FORTY-SECOND ANNUAL MATHEMATICS CONTEST
Sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

Geometry
1998

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Scoring formula: 4R - W + 40

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft head (No. 2 lead or softer). This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly 80 minutes to work.

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1. Let p be the statement "Art is eternal" and q be the statement "life is short." If the compound statement "Art is not eternal nor life is short" is false, then

(a) p is false and q is true
(b) both p and q are true
(c) p is true and q is false
(d) both p and q are false
(e) the truth of p and q cannot be determined

2. If the statement "p implies q" is true then which of the following must be true?

(a) the negation of "p implies q"
(b) the converse of "p implies q"
(c) the inverse of "p implies q"
(d) the contrapositive of "p implies q"
(e) the negation of the converse of "p implies q"

3. A 25 foot ladder rests on a horizontal floor and leans against a vertical wall with the base of the ladder 20 feet from the wall. The ladder slides until the base is 24 feet from the wall. What is the location of the point common to both ladder positions, relative to where the wall and floor meet?

(a) 3 feet up and 7 feet out
(b) 3 feet up and 16 feet out
(c) 74/13 feet up and 144/11 feet out
(d) 4 feet up and 144/11 feet out
(e) 21/11 feet up and 192/11 feet out

4. The sum of the measures of the interior angles of a regular polygon is 1800°. How many sides does the polygon have?

(a) 12   (b) 10   (c) 9   (d) 8   (e) 5

5. Two secants drawn to a circle from an external point P form an angle of 30 degrees. The larger intercepted arc is 80 degrees. The smaller arc is

(a) 20 degrees   (b) 25 degrees   (c) 40 degrees

(d) 50 degrees   (e) 30 degrees

6. The graph of which of the following can include the points (1,-4), (4,-4), (1,10), (4,10), and (3,5)?

(a) a circle   (b) an ellipse   (c) a hyperbola

(d) a parabola   (d) two intersecting lines

7. A boat heads due east across a river at 6 miles per hour and a 6 mile per hour current is flowing from north to south. To an observer on land the apparent speed and course are:

(a) 6 mph to the southeast
(b) 12 mph to the southeast
(c) 6 mph to the southeast
(d) $6\sqrt{2}$ mph to the east
(e) $6\sqrt{2}$ mph to the southeast

8. The median of a trapezoid is parallel to both bases and its length is equal to:

(a) the sum of the lengths of the bases
(b) half the sum of the lengths of the four sides
(c) half the sum of the lengths of the bases
(d) the length of the smaller base
(e) the length of the larger base

9. If the line segment joining the midpoints of two sides of an equilaternal triangle measures 12, find the measure of the perimeter of the triangle.

(a) 24   (b) 36   (c) 48   (d) 72   (e) 144

10. Suppose the radius of a circle is increased from 4000 to 4001 miles. By how many square miles does the area increase?

(a) 8001π (b) 4001π (c) 1   (d) 4001   (e) π

11. An icosahedron is a solid polyhedron with 20 faces, each an equilateral triangle. How many edges does an icosahedron have.

(a) 10   (b) 12   (c) 20   (d) 24   (e) 30

12. The shadow of a tall tree is 39 feet long and the shadow of a short tree is 3 feet long. If the short tree has a height of 5 feet, what is the height of the tall tree?

(a) 41 feet   (b) 46 feet   (c) 65 feet   (d) 117 feet   (e) 195 feet

13. A rectangular corner lot is 120 feet by 160 feet. How much distance is saved by walking along a diagonal of the lot rather than along the outside?

(a) 80 feet   (b) 200 feet   (c) 40 feet   (d) 40√2 feet   (e) 280 feet

14. Assume that the earth is a perfect sphere with a radius of 4000 miles. From a spacecraft 100 miles above, how far is the horizon?

(a) 100 miles　　　　(b) 141 miles　　　　(c) 638 miles　　　　(d) 900 miles　　　　(e) 25,133 miles

15. The ratio of the area of one equilateral triangle to another is nine. What is the ratio of the lengths of their altitudes?

(a) 3　　　　(b) 9　　　　(c) $(\sqrt{3})/2$

(d) $(3\sqrt{3})/4$　　　　(e) $(9\sqrt{3})/4$

16. What is the maximum number of regions that the plane can be divided into by three lines?

(a) 6　　　　(b) 7　　　　(c) 8　　　　(d) 9　　　　(e) 10

17. A regular hexagon with sides of length 1 lies within an equilateral triangle with sides of length 3 as shown. What is the area of the hexagon?

(a) $\dfrac{3\sqrt{3}}{4}$　　　　(b) $\dfrac{3\sqrt{3}}{2}$　　　　(c) 6

(d) $6\sqrt{3}$　　　　(e) $3\sqrt{2}$

18. If the ratio of the surface area of Cube A to the surface area of cube B is 4, then the ratio of the volumes is:
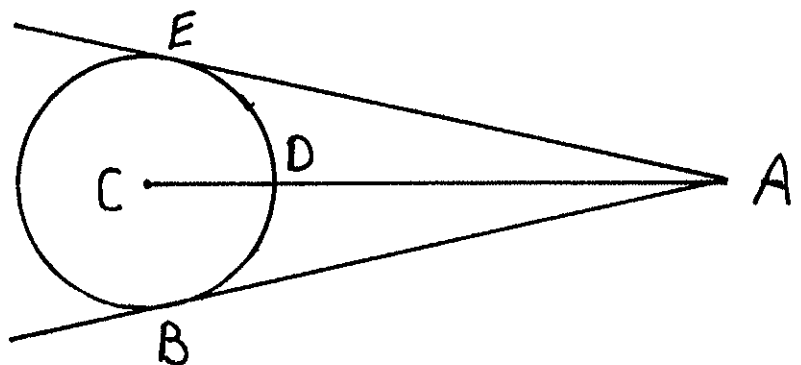
(a) 8　　　　(b) $4^{1/2}$　　　　(c) 4　　　　(d) $4^{3/2}$　　　　(e) 4^2

19. The vertices of a triangle have coordinates (8, 0), (12, 8), and (0, 12). The area of this triangle is:

(a) $8\sqrt{3}$　　　　(b) 7　　　　(c) $\sqrt{10} + \sqrt{5} + \sqrt{3}$

(d) $8\sqrt{2}$　　　　(e) 56

20. In the figure the points A, D, and the center of the circle C are collinear. $\overline{AB}$ and $\overline{AE}$ are tangent to the circle. If AB=b units and AD=a units, what is the radius of the circle?

(a) $\dfrac{b^2 - a^2}{2a}$　　　　(b) $\dfrac{a^2 + b^2}{2}$

(c) $\dfrac{b - a}{2}$　　　　(d) $a^2 - b^2$

(e) $a^2 + b^2$

21. The vertices of a trapezoid have coordinates (2, 0), (5, 0), (2, 4), and (5, 8). The area of this trapezoid is:

(a) 18   (b) $12\sqrt{2}$ (c) 20   (d) 12   (e) 5

22. An equilateral triangle is inscribed in a circle of radius 12. Which of the following most closely · approximates the area of one of the three sectors inside the circle and outside the trialgle?

(a) 74.2   (b) 78.5   (c) 83.6   (d) 88.4   (e) 92.5

23. A room measures 9 feet by 12 feet and is 8 feet high. How many feet is it from the floor at one corner to the ceiling at the opposite corner?

(a) 289   (b) 29   (c) $4\sqrt{13}$ (d) 17   (e) 15

24. Circle B lies within Circle A, is tangent to Circle A, and passes through the center of Circle A. What is the value of the ratio of the circumferences of these two circles?

(a) $2/\pi$ (b) $\pi/2$ (c) 4   (d) π (e) 2

25. A barn with a flat roof is rectangular in shape, 10 yards wide, 13 yards long, and 15 feet high. It is to be painted inside and outside and on the ceiling, but not on the roof or the floor. In square yards, what is the total area to be painted?

(a) 360   (b) 460   (c) 490   (d) 590   (e) 720

26. A coin of radius 12 mm lies tangent to another coin of 15 mm on a table. While holding the larger coin fixed, the smaller coin is rolled along the edge of the larger coin until the two tangent points are together again. How many times is the smaller coin rolled around the larger coin?

(a) 27   (b) 4   (c) 3   (d) 2   (e) 1

27. The number of distinct points common to the graphs of $x^2 + y^2 = 16$ and $x^2 + 4y = 16$ is:

(a) 0   (b) 1   (c) 2   (d) 3   (e) 4

28. An isosceles right triangle is inscribed in a circle of radius r. What is the area of this isosceles triangle?

(a) r^2 (b) $2r + 2r\sqrt{2}$ (c) $2r + \sqrt{2}$ (d) $4r$ (e) 34

29. Given 12 points in a plane,, no three of which are collinear, the number of lines determined by the points is:

(a) 24   (b) 54   (c) 66   (d) 120   (e) 132

30. Using the given figure, find a formula for x in terms of w, y, and z.

(a) $x = y - z + w$ (b) $x = y + z - w$ (c) $x = 180 - y + z + w$

(d) $x = 180 + y - z - w$ (e) $x = w + z - y$
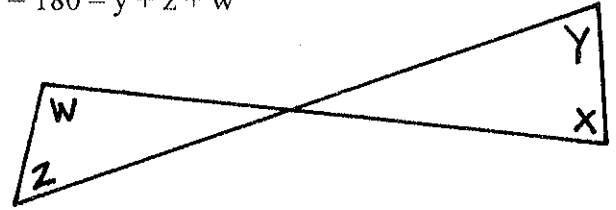
31. Circles of radii 3, 4, and 5 are mutually tangent to each other in the plane. What is the perimeter of the triangle whose vertices are the three centers of these three circles?
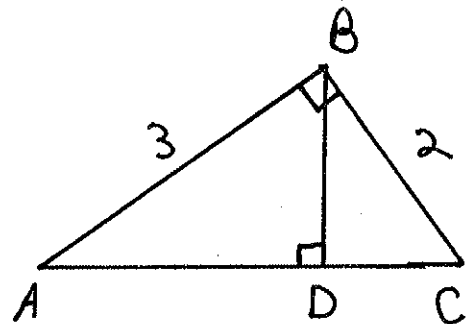
(a) 4π (b) 12   (c) $12\sqrt{2}$ (d) 24   (e) 60

32. The angle formed by two tangents drawn to a circle of radius 1 is 60°. What is the distance between the two points of tangency?

(a) 2   (b) $3\sqrt{2}$ (c) $2\sqrt{3}$ (d) $(3\sqrt{2})/2$ (e) $\sqrt{3}$

33. Find the length of AD in the figure.

(a) $9/\sqrt{13}$ (b) 5/2   (c) $2\sqrt{3}$

(d) $2\sqrt{2}$ (e) $\sqrt{6}$

34. Two sides of a square are radii of a circle. What is the ratio of the area of the circle to the area of the square?

(a) 1   (b) 2   (c) 4   (d) π (e) $\pi/2$

35. What is the ratio of the area of a square inscribed in a semicircle to the area of a square inscribed in the entire circle?

(a) 1/3   (b) 2/5   (c) 1/2   (d) 2/3   (e) 3.4

36. How fast in feet per minute will a cyclist travel on a bicycle with 26 inch diameter wheels if the gear with the pedals has diameter 10 inches, the gear on the back wheel has diameter 4 inches, and she moves the pedals around at the rate of 44 revolutions per minute? (Round to the nearest 10 ft/min.)

(a) 630   (b) 680   (c) 780   (d) 710   (e) 750

37. For any given scalene triangle, which three points are collinear?

(a)   incenter, orthocenter and circumcenter
(b)   orthocenter, incenter and centroid
(c)   incenter, circumcenter and centroid
(d)   orthocenter, circumcenter and centroid
(e)   no three of these points are always collinear in a scalent triangle

38. Two points located on the hypotenuse of right triangle ABC (where C is the right angle) divide the hypotenuse into three equal parts. If the segment drawn between one of these points and the vertex C measures 7 units and the segment drawn between the other point and C mesures 9 units, what is the length of the hypotenuse?

(a) $3\sqrt{65}$ (b) $3\sqrt{17}$ (c) $3\sqrt{26}$ (d) $16\sqrt{3}$ (e) $7\sqrt{15}$.

39. The graph of the line defined by $y = x + 3$ is reflected about graph of the line defined by $y = x$. The resulting graph is of the line $y =$:

(a) $3x$ (b) $x - 3$ (c) $x + 3$ (d) $x - 1/3$ (e) $x + 1/3$

40. The graph of $(x - y - 1)^2 + (x + y - 7)^2 = 0$ is

(a) one point   (b) two points   (c) two lines

(d) a parabola   (e) a circle