home plate. How far in feet is the pitcher's mound from third base?**

(a) $30\sqrt{5}$ (b) $45\sqrt{2}$ (c) $30\sqrt{13-6\sqrt{2}}$

(d) $90\sqrt{2}-60$ (e) $30\sqrt{9-6\sqrt{2}}$

33. In the picture below the image of B is G and the image of E is J under a glide reflection. The image of A is located at

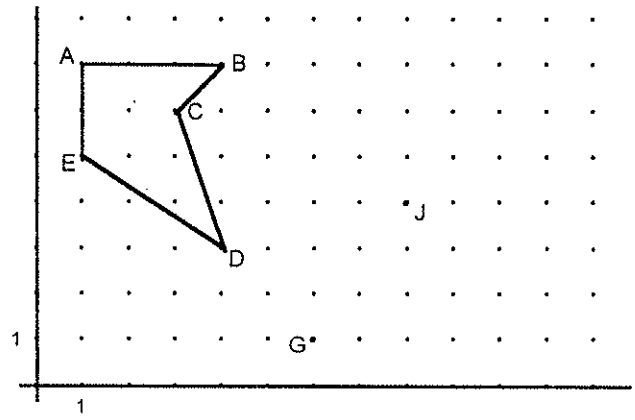
(a) (6, 4)

(b) (8, 2)

(c) (7, 3)

(d) (4, -1)

(e) (7, -1)



34. Two parallel lines are cut by a transversal at points A and B. Point C is the intersection of the angle bisectors of two interior angles on the same side of the transversal. $\angle ACB$ is

(a) an acute angle

(b) an obtuse angle

(c) a right angle

(d) a straight angle

(e) sometimes an acute angle and sometimes an obtuse angle

35. The sides of a triangle are 16 inches, 30 inches, and 34 inches. The length of the altitude to the longest side in inches is

(a) $15\sqrt{2}$ (b) $\sqrt{11}$ (c) $8\sqrt{2}$ (d) $\frac{240}{17}$ (e) $\sqrt{67}$

36. The length of the side of a square is the same as the length of the side of an equilateral triangle. If the square has an area of 24 square inches, then the area (in square inches) of a circle whose diameter is the altitude of the triangle will be

(a) 18π (b) 6π (c) $\frac{9}{2}\pi$ (d) $\frac{8}{3}\pi$ (e) $\frac{3}{2}\pi$

37. Consider the following four statements.

(i) The median to the base of an isosceles triangle bisects the vertex angle.
(ii) If the bisector of an angle of a triangle is also the altitude to the opposite side, then the other two sides of the triangle are congruent.
(iii) If the segment joining the midpoints of two sides of a triangle is parallel to the third side, then the triangle is isosceles.
(iv) If the median to a side of a triangle is also an altitude to that side, then the triangle is isosceles.

(a) All of these statements are true.
(b) None of these statements are true.
(c) Only one of these statements is true.
(d) Only two of these statements are true.
(e) Only three of these statements are true.

38. A regular dodecagon is inscribed in a circle with radius 4 centimeters. The length of a side of the dodecagon in centimeters will be

(a) $4\sqrt{2-\sqrt{3}}$ (b) $2(\sqrt{5}-1)$ (c) $4\sqrt{2}$

(d) $2\sqrt{10-2\sqrt{5}}$ (e) $4\sqrt{2-\sqrt{2}}$

39. A square, a rectangle (which is not a square), a parallelogram (which is not a rectangle), an ellipse (which is not a circle), and a circle each have the same perimeter. The one with the largest area is the

(a) square (b) rectangle (c) parallelogram (d) ellipse (e) circle

40. The coordinates of the midpoints of the sides of a triangle are (-4, 1), (2, -3), and (5, 4). The vertices of the triangle are

(a) (2, 1), (-10, 1), and (14, -7)
(b) (11, 0), (-7, -6), and (-1, 8)
(c) (3, -5), (1, -1), and (9, 9)
(d) (3, -5), (-11, 7), and (15, -13)
(e) (-8, 2), (4, -6), and (10, 8)