THIRTY-FIFTH ANNUAL MATHEMATICS CONTEST
sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

Geometry 1991

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Scoring formula: 4R - W + 40

Edited by: Billy Smith, Roane State
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DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer, and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all the questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers, so it is advisable not to do wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet. The answer sheets will not be returned to you. If you wish to have a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet and begin. The working time for the entire test is 80 minutes.

Contributors to TMTA for Annual Mathematics Contest:

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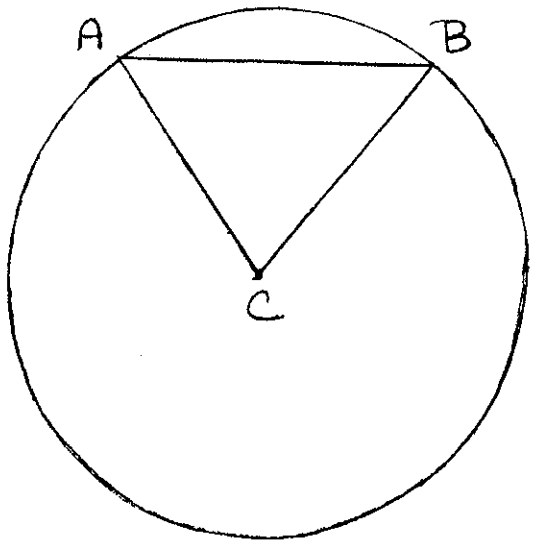
GEOMETRY

1 . $\angle ACB$ is a central angle of 60° in a circle of radius 10 in. The length of chord $\overline{AB}$ is



a) 10 in.

b) 2π in.

c) $\frac{10}{3}\pi$ in.

d) $10\sqrt{3}$ in.

e) 4.67 in.

2 . A central angle of 72° in a circle of radius 10in subtends an arc of

a) 720 in.

b) 4π in.

c) 2π in.

d) 10 in.

e) 4 in.

3 . A right triangle has sides of 3 cm and 4 cm. The length of the altitude from the right angle to the hypotenuse is

a) 4 cm.

b) 5 cm.

c) $\sqrt{12}$ cm.

d) $\frac{4}{3}$ cm.

e) $\frac{12}{5}$ cm.

4 . The product of the lengths of the diagonals of a rhombus is 12. The sum of the squares of the lengths of the diagonals is 25. The area of the rhombus is

a) 6.
b) 5.
c) $3\sqrt{2}$.
d) $4\sqrt{2}$.
e) $\frac{25}{4}$.

5 . A rectangle has adjacent sides of 2 cm and 3 cm respectively. The length of a diagonal of the rectangle is

a) $\frac{5}{2}$ cm.
b) $\sqrt{13}$ cm.
c) $\sqrt{6}$ cm.
d) $\sqrt{5}$ cm.
e) 13 cm.

6 . A vertex angle of a regular octagon has measure

a) 120°.
b) 45°.
c) 108°.
d) 90°.
e) 135°.

7 . Triangles A and B are similar triangles. The ratio of the length of a side of triangle A to the corresponding side of triangle B is 1.5. If the area of triangle B is 3 square units, the area of triangle A is

a) 4.5 square units.
b) 2 square units.
c) 1.33 square units.
d) 6.75 square units.
e) 3 square units.

8 . A circle is inscribed in a square. The ratio of the area of the circle to the area of the square is

a) one.

b) the same as the ratio of the circumference of the circle to the perimeter of the square.

c) twice the ratio of the circumference of the circle to the perimeter of the square.

d) pi.

e) $\sqrt{2}$ times the ratio of the perimeter of the circle to the perimeter of the square.

9 . ABD is a straight line. If $\angle ABC$ is equal to the vertex angle of an isosceles triangle then each base angle of the isosceles triangle equals

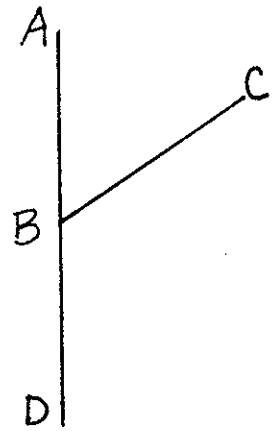
a) $180° - m\angle CBD$.

b) $60°$.

c) $90° - m\angle ABC$.

d) $\frac{1}{2}m\angle ABC$.

e) $\frac{1}{2}m\angle CBD$.



10 . C is the center of a circle and α and β are any two angles as indicated in the figure. Then

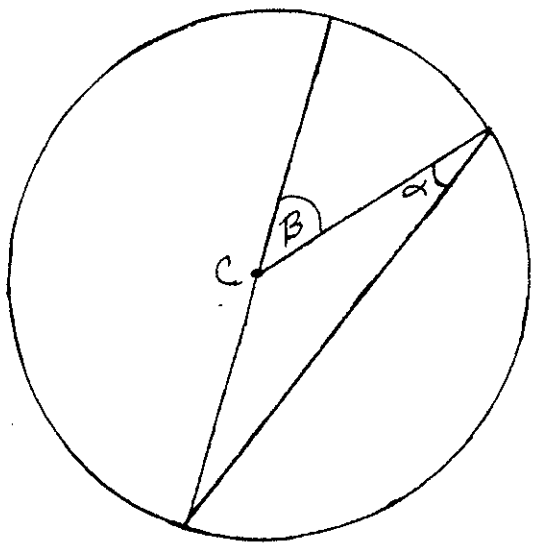
a) $\alpha \cong \beta$.

b) $\alpha \cong 2\beta$.

c) $\beta \cong 2\alpha$.

d) $\alpha + \beta \cong 90°$.

e) $\alpha + \beta \cong 180°$.



11 . The figure consists of a semi-circle with a diameter AB and a line segment DE perpendicular to AB. For any line segments a, b, c, and d, as shown

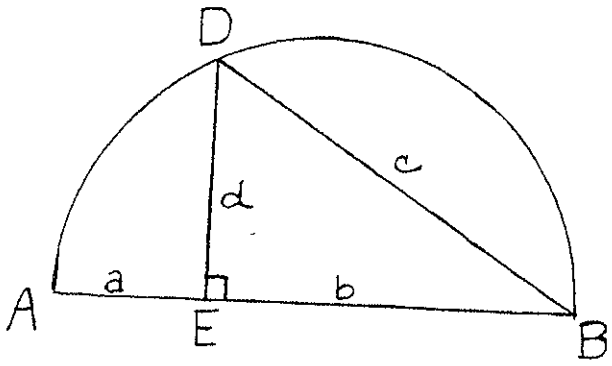
a) $d = \sqrt{c}$.

b) $d = \sqrt{ab}$.

c) $c = ab$.

d) $b = ac$.

e) $a = \sqrt{bc}$.



12 . If all vertices of a regular hexagon are connected by line segments (including the sides of the hexagon), the number of distinct segments produced is

a) 21.

b) 36.

c) 15.

d) 120.

e) 10.

13 . x, y, z are the measures of the angles shown in the figure. The sum of y and z in terms of x is

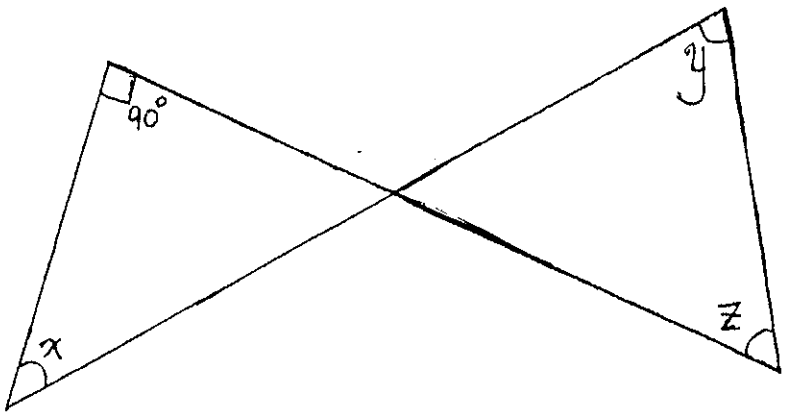
a) $2x$.

b) $90° + x$.

c) $180° - x$.

d) $180° - 2x$.

e) $90° - x$.



14 . Given $\overline{AC} = \overline{BC}$. Measure of angle A is 40°. The measure of angle C is

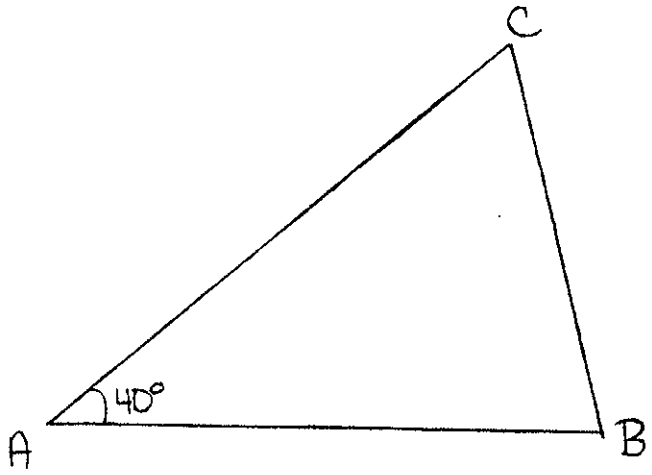
a) 50°.

b) 60°.

c) 80°.

d) 100°.

e) 140°.



15 . x, y, z are the measures of the angles shown. $\ell_1 \| \ell_2$. The measure of x is

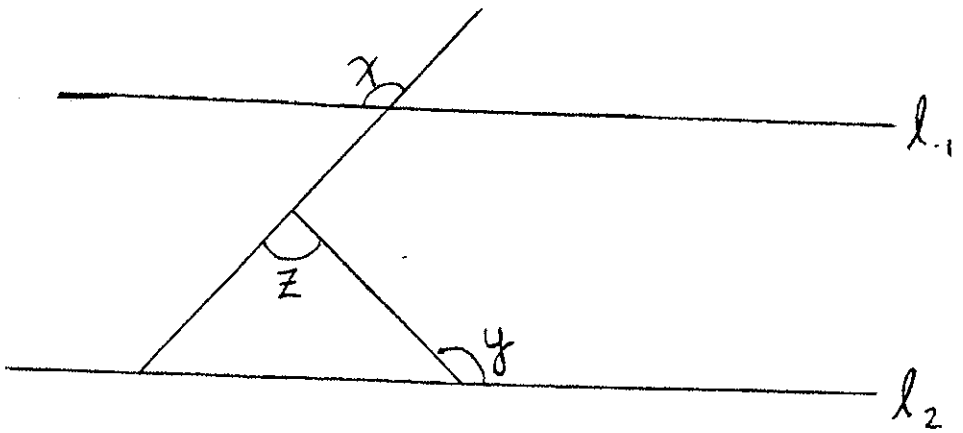
a) $180° - y$.

b) $180° - z$.

c) $180° - z + y$.

d) $180° + z - y$.

e) $z + y - 180°$.



16 . In a right triangle ABC, Angle B is a right angle. The altitude $\overline{BD}$ is drawn. If $\overline{AB} = 4$ and $\overline{AD} = 3$, then the length of $\overline{AC}$ is

a) 3.

b) 4.

c) $\frac{9}{4}$.

d) 12.

e) $\frac{16}{3}$.

17 . An archeologist found a piece of the rim of an old wheel in an ancient ruin in Greece. She needed to determine the diameter of the wheel in order to restore it. She marked three points A, B, and C, on the rim so that $\overline{AB}$ was congruent to $\overline{AC}$. If $\overline{AB}$ = 15 in. and $\overline{BC}$ = 24 in., what was the diameter of the wheel?

a) 50 in.

b) 12.5 in.

c) 25 in.

d) 32 in.

e) 30.4 in.

18 . Circle A is inscribed in a square of side 5 cm. Circle B is circumscribed about the same square. The ratio of the radius of circle A to the radius of circle B is

a) $\frac{1}{2}$ cm.

b) $\frac{\sqrt{3}}{2}$ cm.

c) $\frac{\sqrt{5}}{5}$ cm.

d) $\frac{\sqrt{2}}{2}$ cm.

e) $5\sqrt{2}$ cm.

19 . Figure $ABCDE$ is a regular pentagon. The measure of angle CBD is

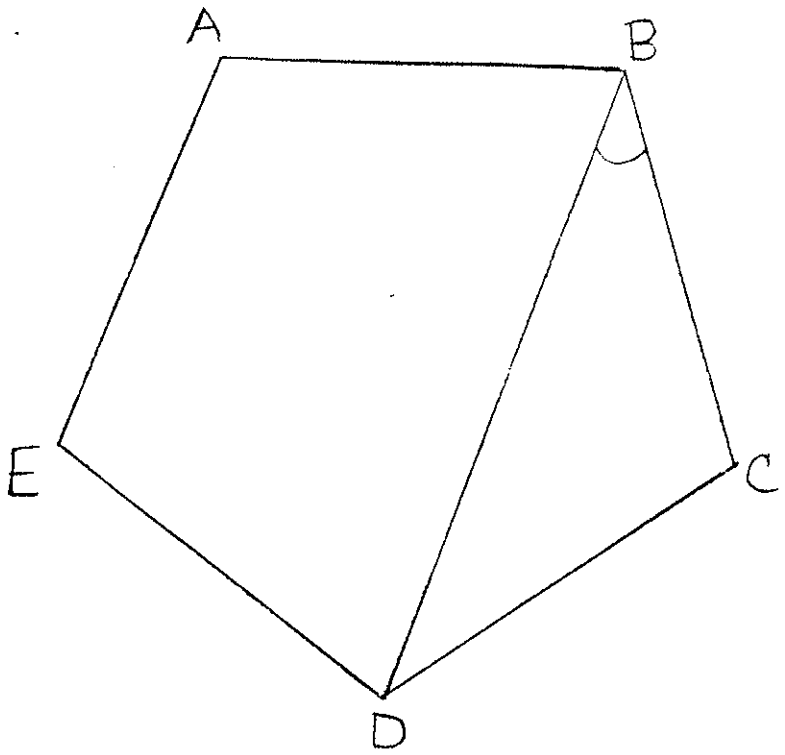
a) 45°.

b) 30°.

c) 108°.

d) 36°.

e) 60°.



20 . The solid called a regular dodecahedron has 12 faces which are congruent regular pentagons. How many distinct edges does a regular dodecahedron have?

a) 10

b) 20

c) 30

d) 40

e) 50

21 . A square is inscribed in a circle. The length of each side of the square is 2. The radius of the circle is

a) 2

b) 4

c) $\sqrt{2}$

d) $\sqrt{3}$

e) $\sqrt{1}$

22 . In the figure shown, the value of d^2 is

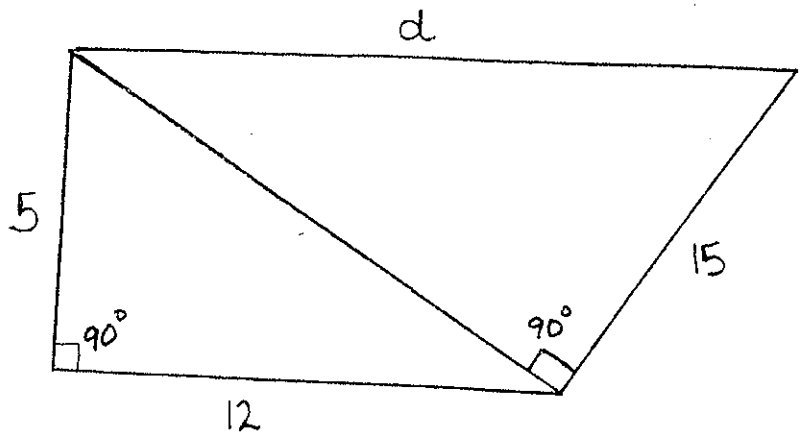
a) 32

b) 308

c) 324

d) 394

e) 428



23 . In the figure shown, the value of x is

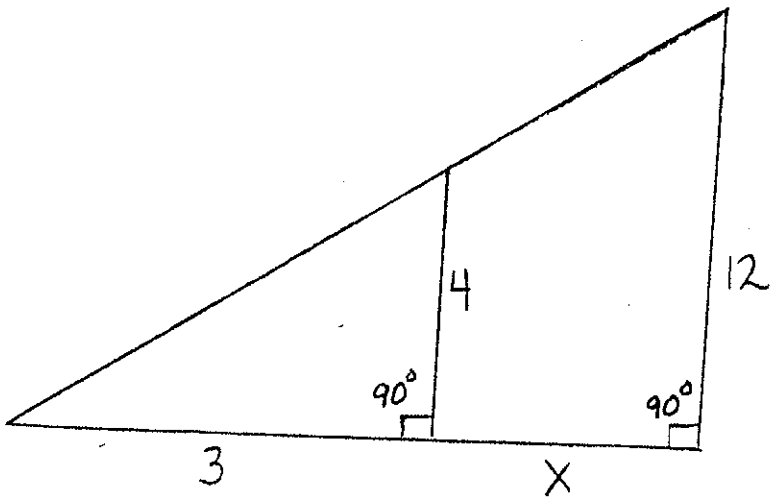
a) 9.

b) 11.

c) 8.

d) 6.

e) 2.



24 . Given $\ell_1 \| \ell_2$ as shown in the figure, then $m\angle x$ is

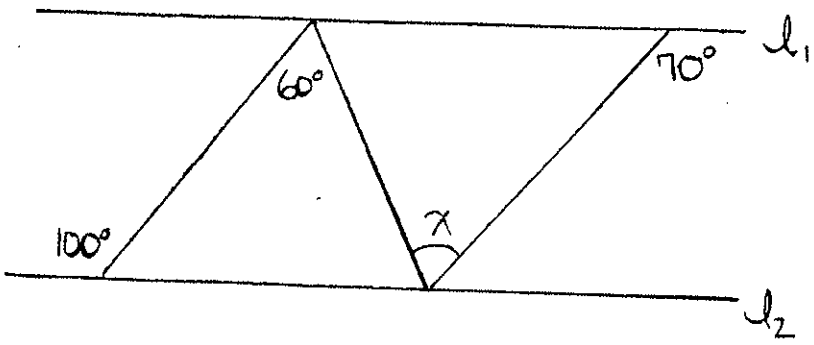
a) 30°.

b) 70°.

c) 60°.

d) 100°.

e) 10°.



25 . In the given figure, which one of the following is necessarily true?

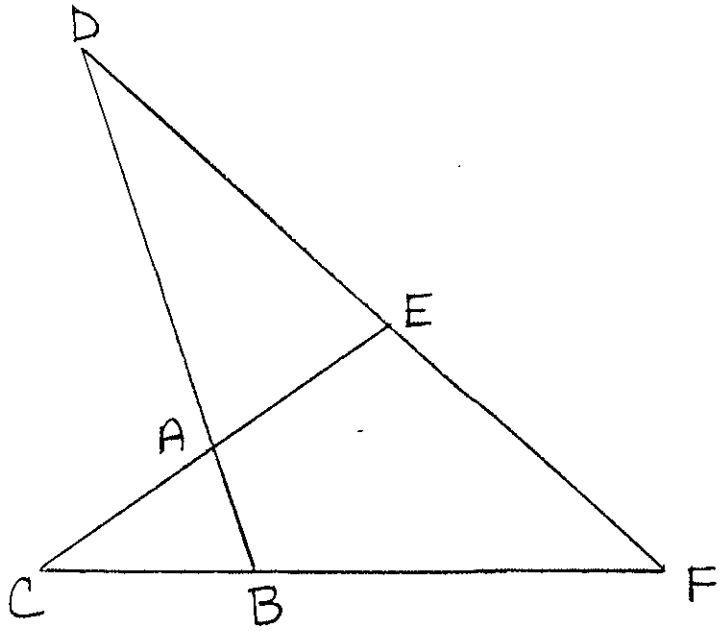
a) $m\angle DAE + m\angle CAB = 180°$

b) $m\angle EDA = m\angle ABC$

c) $m\angle ABF - m\angle DAE = m\angle ACB$

d) $m\angle BDF + m\angle DFB = m\angle FBD$

e) $m\angle ABF + m\angle DAE = 180° - m\angle ACB$



26 . An equilateral triangle has a side of 4 in. The radius of a circle inscribed in the triangle is

a) $2\sqrt{2}$ in.

b) $2\sqrt{3}$ in.

c) $\frac{\pi}{3}$ in.

d) $\frac{2\sqrt{3}}{3}$ in.

e) $\frac{2}{3}$ in.

27 . In triangle ABC; E is the midpoint of $\overline{BC}$, D is the midpoint of $\overline{AC}$, $\overline{AE}$ and $\overline{BD}$ intersect at G, $\overline{AE} = 6$ mm, and $\overline{BD} = 8$ mm. Then $\overline{EG} =$

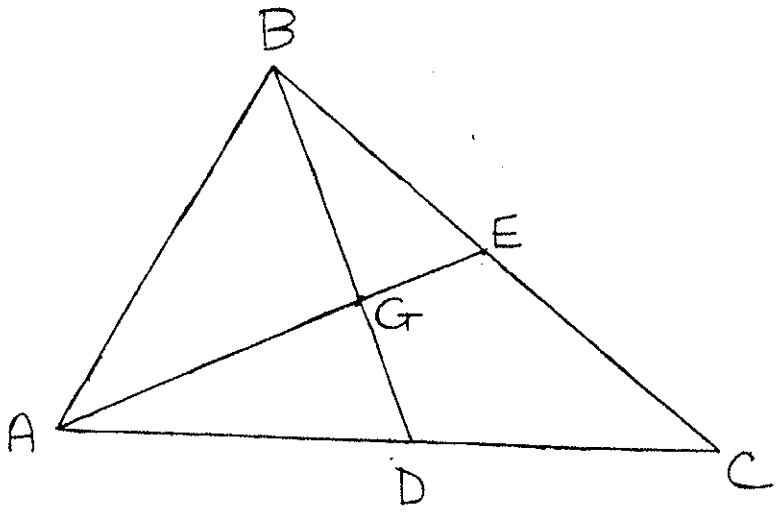
a) 3 mm.

b) 4 mm.

c) $\sqrt{6}$ mm.

d) 2 mm.

e) $2\sqrt{3}$ mm.



28 . In the diagram, 0 is the center of the circle Given:

$m\widehat{AD} = 92°$, $m\widehat{CD} = 103°$, $m\widehat{BC} = 41°$. The $m\angle ECA$ is

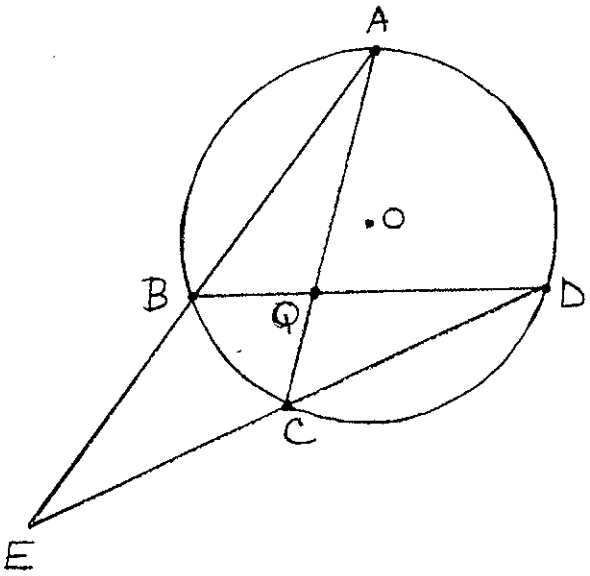
a) 96°.

b) 134°.

c) 101°.

d) 130.5°.

e) 150°.



29 . Two concentric circles have radii of 2 inches and 6 inches. Line m is tangent to the smaller circle. How many points are equidistant from the circles and 2 inches from m?

a) 0

b) 3

c) 2

d) 1

e) 4

30 . In the given diagram, 0 is the center of the circle, $\overline{XA}$ is tangent to $\odot 0$ at X. If $\overline{OM} = 8$ and $\overline{MY} = 9$, then $\overline{XZ}$ equals

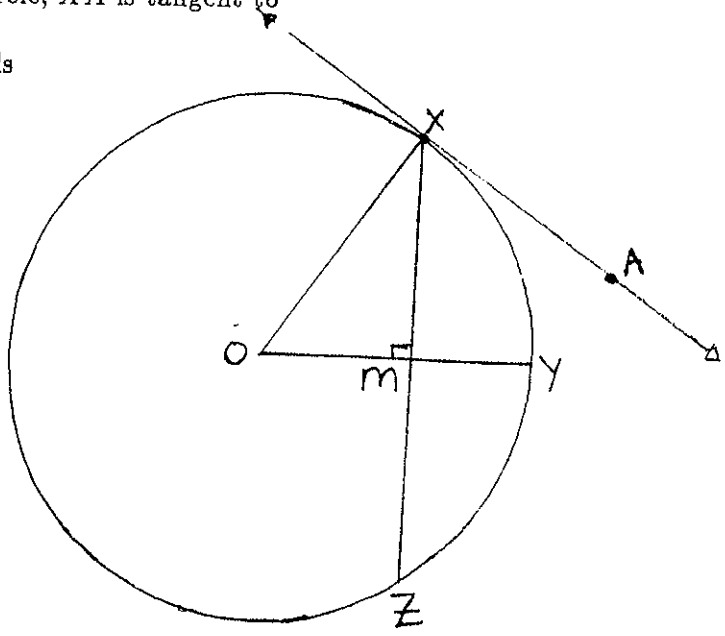
a) $6\sqrt{2}$.

b) $2\sqrt{17}$.

c) $\sqrt{145}$.

d) 45.

e) 30.



31 . In the diagram, 0 is the center of the circle, $\angle 0$ is a right angle, and the radius is 6. The area of the shaded region is

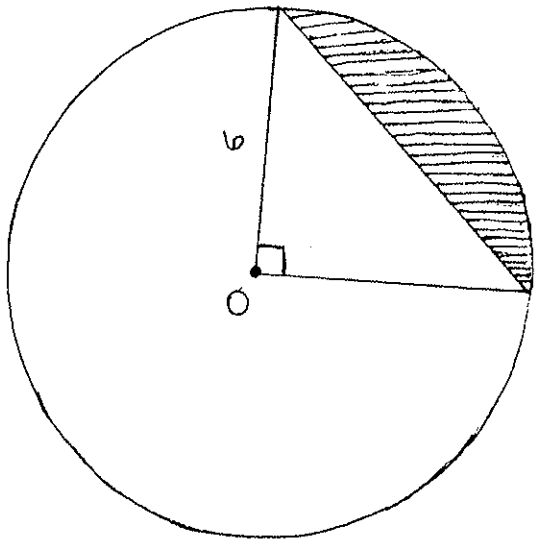
a) $9\pi - 36$.

b) $12\pi - 36$.

c) $9\pi - 18$.

d) $12\pi - 18$.

e) $9\pi + 18$.



32 . $ABCD$ is a trapezoid. Given: $\overline{AB} \| \overline{CD}$, $\overline{DC} = 8m$, $\overline{AB} = 12m$, $m\angle A = m\angle B = 45°$. The area of trapezoid $ABCD$ is

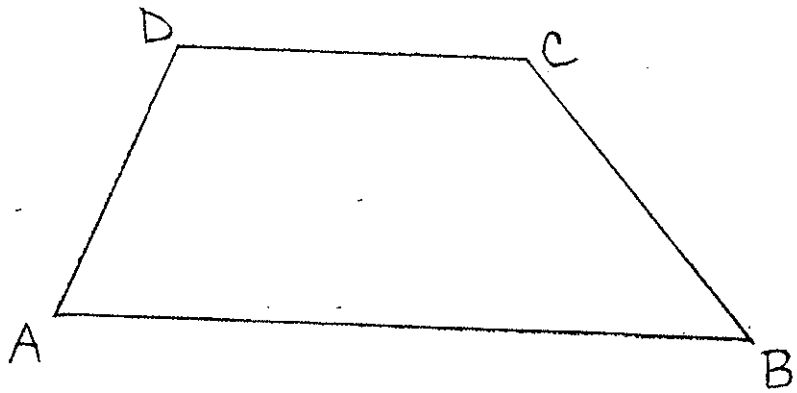
a) 20 m^2.

b) 40 m^2.

c) $20\sqrt{2}$ m^2.

d) 32 m^2.

e) 24 m^2.



33 . In triangle ABC, BD is the bisector of $\angle ABC$. If $\overline{AB} = 3$ units, $\overline{BC} = 4$ units, and $\overline{AC} = \frac{14}{3}$ units, then the length of $\overline{AD}$ is

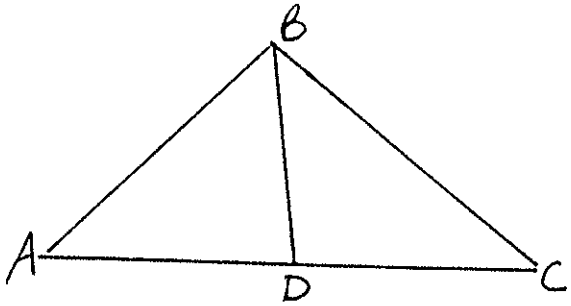
a) 2 units.

b) 3 units.

c) $2\sqrt{3}$ units.

d) $\sqrt{5}$ units.

e) 5 units.



34 . In the figure; circles 0 and P are tangent at B, line ℓ is tangent to circle 0 at A and tangent to circle P at C. $\overline{AD} = \overline{DC} = \overline{DB}$. The measure of angle ABC is

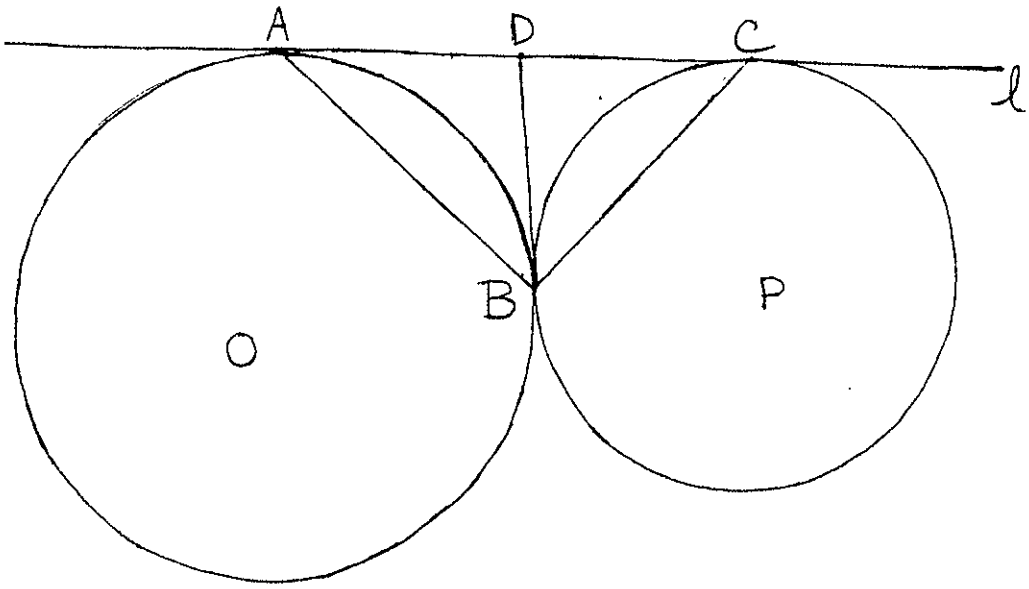
a) $120° - 45°$.

b) $75° + 30°$.

c) $120°$.

d) $60° + 30°$.

e) unknown.



35 . The area of an equilateral triangle is 21.22 sq. in. (correct to 2 decimal places). The length of a side of the triangle (correct to 2 decimal places) is

a) 4.61 in.

b) 10.61 in.

c) 7.00 in.

d) 5.32 in.

e) 5.20 in.

36 . C is the center of circle 0, whose radius is 2 cm and $\overline{AD}$ is an external tangent to the circle. If $\overline{AD} = 7$ cm and $m\angle DAB = 55°$ then $m\angle ECD$ is

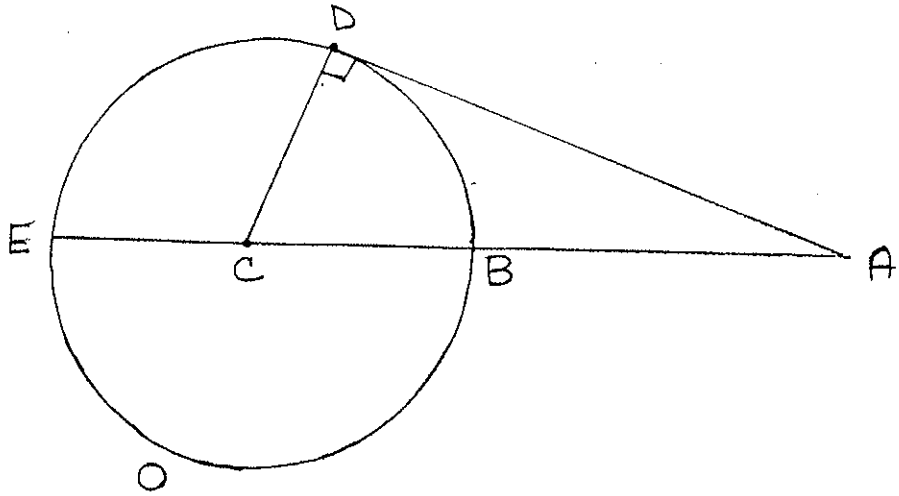
a) 120°.

b) 27.5°.

c) 90°.

d) 155°.

e) 145°.



37 . C is the center of circle 0 whose radius is 2 cm. $\overline{AD}$ is an external tangent to the circle. If $\overline{AB} = 5$ cm, then the length of $\overline{AD}$ is

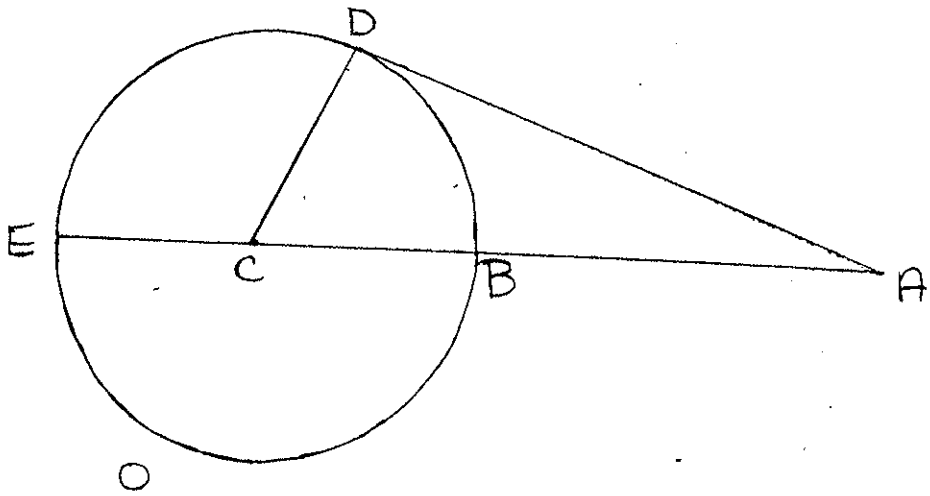
a) $3\sqrt{5}$ cm.

b) $\sqrt{35}$ cm.

c) $2\sqrt{5}$ cm.

d) $\sqrt{29}$ cm.

e) 45 cm.



38 . A pair of parallel planes A and B are 4 in. apart. Point P is 1 in. from A and 3 in. from plane B. The set of points whose locus is equidistant from planes A and B and 2 in. from point P is

a) a plane.

b) a circle.

c) a sphere.

d) a point.

e) a line.

39 . Two tangent segments to a circle from an external point P determine an angle of 60°. If the diameter of the circle is 10 units, how long is one of the tangent segments?

a) $5\sqrt{3}$.

b) 10.

c) $10\sqrt{3}$.

d) $10\sqrt{2}$.

e) 20.

40 . $ABCD$ is a parallelogram. $m\angle BAC = 35°$, $m\angle AEB = 120°$, and $m\angle BAD = 120°$. Then $m\angle DBC =$

a) 60°.

b) 55°.

c) 35°.

d) 85°.

e) 25°.