THIRTY-EIGHTH ANNUAL MATHEMATICS CONTEST
sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

Geometry 1994

Prepared by the Mathematics Dept.
Tennessee Technological University
Coordinated by Richard Savage

Scoring formula: 4R - W + 40

Edited by: Larry Bouldin, Roane State
Community College, Harriman, TN

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer, and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all the questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers, so it is advisable not to do wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet. The answer sheets will not be returned to you. If you wish to have a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet and begin. The working time for the entire test is 80 minutes. The use of calculators is prohibited.

NOTE: 1995 Contest date, April 4

Contributors to TMTA for Annual Mathematics Contest:

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GEOMETRY

1. $\overline{AB}$ and $\overline{CD}$ are chords of circle 0, intersecting at point E. If $m\widehat{BD} = 81°$ and $m\angle DEB = 52°$, what is $m\widehat{AC}$?

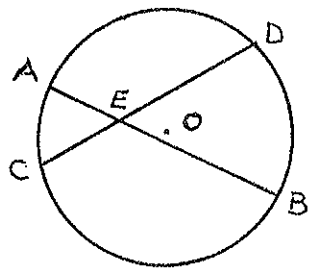
(a) 185°

(b) 23°

(c) 133°

(d) 66.5°

(e) 52°



2. How many faces does a tetrahedron have?

(a) 4

(b) 6

(c) 3

(d) 8

(e) 5

3. $\overline{AB}$ and $\overline{CD}$ are chords of circle O, intersecting at point E. If $AE = 2$, $EB = 3$, and $CE = \frac{3}{2}$, find ED.

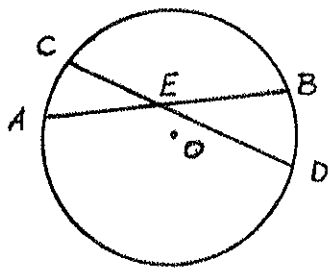
(a) $\frac{7}{2}$

(b) $\frac{3}{2}\sqrt{6}$

(c) 4

(d) 9

(e) $\frac{13}{4}$



4. Point P is the midpoint of side AB on parallelogram $ABCD$. The area of $\triangle PBC$ is what fraction of the area of the parallelogram.

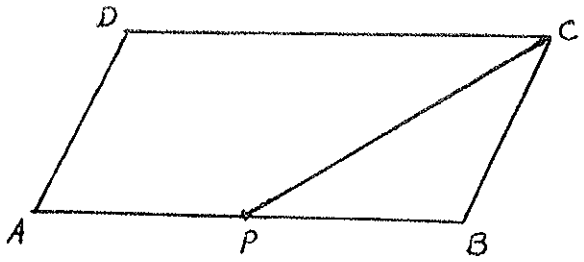
(a) $\frac{1}{8}$

(b) $\frac{1}{5}$

(c) $\frac{1}{4}$

(d) $\frac{1}{3}$

(e) $\frac{1}{2}$



5. In $\triangle ABC$, $\overline{DC}$ bisects $\angle ACB$. If $AC = 6$, $BC = 5$, and $AB = 8$, What is BD?

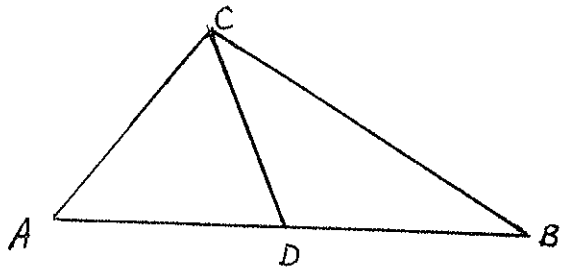
(a) $3\frac{7}{11}$

(b) $4\frac{4}{11}$

(c) $6\frac{2}{3}$

(d) $9\frac{3}{5}$

(e) $3\frac{3}{4}$



6. $\triangle ABC$ is an equilateral triangle inscribed in a circle of radius 1. Find the shaded area.

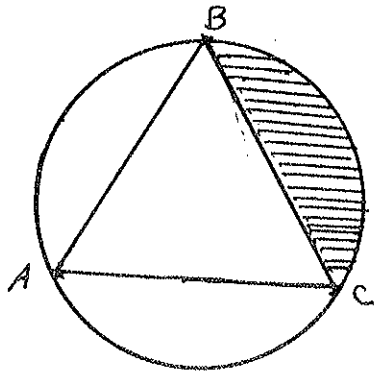
(a) $\pi - 3$

(b) $\frac{4\pi - 3\sqrt{3}}{12}$

(c) $\frac{8\pi - 3\sqrt{3}}{24}$

(d) $\frac{4\pi - 3\sqrt{3}}{4}$

(e) $\frac{8\pi - 3\sqrt{3}}{8}$



7. In $\triangle ABC$, $AB = 3$, $BC = 2$, and D is the midpoint of $\overline{AC}$. Which of the following statements must be true?

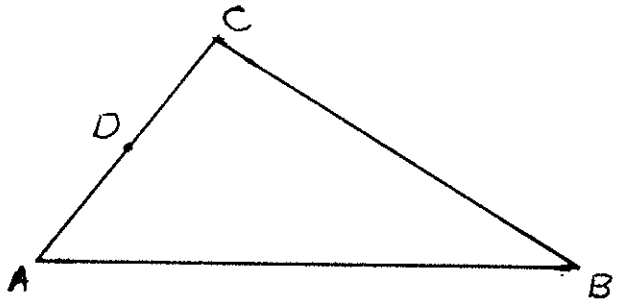
(a) $\overline{BD} \perp \overline{AC}$

(b) $\angle ABD \cong \angle CBD$

(c) $BD = 2$

(d) $m\angle C > m\angle A$

(e) $m\angle C < m\angle A$



8. Suppose that A, B, and C are centers of three mutually tangent circles with radii 3, 4, and 2 centimeters. The area of the triangle in square centimeters is:

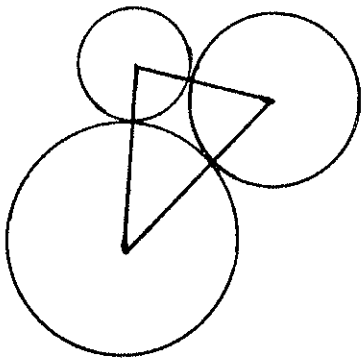
(a) $6\sqrt{6}$

(b) $8\sqrt{6}$

(c) $9\sqrt{6}$

(d) $12\sqrt{6}$

(e) 15



9. Using the given figure, find a formula for x in the terms of y, z, and w.

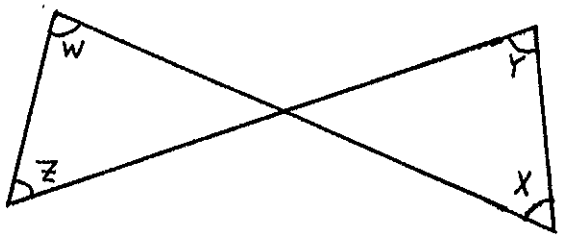
(a) $x = y - z + w$

(b) $x = y + z - w$

(c) $x = 180° - y + z + w$

(d) $x = 180° + y - z - w$

(e) $x = w + z - y$



10. Find the area of $\triangle ABC$ of the figure.

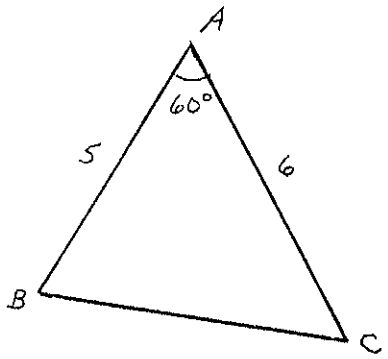
(a) $\frac{15}{2}$

(b) $\frac{15\sqrt{3}}{2}$

(c) 15

(d) $15\sqrt{3}$

(e) $\frac{33\sqrt{3}}{8}$



11. If the complements of angles A and B are complementary, then the supplements of angles A and B

(a) are congruent.

(b) are supplementary.

(c) are complementary.

(d) differ by 90°.

(e) add up to 270°.

12. (Assume in this problem that the surface of the earth is a sphere). A man walks due north for 10 miles from point A then turns and walks 10 miles due east to point B. Let d denote the distance (as measured on the sphere) from point A to point B. Then

(a) $d < 10$

(b) $d > 10$ and depends upon the starting point A

(c) $d \geq 10$ and $d = 10$ for some starting points A

(d) $d = 10\sqrt{2}$

(e) d may be less than, equal to, or more than 10 depending on the starting point A.

13. In quadrilateral $ABCD$, $\overline{CD} \perp \overline{BC}$, $\overline{BC} \parallel \overline{AD}$, $BC = 8$, $AD = 10$ and $CD = 3$. What is the area of $ABCD$?

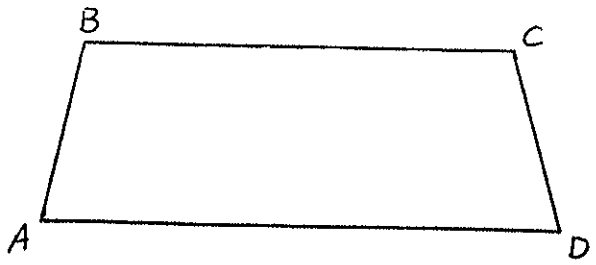
(a) 54

(b) 30

(c) 27

(d) 240

(e) 55



14. $m\angle EAD = m\angle DAB$ and $m\angle FBD = m\angle DBA$. Find the measure of $\angle ADB$.

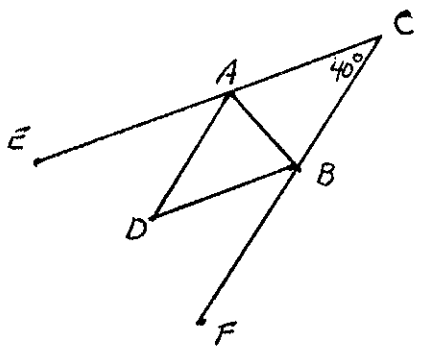
(a) 50°

(b) 60°

(c) 70°

(d) 80°

(e) 90°



15. The area of a triangle is A. If each side of the triangle is tripled the area of the resulting triangle is

(a) $3A$

(b) $3A^2$

(c) $9A$

(d) $9A^2$

(e) $27A$

16. In $\triangle ABC$, M is the midpoint of BC and $ADME$ is a parallelogram. If the shaded area is 1, what is the area of $\triangle ABC$?

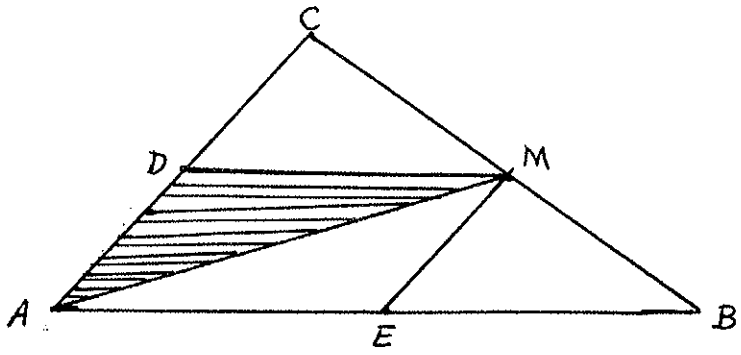
(a) $2\sqrt{2}$

(b) 3

(c) $2\sqrt{3}$

(d) 4

(e) 5



17. $\overline{AB} \parallel \overline{DE}$ and $\overline{AD} \parallel \overline{BF}$. If $m\angle ABD = 40°$ and $m\angle ECF = 20°$, what is $m\angle ADB$?

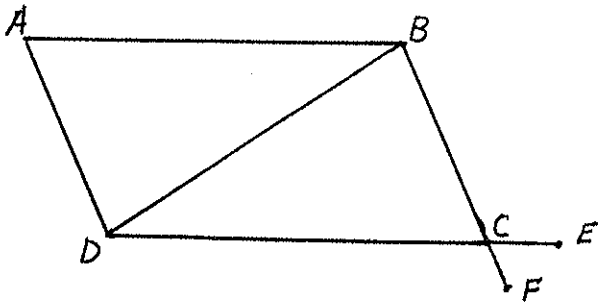
(a) 120°

(b) 20°

(c) 40°

(d) 60°

(e) 160°



18. If A and B are points on circle O, $m\angle AOB = 50°$, and $OA = 5$, what is the area of sector AOB?

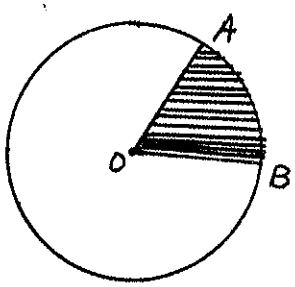
(a) 25π

(b) $\frac{125}{18}\pi$

(c) $\frac{25}{18}\pi$

(d) $\frac{25}{9}\pi$

(e) $\frac{125}{36}\pi$



19. In the figure, the semicircle centered at O has radius 1, $\overline{CO} \perp \overline{AB}$, and AEC forms a semicircle with diameter $\overline{AC}$. Find the area of the curved region $AECD$.

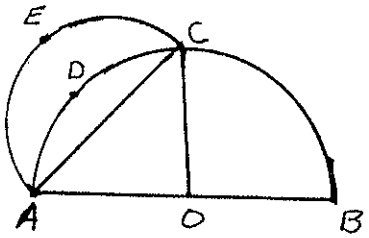
(a) $\frac{1}{2}$

(b) $\frac{\pi}{2} - 1$

(c) $\frac{\pi}{4} - \frac{1}{2}$

(d) $\frac{\pi}{4} - (\frac{\sqrt{2}-1}{2})$

(e) $\frac{3\pi+1}{2}$



20. $\overline{AB} \perp \overline{BC}$, $\overline{AC} \perp \overline{BD}$, $AB = 8$ and $AD = 5$. What is BC?

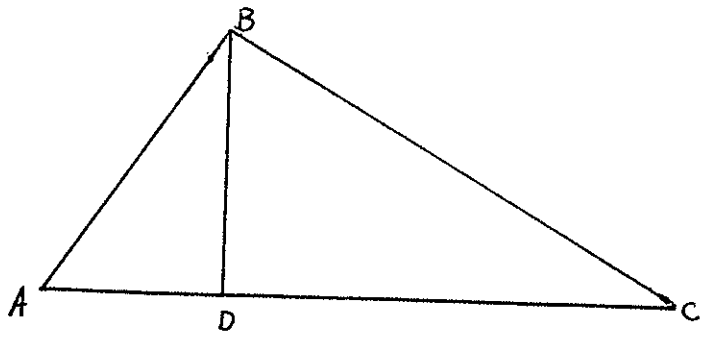
(a) $\sqrt{39}$

(b) $\frac{8\sqrt{39}}{5}$

(c) $\frac{64}{5}$

(d) $2\sqrt{10}$

(e) $\sqrt{89}$



21. What is the ratio of the area of a square inscribed in a semicircle to the area of a square inscribed in the entire circle?

(a) $\frac{1}{3}$

(b) $\frac{2}{5}$

(c) $\frac{1}{2}$

(d) $\frac{2}{3}$

(e) $\frac{3}{4}$

22. The hypotenuse of a right triangle is divided into segments of length 3 and 5 by an altitude whcih extends from the right angle. What is the area of the triangle?

(a) 17

(b) $\frac{64}{15}$

(c) $4\sqrt{15}$

(d) $\frac{\sqrt{34}}{2}$

(e) $\frac{\sqrt{34}}{30}$

23. The two larger circles are tangent and have equal radii. They are each tangent to the smaller circle which has a radius of 4. All three circles are tangent to the line. What is the radius of the larger circles.

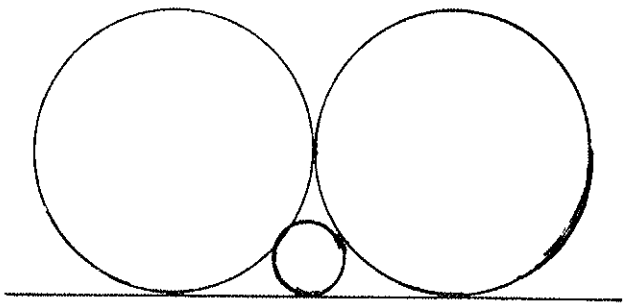
(a) 12

(b) $11\sqrt{2}$

(c) 16

(d) $12\sqrt{2}$

(e) $14\sqrt{2}$



24. Find the altitude from B to $\overline{AC}$ of the triangle ABC, given that its area is 2 in ^{2} and $AB = BC$.

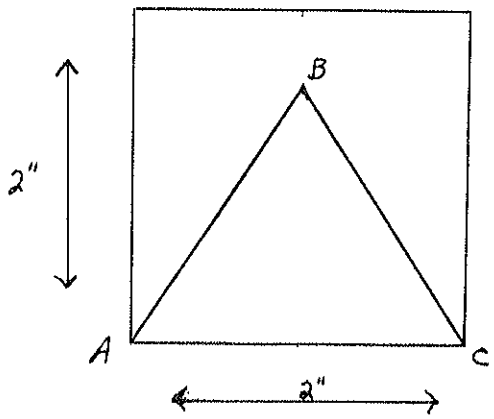
(a) $\sqrt{2}$ in.

(b) $\frac{\sqrt{2}}{2}$ in.

(c) $\sqrt{3}$ in.

(d) 2 in.

(e) 1 in.



25. ABCD is a square of area 1. $\overline{EF}$, and $\overline{GH}$ are parallel to the diagonal and divide the square into three regions of equal area. Find the length of $\overline{EF}$.

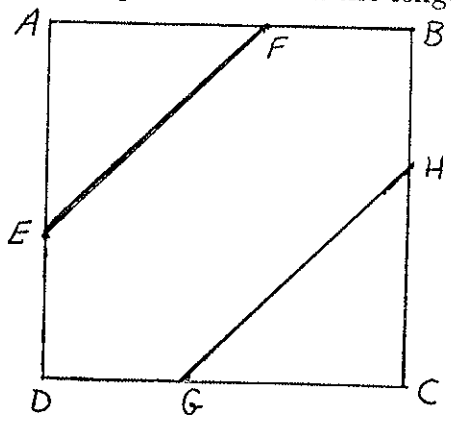
(a) $\frac{1}{3}$

(b) $\frac{1}{2}$

(c) $\frac{\sqrt{3}}{3}$

(d) $\frac{\sqrt{6}}{3}$

(e) $\frac{2\sqrt{3}}{3}$



26. The area of a triangle is 2 square feet. If the length of the base of the triangle is 20 inches greater than the length of the altitude, how long is the base of the triangle?

(a) $10 + 2\sqrt{26}$ inches

(b) $10 - 2\sqrt{26}$ inches

(c) 16 inches

(d) $10 + \sqrt{102}$ inches

(e) 36 inches

27. Find r such that the areas of the washer and hole are equal.

(a) $\frac{\sqrt{2}}{2}$

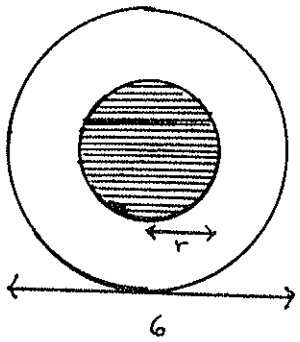
(b) $\frac{\sqrt{3}}{2}$

(c) $\sqrt{2}$

(d) $\frac{3}{2}$

(e) $\frac{3\sqrt{2}}{2}$

(r denotes the radius of the hole)



28. Suppose that $\triangle ABC$ is an isosceles right triangle with right angle at B as shown in the figure. The perimeter of rectangle BDEF inscribed in the triangle is equal to

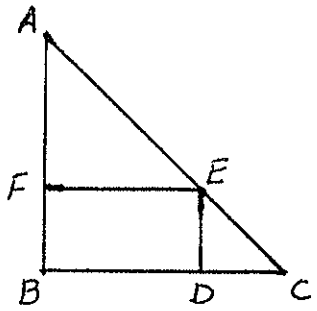
(a) one half of the perimeter of $\triangle ABC$.

(b) twice the length of $\overline{BC}$.

(c) twice the length of $\overline{AC}$.

(d) the sum of the lengths of $\overline{AC}$ and $\overline{BC}$.

(e) cannot be determined from the given information.



29. Given $XY = WZ$, what other equality can be deduced?

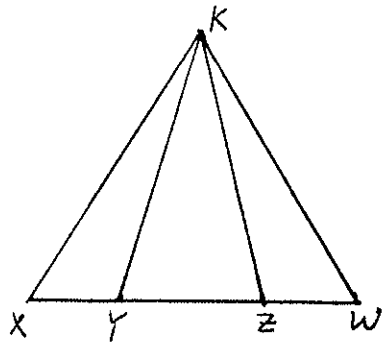
(a) $m\angle X = m\angle W$

(b) $XK = WK$

(c) $XZ = YW$

(d) $KY = KZ$

(e) $m\angle XKY = m\angle WKZ$



30. If $\angle ABC \cong \angle BDE$ and $\angle CBD \cong \angle EDF$, which of the following pairs of segments must be parallel?

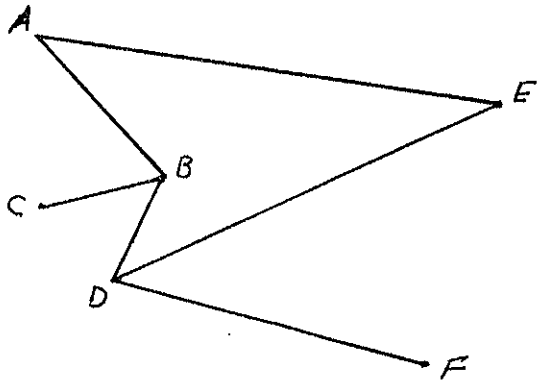
(a) $\overline{AB}$ and $\overline{DE}$

(b) $\overline{BC}$ and $\overline{DE}$

(c) $\overline{BC}$ and $\overline{DF}$

(d) $\overline{AB}$ and $\overline{DF}$

(e) $\overline{AE}$ and $\overline{DF}$



31. If $\triangle ABD \cong \triangle BAE$, A, D, and C are collinear, and B, E, and C are collinear, which of the following statements is a valid conclusion?

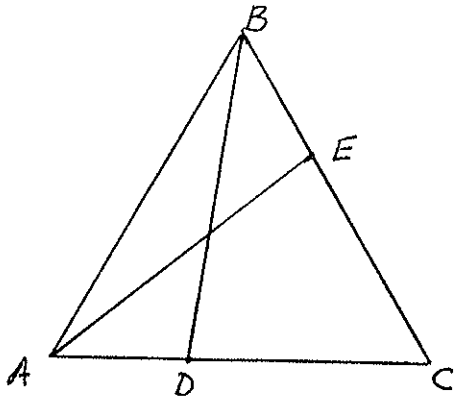
(a) $\triangle ABC$ is equilateral

(b) $\overline{AB} \parallel \overline{DE}$

(c) $\angle EDC \cong \angle ECD$

(d) $\overline{AE} \perp \overline{BC}$

(e) $\overline{DE} \perp \overline{BC}$



32. A right circular cone is constructed by removing $\frac{1}{4}$ of a circle of radius 20 and then taping the two straight sides together. How high is the resulting cone?

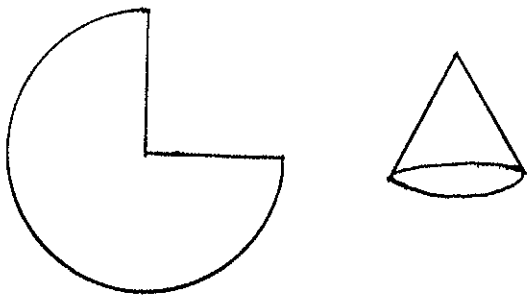
(a) 13

(b) $5\sqrt{7}$

(c) 15

(d) $7\sqrt{5}$

(e) 20



33. Suppose A, B, and C are collinear, $AB = 2$, $BC = 1$, $\overline{AC} \perp \overline{DE}$, $\overline{AC} \perp \overline{BF}$, and $\overline{CD} \cong \overline{CE}$. Find CG.

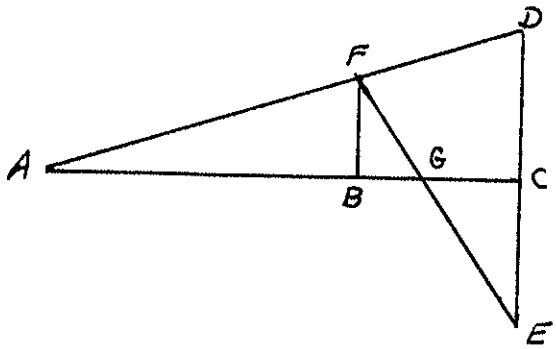
(a) $(\sqrt{5}-1)/2$

(b) $\frac{3}{5}$

(c) $\frac{2}{3}$

(d) $\frac{1}{2}$

(e) Cannot be determined from the given information.



34. Two circles with radii 2 and 3 are externally tangent. A third larger circle is circumscribed about these two as shown. Find the ratio of the area of the smallest circle to the area of the shaded region.

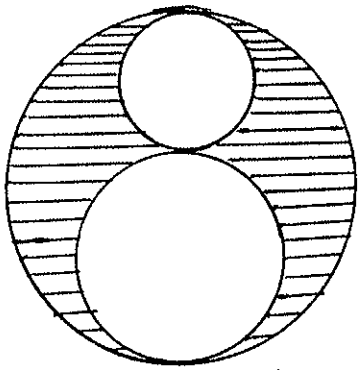
(a) $\frac{1}{9}$

(b) $\frac{2}{9}$

(c) $\frac{1}{4}$

(d) $\frac{1}{3}$

(e) $\frac{2}{5}$



35. A circle of radius 1 is inscribed in an isosceles right triangle. Find the area of the triangle.

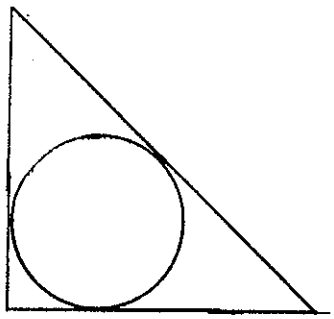
(a) $3+2\sqrt{2}$

(b) 4

(c) $\frac{3}{2}+\sqrt{2}$

(d) $6+4\sqrt{2}$

(e) $12+8\sqrt{2}$



36. $\triangle ABC$ is equilateral, $\overline{BF} \perp \overline{AC}$, and $\overline{DE} \parallel \overline{AC}$. If $BF = 10$ and $DG = 3$, what is the area of quadrilateral $ACED$?

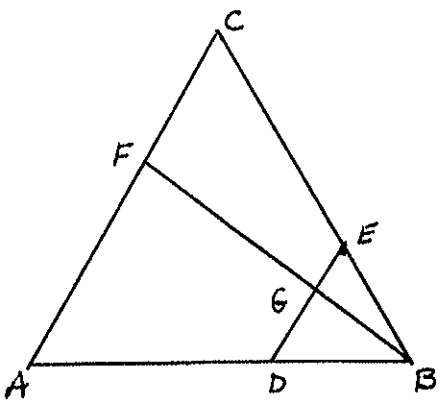
(a) $\frac{73\sqrt{3}}{3}$

(b) 48

(c) 32

(d) $30+9\sqrt{3}$

(e) $\frac{50\sqrt{3}}{3}$



37. Square ABCD has sides of length 6, and both $\widehat{AED}$ and $\widehat{AEB}$ are semicircles. What is the area of the shaded region?

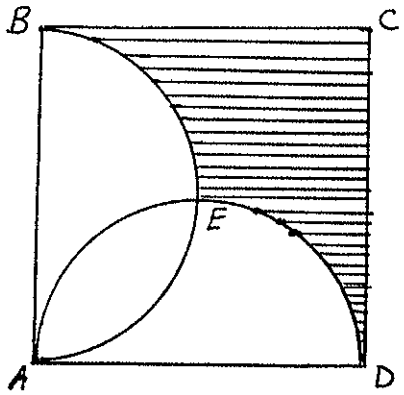
(a) $27 - \frac{9\pi}{2}$

(b) $36 - 9\pi$

(c) $36 - 6\pi$

(d) $24 - 3\pi$

(e) $36 - \frac{45\pi}{4}$



38. Rectangle ABCD is made up of four congruent sub-rectangles. If $AI/AH = \frac{3}{2}$, find AC/AJ.

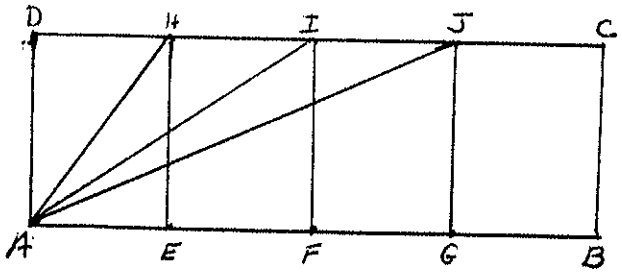
(a) $\frac{3}{2}$

(b) $\frac{9}{4}$

(c) 3

(d) $\sqrt{87/52}$

(e) $\sqrt{117/68}$



39. The radius of the circle with center at C as shown in the figure is r units. Suppose $\overline{AD}$ is tangent to the circle. If the length of AB is x units, then the ratio of the area of the circle to the area of $\triangle ACD$ is:

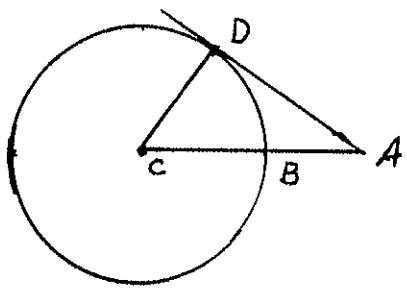
(a) $\pi r^3\sqrt{x^2+2rx}$

(b) $\frac{\pi r}{\sqrt{x^2+2rx}}$

(c) $\frac{\pi r^3\sqrt{x^2+2rx}}{2}$

(d) $\frac{2\pi r}{\sqrt{x^2+2rx}}$

(e) $\frac{\pi r}{\sqrt{x^2+rx}}$



40. Given $RQ = US$ and $\angle RUX \cong \angle S$, which of the following would imply that $\triangle RUX \cong \triangle QSY$.

(a) $\angle T \cong \angle XZY$

(b) $RX = QY$

(c) $XZ = YZ$

(d) $\overline{QY} \perp \overline{XU}$

(e) $QZ = UZ$