THIRTY-SEVENTH ANNUAL MATHEMATICS CONTEST
sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

Geometry 1993

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Coordinated by Theresa Rushing

Scoring formula: 4R - W + 40

Edited by: Larry Bouldin, Roane State
Community College, Harriman, TN

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer, and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all the questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers, so it is advisable not to do wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet. The answer sheets will not be returned to you. If you wish to have a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet and begin. The working time for the entire test is 80 minutes.

Contributors to TMTA for Annual Mathematics Contest:

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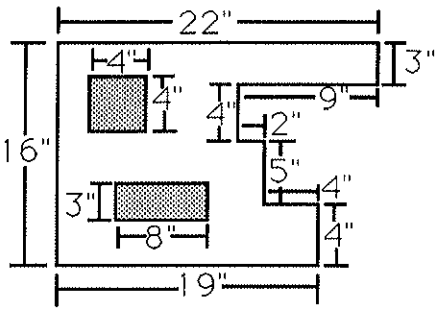
NOTE: 1994 CONTEST DATE--APRIL 12

1. A, E, and K are three points of a line. The coordinates of A and K are $\sqrt{2}$ and $-\sqrt{18}$. If AE = EK, what is the coordinate of E?

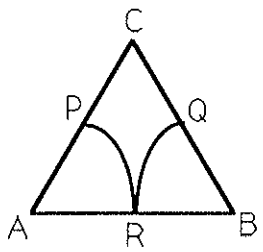
(a) $-\sqrt{2}$
(b) $-2\sqrt{2}$
(c) -6
(d) $-4\sqrt{2}$
(e) $-5\sqrt{2}$

2. The figure is the face of a certain machine part. To compute the cost of painting a number of these parts, one must know the area of a face. The shaded regions are not to be painted. Find the area of the region to be painted.



(a) 227 in^2
(b) 229 in^2
(c) 235 in^2
(d) 352 in^2
(e) 418 in^2

3. The length of a side of the equilateral triangle $\triangle ABC$ is 6 cm, and P, Q, R are midpoints of the sides. Arcs PQ, PR, QR have the vertices of the triangle as their centers. Find the area of the shaded region PQR.

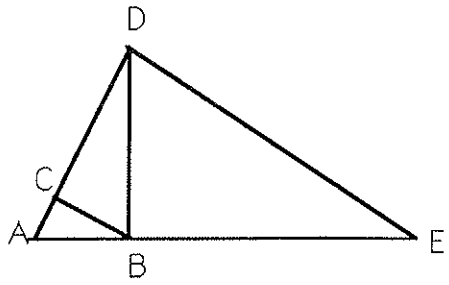


(a) $9(\sqrt{3} - \frac{\pi}{2})$ cm^2
(b) $(9\sqrt{3} - \frac{\pi}{2})$ cm^2
(c) $36\ (\sqrt{3} - \frac{\pi}{2})$ cm^2
(d) $9(\sqrt{3} - \frac{\pi}{3})$ cm^2
(e) $9\ (\sqrt{3} - \pi)$ cm^2

4. Each exterior angle of a regular polygon cannot be

   (a) $20°$
   (b) $30°$
   (c) $35°$
   (d) $45°$
   (e) $60°$

5. If, in the figure to the right, $\overline{AD} \perp \overline{DE}$, $\overline{DB} \perp \overline{AE}$, $\overline{BC} \perp \overline{AD}$, $AC = 3$ cm, and $AB = 5$ cm, then BE is



   (a) $\frac{4\sqrt{10}}{3}$ cm
   (b) $\frac{16}{3}$ cm
   (c) $\frac{20}{3}$ cm
   (d) $\frac{32}{9}$ cm
   (e) $\frac{80}{9}$ cm

6. The set of vertices of the right angle of a triangle having a given hypotenuse lies on

   (a) a rectangle with the hypotenuse as a diagonal.
   (b) a sphere having the hypotenuse as a diameter.
   (c) a parallelepiped with the hypotenuse as diagonal and sides of same length as the sides of the triangle.
   (d) a cone whose apex is at the vertex of the right angle.
   (e) a cylinder whose height is the hypotenuse.

7. A barn with a flat roof is rectangular in shape, 10 yards wide, 13 yards long, and 5 yards high. It is to be painted inside and outside, and on the ceiling, but not on the roof or floor. The total number of square yards to be painted is

   (a) 360
   (b) 460
   (c) 490
   (d) 590
   (e) 720

8. Of the following sets of data, the only one that does not determine the shape of a triangle is

   (a) the ratio of two sides and the included angle
   (b) the ratios of the three altitudes
   (c) the ratios of the three medians
   (d) the ratio of the altitude to the corresponding base
   (e) two angles

9. If $\triangle ABC$ is inscribed in a semicircle whose diameter is $\overline{AB}$, then

   (a) $AC + BC = AB$
   (b) $AC + BC = AB\sqrt{2}$
   (c) $AC + BC \geq AB\sqrt{2}$
   (d) $AC + BC \leq AB\sqrt{2}$
   (e) $AC + BC = AB^2$

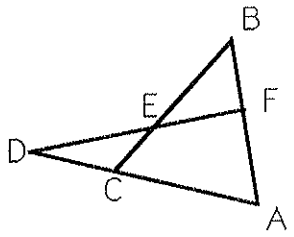
10. A circle and a square have the same perimeter. Then

   (a) their areas are equal
   (b) the area of the circle is greater
   (c) the area of the square is greater
   (d) the area of the circle is π times the area of the square
   (e) the area of the square is π times the area of the circle

11. Which of the following is false?

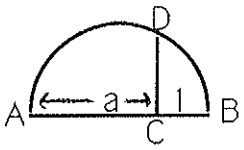
   (a) Doubling the length of a rectangle doubles its area.
   (b) Doubling the altitude of a triangle doubles its area.
   (c) Doubling the height of a cylinder doubles its volume.
   (d) Doubling the radius of a circle doubles its area.
   (e) Doubling both bases of a trapezoid doubles its area.

12. $\overline{DF}$ and $\overline{CB}$ intersect at E, $\overline{DA}$ and $\overline{CB}$ intersect at C, $\overline{AB}$ and $\overline{DF}$ intersect at F, $\overline{DF} \perp \overline{BA}$, $m\angle FEC = 160°$. $\angle A \cong \angle B$. $m\angle ECA =$



(a) 20°
(b) 40°
(c) 60°
(d) 70°
(e) 140°

13. Given a semicircle with diameter $\overline{AB}$, $\overline{CD} \perp \overline{AB}$. The measure of $\overline{CD}$ is



(a) 1
(b) a
(c) a - 1
(d) $\sqrt{a}$
(e) $\sqrt{a} - 1$

14. The number of distinct points common to the graphs of $x^2 + y^2 = 16$ and $x^2 + 4y = 16$ is

(a) 0
(b) 1
(c) 2
(d) 3
(e) 4

15. The perimeter of an isosceles right triangle is 4p. The area is

(a) $(3 - 2\sqrt{2})p$
(b) $\dfrac{4p}{2 + \sqrt{2}}$
(c) $\dfrac{4p^2}{3 + 2\sqrt{2}}$
(d) $\dfrac{4p^2}{3 - 2\sqrt{2}}$
(e) $2p^2$

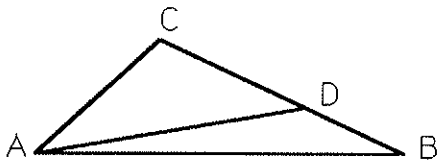
16. The base of a triangle is twice as long as the side of a square and their areas are the same. The ratio of the altitude of the triangle to the side of the square is

(a) $\frac{1}{4}$
(b) $\frac{1}{2}$
(c) $\frac{1}{1}$
(d) $\frac{2}{1}$
(e) $\frac{4}{1}$

17. The ratio of the areas of two circles is 1:5. If the radius of the smaller circle is r, then the difference between the radii is best approximated by

(d) 1.24
(b) 1.24r
(c) $\sqrt{5}$
(e) $\sqrt{5}r$
(a) 4r

18. In $\triangle ABC$, $\overline{AC} \cong \overline{CD}$ and $m\angle CAB - m\angle ABC = 40°$. Then $m\angle BAD =$



(a) 15°
(b) 20°
(c) 30°
(d) 35°
(e) 40°

19. Draw a curve with the aid of two tacks and a piece of string. Stick the two tacks into a piece of paper in such a way that they are closer together than the length of the string. Tie the two ends of the string to the two tacks. Keeping the string taut with a pencil, trace out a curve (to make a complete loop, lift the string over one of the tacks when necessary). It can be proved that this curve is

(a) a circle
(b) an ellipse
(c) either a circle or an ellipse
(d) a hyperbola
(e) a parabola

20. The volume of a rectangular solid each of whose side, front, and bottom faces are $50\ in^2$, $32\ in^2$, and $25\ in^2$, respectively is

(a) $200\ in^3$
(b) $321\ in^3$
(c) $800\ in^3$
(d) $1600\ in^3$
(e) $40000\ in^3$

21. A 50-foot ladder is placed against a vertical wall of a building. The bottom of the ladder is 14 feet from the base of the building. If the top of the ladder slips 8 feet, then the bottom of the ladder will slide

(a) 4 ft
(b) 8 ft
(c) 9 ft
(d) 10 ft
(e) 16 ft

22. A regular hexagon is inscribed in a circle of radius 8 inches. The area enclosed by the hexagon is

(a) $32\sqrt{3}\ in^2$
(b) $48\ in^2$
(c) $96\ in^2$
(d) $96\sqrt{3}\ in^2$
(e) $192\sqrt{3}\ in^2$

23. A right circular cone has for its base a circle having the same radius as a given sphere. The volume of the cone is $\frac{2}{3}$ that of the sphere. The ratio of the altitude of the cone to the radius of its base is

(a) $\frac{1}{1}$
(b) $\frac{1}{2}$
(c) $\frac{4}{1}$
(d) $\frac{4}{3}$
(e) $\frac{8}{3}$

24. What is the area of a triangle with sides 15 cm, 15 cm, 24 cm?

(a) 54 cm^2
(b) 108 cm^2
(c) 110 cm^2
(d) 180 cm^2
(e) 216 cm^2

25. Two similar pyramids have volumes 27 cm^3 and 125 cm^3. If the pyramid with the smaller volume has lateral area 18 cm^2, what is the lateral area of the pyramid with the larger volume?

(a) 25 cm^2
(b) 30 cm^2
(c) 50 cm^2
(d) 60 cm^2
(e) $83\frac{1}{3}$ cm^2

26. How many lines of symmetry does a rhombus have?

(a) none
(b) one
(c) two
(d) four
(e) five

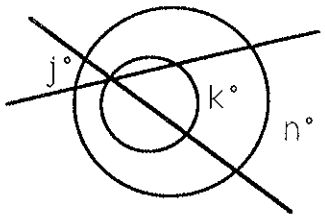
27. What is the given set of points 3 cm from a given point A?

(a) a line
(b) a plane
(c) a cylindrical surface
(d) a sphere
(e) a ray

28. The altitude to the 55 cm hypotenuse of a right triangle divides the hypotenuse into segments 25 cm and 30 cm long. How long is the altitude?

(a) $5\sqrt{30}$ cm
(b) $5\sqrt{55}$ cm
(c) $15\sqrt{3}$ cm
(d) $15\sqrt{5}$ cm
(e) $30\sqrt{5}$ cm

29. The two lines intersect at a point on the circle with the smaller radius. Write an equation involving the arcs j, k, and n.



(a) $k^\circ = j^\circ - n^\circ$
(b) $k^\circ + j^\circ = n^\circ$
(c) $k^\circ = j^\circ + n^\circ$
(d) $\frac{1}{2}k^\circ = j^\circ + n^\circ$
(e) $k^\circ = \frac{1}{2}(j^\circ + n^\circ)$

30. The converse of, "An equilateral triangle is isosceles," is

(a) A triangle that is not equilateral is not isosceles.
(b) A triangle that is not isosceles is not equilateral.
(c) An equilateral triangle is not isosceles.
(d) An isosceles triangle is not equilateral.
(e) An isosceles triangle is equilateral.

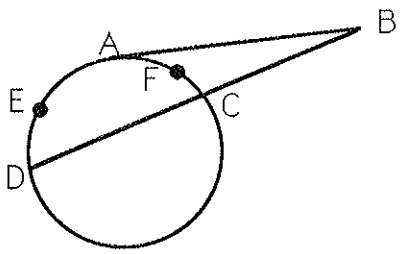
31. Trapezoid ABCD, with $\overline{AB}$ parallel to $\overline{CD}$, has median $\overline{XY}$. AB = 12 cm and XY = 17 cm. Find CD.

(a) 14.5 cm
(b) 22 cm
(c) 24 cm
(d) 29 cm
(e) 34 cm

32. If the <u>indirect method</u> of proof is used to prove: "If two coplanar lines, L_1 and L_2, are perpendicular to line L_3, then L_1 and L_2 are parallel," then an appropriate hypothesis is

(a) L_1 and L_2 are not perpendicular to L_3.
(b) L_1 and L_2 are parallel.
(c) L_1 and L_2 are not parallel.
(d) Either L_1 and L_3 are skew lines or L_2 and L_3 are skew lines.
(e) L_3 is not in the same plane as L_1 and L_2.

33. In the accompanying figure, AB is tangent to the circle at A, and BD is a secant intersecting the circle at D and C. $m\angle ABD = 32°$ and m(arc AED) = 88°. Which one of the following is m(arc AFC)?



(a) 24°
(b) 28°
(c) 30°
(d) 56°
(e) 60°

34. In $\triangle ABC$, AC = 24 cm, BC = 10 cm, and AB = 26 cm. The radius of the inscribed circle is

(a) 4 cm
(b) 8 cm
(c) 11 cm
(d) 13 cm
(e) 26 cm

35. In $\triangle ABC$, AB = 12 cm, AC = 7 cm, and BC = 10 cm. If AB and AC are doubled and BC remains the same, then

(a) the area is doubled
(b) the altitude to BC is doubled
(c) the area is quadrupled
(d) the median to BC is doubled
(e) the area of the triangle is 0

36. Given 12 points in a plane, no three of which are collinear, the number of lines the points determine is

(a) 24
(b) 54
(c) 66
(d) 120
(e) 132

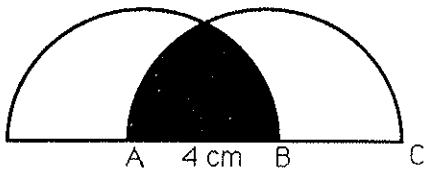
37. The base of a triangle is 15 cm. Two lines are drawn parallel to the base. The lines intersect the other two sides and divide the triangle into three equal areas. The length of the parallel segment closer to the base is

(a) $2.5\sqrt{3}$ cm
(b) $4\sqrt{3}$ cm
(c) 7.5 cm
(d) 10 cm
(e) $5\sqrt{6}$ cm

38. If the longer base of an isosceles trapezoid has the same length as a diagonal and the shorter base has the same length as the altitude, then the ratio of the smaller base to the longer base is

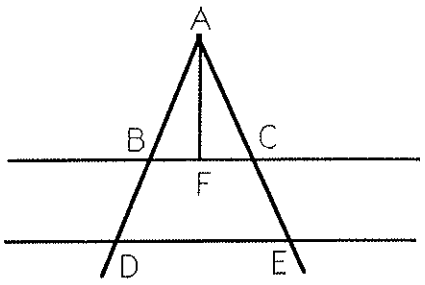
(a) $\frac{1}{2}$
(b) $\frac{2}{3}$
(c) $\frac{2}{5}$
(d) $\frac{3}{4}$
(e) $\frac{3}{5}$

39. Suppose semicircles with centers A and B each have radius 4 cm. The area of the shaded portion is



(a) $\frac{16\pi}{3}$ cm^2
(b) $(\frac{16\pi}{3} - 4\sqrt{3})$ cm^2
(c) $\frac{8\pi}{3}$ cm^2
(d) $(\frac{8\pi}{3} - 4\sqrt{3})$ cm^2
(e) $(\frac{2\pi}{3} - 2\sqrt{3})$ cm^2

40. Suppose the area of $\triangle ABC$ is $10\ cm^2$ and the area of $\triangle ADE$ is $50\ cm^2$. $BC \parallel DE$ and $\overline{AF} \perp \overline{BC}$. The distance between BC and DE is 5 cm. The length of $\overline{AF}$ is



(a) 1.25 cm
(b) $1.25\ (5 + \sqrt{5})$ cm
(c) $1.25\ (\sqrt{5} + 1)$ cm
(d) $2.5\ (\sqrt{5} + 1)$ cm
(e) $(\sqrt{5} - 1)$ cm