

FORTIETH ANNUAL MATHEMATICS CONTEST

sponsored by

THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

Geometry 1996

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Scoring formula:  $4R - W + 40$

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DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft head (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly 80 minutes to work.

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Contributors to TMTA for the Annual Mathematics Contest:

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TRW Commercial Steering Division, Lebanon, Tennessee  
Wright Industries, Inc., Nashville, Tennessee



TMTA CONTEST - GEOMETRY

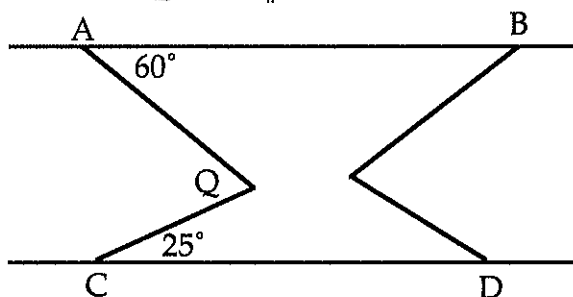
1. The measure of each of two angles of a triangle is  $60^\circ$ , and the length of the included side is 4 inches. The area of the triangle in square inches is:

a.  $8\sqrt{3}$       b. 8      c.  $4\sqrt{3}$       d. 4      e.  $2\sqrt{3}$

2. The length of the diagonal of a square is  $x + y$ . The perimeter of a second square with twice the area of the first is:

a.  $(x + y)^2$       b.  $\sqrt{2}(x + y)^2$       c.  $2(x + y)$       d.  $\sqrt{8}(x + y)$       e.  $4(x + y)$

3. Find the measure of  $\angle Q$  if  $\vec{AB} \parallel \vec{CD}$ .



a.  $105^\circ$       b.  $90^\circ$       c.  $85^\circ$       d.  $60^\circ$       e.  $25^\circ$

4. Given the true statement: If a quadrilateral is a square, then it is a rectangle. Of the converse and the inverse of this true statement:

a. only the converse is true  
 b. only the inverse is true  
 c. both are true  
 d. neither is true  
 e. the inverse is true, and the converse is sometimes true

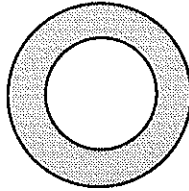
5.  $\overline{XY}$  is the hypotenuse of the right triangle  $XYZ$ . The length of the median  $\overline{XU}$  is 4 and the length of the median  $\overline{YV}$  is 7. The length of  $\overline{XY}$  is:

a. 10      b.  $5\sqrt{3}$       c.  $5\sqrt{2}$       d.  $2\sqrt{13}$       e.  $2\sqrt{15}$

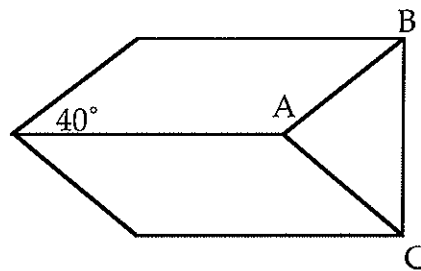
6. A rhombus is formed by two radii and two chords of a circle which has radius 16 meters. The area of the rhombus in square meters is:

a. 128      b.  $128\sqrt{3}$       c. 256      d. 512      e.  $512\sqrt{3}$

7. The shaded region pictured here is bounded by two concentric circles and has an area of  $40\pi$  cm<sup>2</sup>. If the inner circle has a radius of 9 cm, find the radius of the outer circle.



- a. 11 cm    b. 12 cm    c. 13 cm    d. 14 cm    e. 15 cm
8. Given that the two parallelograms pictured here are similar, find the measure of  $\angle ABC$ .



- a.  $50^\circ$     b.  $55^\circ$     c.  $52.5^\circ$     d.  $60^\circ$     e.  $57.5^\circ$
9. A regular octagon is formed by cutting congruent isosceles right triangles from the corners of a square. If the length of the side of the square is 1, the length of the leg of one of the triangles is:

- a.  $\frac{2+\sqrt{2}}{3}$     b.  $\frac{2-\sqrt{2}}{2}$     c.  $\frac{1+\sqrt{2}}{2}$     d.  $\frac{1+\sqrt{2}}{3}$     e.  $\frac{2-\sqrt{2}}{3}$

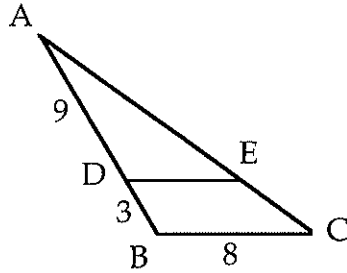
10. The vertices of triangle  $PQR$  have coordinates as follows:  $P(0, a)$ ,  $Q(b, 0)$ ,  $R(c, d)$ , where  $a$ ,  $b$ ,  $c$ , and  $d$  are positive. The origin and the point  $R$  lie on opposite sides of line  $PQ$ . The area of triangle  $PQR$  is:

- a.  $\frac{ab+ac+bc+cd}{2}$     b.  $\frac{ac+bd-ab}{2}$     c.  $\frac{ab-ac-bd}{2}$   
 d.  $\frac{ac+bd+ab}{2}$     e.  $\frac{ac+bd-ab-cd}{2}$

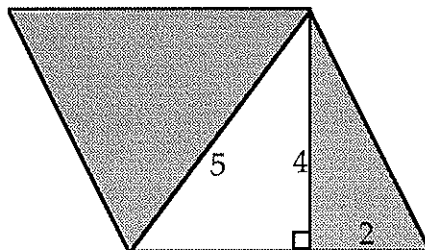
11. If the length of each side of a square is squared, how does the area change?

- a. It doesn't change.    b. It is squared  
 c. It is raised to the power of 4.    d. It is multiplied by 4.  
 e. It doubles.

12. In the figure below,  $\overline{BC} \parallel \overline{DE}$ ,  $AC = 16$ , and other dimensions are as shown. Find  $DE$ .

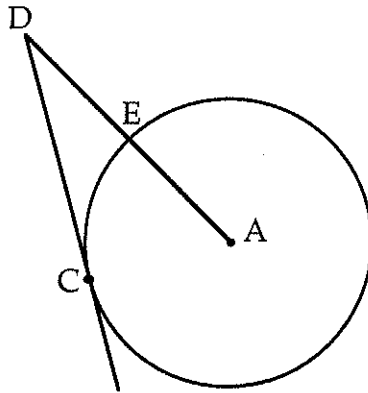


- a. 12      b.  $10\frac{2}{3}$       c.  $14\frac{2}{3}$       d. 6      e.  $21\frac{1}{3}$
13. Which of the following are true?  
 I. Every rectangle is a parallelogram.  
 II. Every rhombus is a square.  
 III. Every square is a rectangle.
- a. I only      b. II only      c. III only  
 d. I and II only      e. I and III only
14. Which of the following is false?
- a. Supplements of congruent angles are congruent.  
 b. Complements of congruent angles are congruent.  
 c. Supplements of right angles are right angles.  
 d. All right triangles are similar to each other.  
 e. All squares are similar to each other.
15. The midpoints of the sides of an equilateral triangle are connected to form a smaller triangle with perimeter 6. What is the perimeter of the original triangle?
- a. 10      b. 12      c. 15      d.  $5\sqrt{3}$       e. 9
16. What is the area of the shaded portion of this parallelogram?

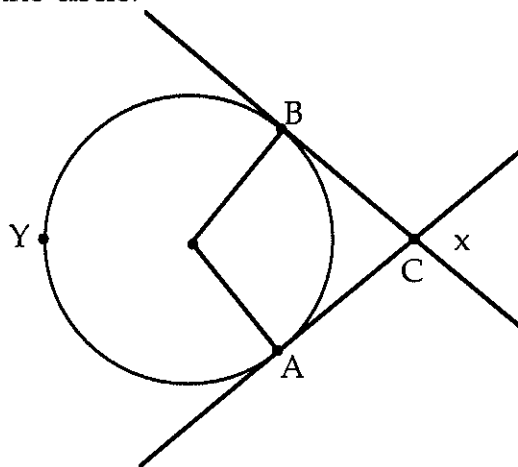


- a. 6      b. 8      c. 14      d. 16      e. 20

17. If the radius of a circle is increased 100%, the area is increased
- a. 100%      b. 200%      c. 300%      d. 400%      e. 600%
18. The volume of a rectangular solid the area of whose side, front, and bottom faces are  $12 \text{ in}^2$ ,  $8 \text{ in}^2$ , and  $6 \text{ in}^2$ , respectively is
- a.  $576 \text{ in}^3$       b.  $24 \text{ in}^3$       c.  $9 \text{ in}^3$       d.  $104 \text{ in}^3$       e.  $96 \text{ in}^3$
19. Given that  $DE = 2$ ,  $CD = 4$ ,  $A$  is the center of the circle, and  $\overleftrightarrow{CD}$  is tangent to the circle, find the radius of the circle.

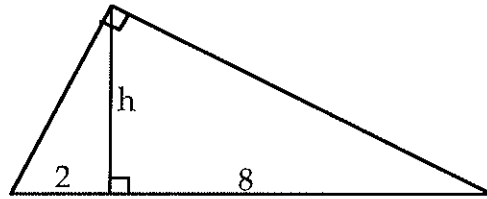


- a. 2      b. 3      c. 4      d. 5      e. 6
20. Find  $x$ , given that the measure of arc  $BYA = 250^\circ$  and that  $\overline{AC}$  and  $\overline{BC}$  are tangent to the circle.

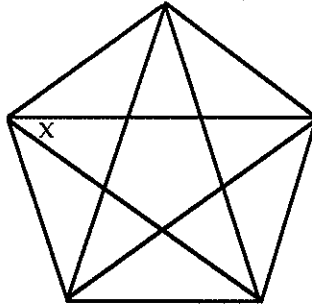


- a.  $250^\circ$       b.  $110^\circ$       c.  $125^\circ$       d.  $75^\circ$       e.  $70^\circ$
21. The area of the largest triangle that can be inscribed in a semi-circle whose radius is  $r$  is
- a.  $r^2$       b.  $r^3$       c.  $2r^2$       d.  $2r^3$       e.  $\sqrt{2}r$

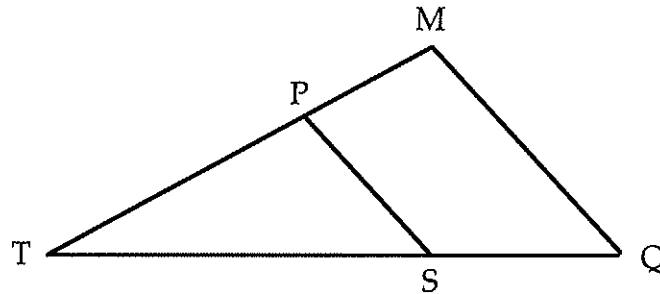
22. Find  $h$ .



- a. 3                      b. 4                      c. 5                      d. 6                      e. 7
23. Find the measure of angle  $x$  in this regular figure.

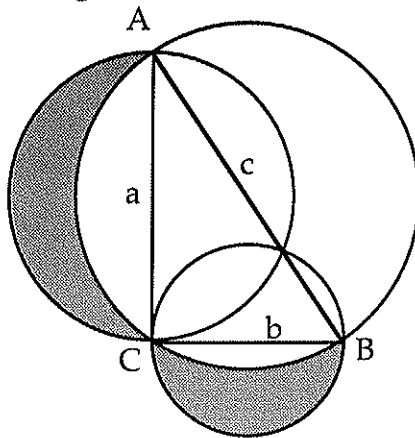


- a.  $36^\circ$                       b.  $72^\circ$                       c.  $108^\circ$                       d.  $60^\circ$                       e.  $30^\circ$
24. Assume  $\triangle TQM$  and  $\triangle TSP$  are similar. If  $TQ = 12$  units,  $TS = 8$  units, and  $TP = 6$  units, what is  $TM$ ?



- a. 6 units                      b. 7.5 units                      c. 9 units                      d. 10 units                      e. 12 units
25. In a quadrilateral  $ABCD$ ,  $AB = BC = CD$ , and also  $AD = AC = BD$  ( $\overline{AC}$  and  $\overline{BD}$  being diagonals of the quadrilateral). Find the measure of the angle  $ABC$ .
- a.  $108^\circ$                       b.  $118^\circ$                       c.  $98^\circ$                       d.  $128^\circ$                       e.  $88^\circ$
26. A 32-foot telephone pole is held in place by a support wire which is attached to the top of the pole and to a stake in the ground 24 feet from the bottom of the pole. How long is the wire?
- a. 30 ft                      b. 40 ft                      c. 50 ft                      d. 56 ft                      e. 60 ft

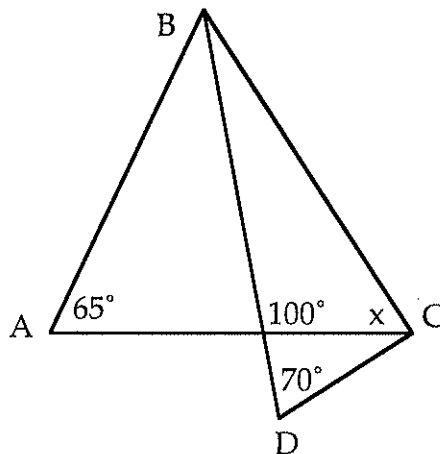
27.  $\overline{AD}$  and  $\overline{BC}$  are diameters of a circle where  $A, B, C,$  and  $D$  lie on the circle,  $E$  is the center of the circle, and one of the arcs from  $B$  to  $D$  has measure  $210^\circ$ . Find the measure of  $\angle BED$ .
- a.  $150^\circ$     b.  $155^\circ$     c.  $160^\circ$     d.  $210^\circ$     e.  $145^\circ$
28. Two points located on the hypotenuse of right triangle  $ABC$  divide the hypotenuse into three equal parts. If the segment drawn between one of these points and the vertex  $C$  measures 7 units and the segment drawn between the other point and  $C$  measures 9 units, what is the length of the hypotenuse  $\overline{AB}$ ?
- a.  $3\sqrt{65}$     b.  $3\sqrt{17}$     c.  $3\sqrt{26}$     d.  $16\sqrt{3}$     e.  $7\sqrt{15}$
29. The vertex angle of an isosceles triangle measures  $120^\circ$  and the equal sides are 10 inches. What is the area of the triangle?
- a.  $50 \text{ in}^2$     b.  $25 \text{ in}^2$     c.  $25\sqrt{3} \text{ in}^2$     d.  $50\sqrt{3} \text{ in}^2$     e.  $25\sqrt{2} \text{ in}^2$
30. Circles are constructed on each side of the three sides of right triangle  $ABC$ . In each case the center of the circle is the midpoint of the side, with the side being a diameter of the circle. If the area of the triangle  $ABC$  is 12 square units, what is the total area of that part of the two smaller circles that lies outside the largest circle? (The area in question is that of the shaded region shown below.)



- a.  $6\pi$     b. 12    c.  $\frac{ab\pi}{2}$     d.  $\frac{ab}{3}$     e.  $12\pi$
31. A cone shaped cup is 12 cm high and 10 cm in diameter. How much paper was used to make the cup?
- a.  $60\pi \text{ cm}^2$     b.  $26\pi \text{ cm}^2$     c.  $32.5\pi \text{ cm}^2$     d.  $65\pi \text{ cm}^2$     e.  $169\pi \text{ cm}^2$

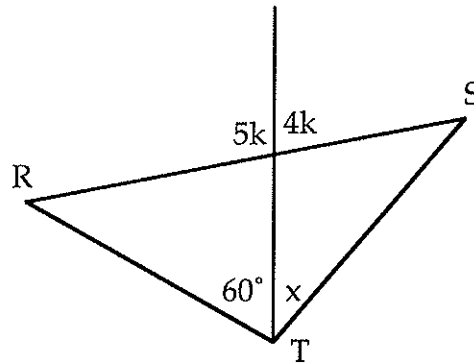


32. The base of a circus tent is a regular octagon with 36 ft sides. The roof of the tent is made of cloth sections which attach each side of the octagon to the top of a center pole. The distance from the top of the center pole to an edge of the octagon is 30 ft. How much cloth was used to make the roof of the tent?
- a. 4000 ft<sup>2</sup>    b. 4320 ft<sup>2</sup>    c. 3600 ft<sup>2</sup>    d. 7200 ft<sup>2</sup>    e. 1200 ft<sup>2</sup>
33. Consider a triangle  $\triangle ABC$  with a point  $P$  inside.  
 $\angle APB = 125^\circ$ ,  $\angle CAP = 25^\circ$ , and  $AP = PC$ . Find the measure of  $\angle BPC$ .
- a.  $100^\circ$     b.  $105^\circ$     c.  $110^\circ$     d.  $115^\circ$     e.  $120^\circ$
34. How many sides does each base of a prism have if the prism has 20 vertices?
- a. 4    b. 5    c. 8    d. 10    e. 20
35. A room is 16 ft long, 14 ft wide, and 9 ft high. It has a door which is 7 ft high and 3 feet wide, and two windows which are 3 ft wide and 6 ft high. (Assume all measurements are to the nearest fourth of an inch.) A roll of wall paper is 21 inches wide and 60 ft long. How many rolls will be needed to paper the walls of the room?
- a. 5 rolls    b. 6 rolls    c. 7 rolls    d. 8 rolls    e. 9 rolls
36.  $BC \perp CD$ . Find  $x$ .

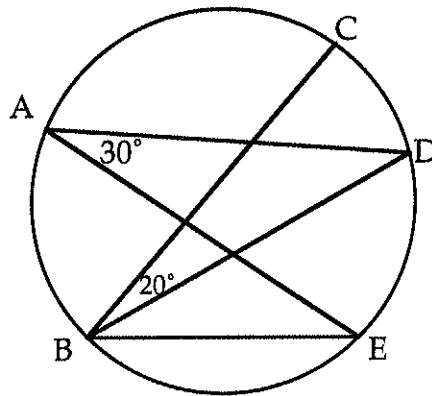


- a.  $20^\circ$     b.  $25^\circ$     c.  $50^\circ$     d.  $60^\circ$     e.  $65^\circ$
37. The area of a square inscribed in a circle of area  $18\pi$  is
- a. 25    b. 30    c. 35    d. 36    e. 40

38.  $RT = TS$  and  $k$  is a constant. Find  $x$ .



- a.  $20^\circ$       b.  $40^\circ$       c.  $50^\circ$       d.  $60^\circ$       e.  $65^\circ$
39. Radians can be used to measure the arc of a circle. One radian is  $\frac{180}{\pi}$  degrees. How many radians are needed for a complete circle?
- a.  $2\pi$       b. 6      c. 6.28      d. 360      e.  $\pi$
40. In this diagram find the measure of  $\angle CBE$ .



- a.  $45^\circ$       b.  $50^\circ$       c.  $55^\circ$       d.  $60^\circ$       e.  $65^\circ$



