

THIRTY-NINTH ANNUAL MATHEMATICS CONTEST

sponsored by

THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

Geometry 1995

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Scoring formula: $4R - W + 40$

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft head (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly 80 minutes to work.

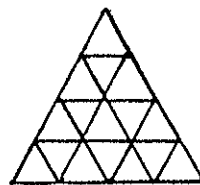
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TMTA CONTEST - GEOMETRY

1. The measure of an exterior angle of a regular dodecagon is:
a. 12° b. 30° c. 24° d. 18° e. 36°
2. Which of the following classes of quadrilaterals contains all the others as special cases?
a. rectangles b. parallelograms c. rhombuses d. squares e. trapezoids
3. For any given scalene triangle, which three of these points are collinear?
a. incenter, orthocenter and circumcenter
b. orthocenter, incenter and centroid
c. incenter, circumcenter and centroid
d. orthocenter, circumcenter and centroid
e. no three of these points are always collinear in a scalene triangle.

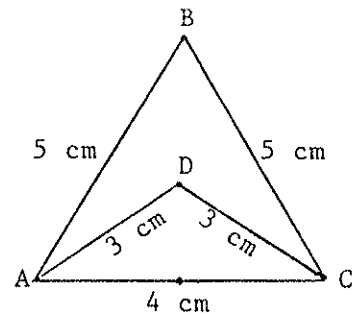
4. The total number of triangles in the given figure is:
a. 27 b. 26 c. 25 d. 17 e. 16



5. Given a polyhedron with 37 edges and 23 vertices, the number of faces is:
a. 16 b. 58 c. 12 d. 62 e. 60
6. Which of the following is not a valid congruence theorem for triangles?
a. SAS b. SSA c. SSS d. ASA e. SAA
7. What is the area of a triangle with sides of measure 11 cm, 17 cm, and 23 cm? (Give the answer correct to the nearest square cm.)
a. 86 cm^2 b. 87 cm^2 c. 88 cm^2 d. 89 cm^2 e. 90 cm^2

8. Refer to the given diagram. Find the area of quadrilateral ABCD.

- a. $2\sqrt{21}$ b. $2\sqrt{21} - 2\sqrt{5}$ c. $2\sqrt{17}$
 d. $2\sqrt{27}$ e. $2\sqrt{27} - 2\sqrt{5}$



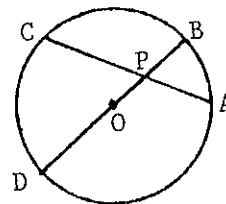
9. Three circles in a plane, each with radius 20, are mutually tangent. What is the area of the region enclosed between (outside of) the three circles?

- a. $400\sqrt{3}$ b. $200\pi - 400\sqrt{3}$ c. 200π d. 400π e. $400\sqrt{3} - 200\pi$

10. The longest stick that can be enclosed in a right rectangular box that is 12 cm wide, 15 cm long, and 9 cm high will measure:

- a. $3\sqrt{41}$ cm b. $3\sqrt{34}$ cm c. $15\sqrt{2}$ cm d. 15 cm e. 36 cm

11. Refer to the diagram of circle O. Given that chord BD is a diameter, if $AP = 15$, $BP = 9$, and $PC = 36$, find the radius of the circle.



- a. 34.5 b. 32.4 c. 33.8 d. 35.3 e. 30.7

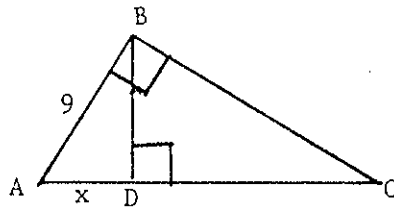
12. Which of the following statements is false?

- a. All squares are rectangles.
 b. Some rectangles are rhombuses.
 c. Some rhombuses are squares.
 d. Every rhombus is a regular polygon.
 e. All regular quadrilaterals are squares.

13. The area of a circle circumscribed about a square with a side of 4 m is:

- a. 8π m² b. 4π m² c. 16 m² d. 16π m² e. 22π m²

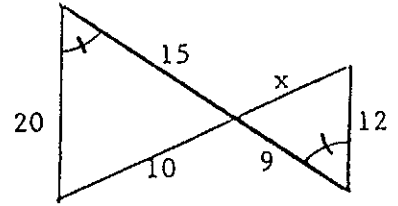
14. How fast (feet per minute) will a cyclist travel on a bicycle with 26 inch diameter wheels if the gear with the pedals has diameter 10 inches, the gear on the back wheel has diameter 4 inches, and she moves the pedals around at the rate of 44 revolutions per minute? (Round to the nearest 10 ft/min.)
- a. 630 ft/min b. 680 ft/min c. 780 ft/min d. 710 ft/min e. 750 ft/min
15. Which of the following statements is false?
- a. The perimeter of a rectangle may be equal to its area.
 b. The perimeter of a rectangle may be greater than its area.
 c. The perimeter of a rectangle may be less than its area.
 d. If the perimeter of a rectangle is equal to its area, then the length of the rectangle must be equal to the width of the rectangle.
 e. If the perimeter of a rectangle is equal to its area, then either the length or the width of the rectangle must be greater than 2.
16. Find the coordinates of endpoint R of segment RS, given endpoint S(5,6) and midpoint M(-2,-3).
- a. (-12,-9) b. (12,15) c. (-10,-8) d. (-5,-9) e. (-9,-12)
17. The standard method for bisecting an angle with straightedge and compass is justified by:
- a. SAS b. ASA c. SSS d. AAS e. the Pythagorean Theorem
18. The area of $\triangle ABC$ is 45 square units. Determine x to the nearest whole unit.



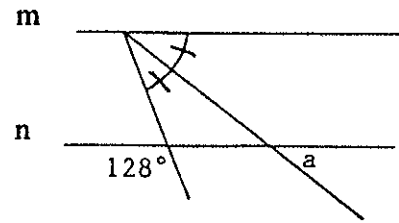
19. The volume of a right circular cone is 512π cubic centimeters. If its height is 3 times the radius of its base, what is the radius of the base?
- a. 13 cm b. 20 cm c. 8 cm d. 40 cm e. $\sqrt{7}$ cm

20. Each side of ΔABC is 6 cm. What is the area of its circumscribed circle?
- a. $6\pi \text{ cm}^2$ b. $15\pi \text{ cm}^2$ c. $9\pi \text{ cm}^2$ d. $12\pi \text{ cm}^2$ e. $8\pi \text{ cm}^2$

21. In the diagram given, determine the value of x .
- a. 7 b. 4 c. 3 d. 11 e. 6



22. Given the following diagram with parallel lines m and n , find the measure of angle a .
- a. 26° b. 31° c. 15° d. 35° e. 52°

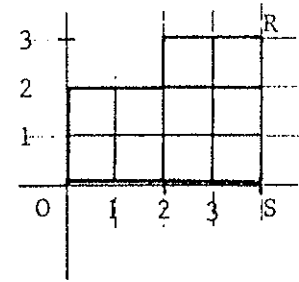


23. Find the radius of a circle with diameter AB and tangent segment BD , given that $AD = 8$ and $BD = \sqrt{15}$.
- a. 7 b. $\sqrt{7}$ c. 3.5 d. $\sqrt{3.5}$ e. $\sqrt{161}$

24. Which statement is logically equivalent to, 'All squares are rectangles.'
- a. Some squares are not rectangles.
 b. Polygons that are not rectangles are not squares.
 c. Some rectangles are not squares.
 d. All rectangles are squares.
 e. Polygons that are not squares are not rectangles.

25. Given points $A(2,5)$, $B(7,10)$, and $C(13,4)$ on a rectangular coordinate system, find the area of ΔABC .
- a. 24 b. 32 c. 18 d. 30 e. 27

26. The given figure consists of ten congruent squares. Find the y coordinate of the point, P on segment RS such that the segment OP divides the given figure into two parts of equal area.



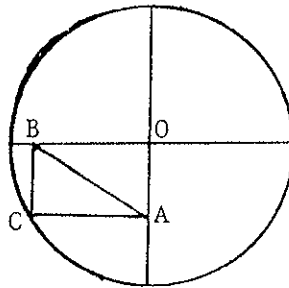
- a. 2 b. 3 c. 2.5 d. $2 \frac{1}{4}$ e. $2 \frac{3}{8}$

27. Find the equation of the image of the line defined by $y = 3x + 1$ when it is reflected about the x-axis.

- a. $y = -(1/3)x - 1$ b. $y = -3x + 1$ c. $y = 3x - 1$
 d. $y = (1/3)x - 1$ e. $y = -3x - 1$

28. In the given figure, if quadrilateral ACBO is a rectangle and the diameter of the circle is 10 cm, find the length of segment AB.

- a. 10 cm
 b. $\sqrt{10}$ cm
 c. $\sqrt{50}$ cm
 d. 5 cm
 e. $\sqrt{5}$ cm



29. If the edge of a cube is doubled, how does the volume change?

- a. It doubles b. It is multiplied by 8 c. It triples
 d. It is multiplied by 6 e. It is multiplied by 4

30. How many line segments are determined by 6 collinear points?

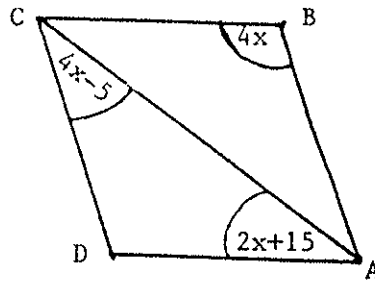
- a. 5 b. 10 c. 15 d. 20 e. 30

31. The area of a regular octagon inscribed in a circle with radius 6 m is:

- a. $64\sqrt{3}$ m² b. $72\sqrt{2}$ m² c. 112 m² d. $44\sqrt{6}$ m² e. $60\sqrt{3}$ m²

32. In the diagram, if ABCD is a rhombus, find the measure of $\angle BAD$.

- a. 68° b. 17° c. 112°
 d. 102° e. 117°



33. Circle A has radius 4 and circle B has radius 9. If Circles A and B are externally tangent, find the length of the common external tangent.

- a. 13 b. 12 c. $\sqrt{153}$ d. $\sqrt{97}$ e. 6π

34. Given a regular hexagon with each side of measure 8 cm, find the circumference of the inscribed circle.

- a. 16π cm b. 48π cm c. $4\pi\sqrt{3}$ cm d. $8\pi\sqrt{2}$ cm e. $8\pi\sqrt{3}$ cm

35. If a regular polygon of 20 sides has interior angle with measure x and exterior angle with measure y , find $x - 5y$.

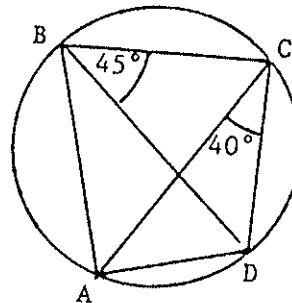
- a. 136° b. 108° c. 90° d. 72° e. 60°

36. How many diagonals does a convex decagon have?

- a. 10 b. 20 c. 35 d. 40 e. 45

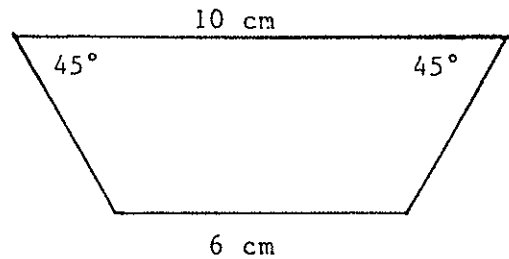
37. Refer to the given diagram. Find the measure of $\angle ADC$.

- a. 95° b. 90° c. 85°
 d. 105° e. 110°



38. To inscribe a circle within any given triangle, one must construct:
- the perpendicular bisectors of two sides.
 - the medians from two angles.
 - the altitudes from two angles.
 - a square on one side of the triangle.
 - the bisectors of two angles.

39. Refer to the given trapezoid. Find the difference when the perimeter is subtracted from the area.



- 32
 - $17 + \sqrt{218}$
 - $4\sqrt{2}$
 - $16 + 2\sqrt{32}$
 - $52 + 2\sqrt{32}$
40. How many edges does a regular dodecahedron have?
- 12
 - 18
 - 24
 - 30
 - 36