

THIRTY-FOURTH ANNUAL MATHEMATICS CONTEST  
sponsored by  
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

Geometry 1990

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Scoring formula:  $4R - W + 40$

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DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer, and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all the questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers, so it is advisable not to do wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet. The answer sheets will not be returned to you. If you wish to have a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet and begin. The working time for the entire test is 80 minutes.

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1. In triangle ABC, let AD, BE, and CF be medians from vertices A, B, and C, respectively, intersecting in point P. If  $\overline{BE} = 9$ , then the length  $\overline{PE}$  is

Choose one of the following:

- a) 3
- b) 4
- c) 4.5
- d) 6
- e) cannot be determined from the information given.

2. Which of the following statements is false?

Choose one of the following:

- a)  $n$  points may be collinear.
- b)  $n$  points may be coplanar.
- c) Four points may be collinear.
- d) Four points must be collinear.
- e) Two points must be collinear.

3. A church steeple casts a shadow of 120 feet while a nearby vertical pole 10 feet high casts a shadow of 8 feet. How high is the steeple?

Choose one of the following:

- a) 96 feet
- b) 136 feet
- c) 150 feet
- d) 160 feet
- e) 170 feet

4. The sum of the interior angles of a heptagon is

Choose one of the following:

- a)  $720^{\circ}$
- b)  $900^{\circ}$
- c)  $1080^{\circ}$
- d)  $1440^{\circ}$
- e)  $1160^{\circ}$

5. ABCDEFGH is a regular octagon with each side of length 2. Find  $m(\overline{AF})$ .

Choose one of the following:

- a)  $2 + \sqrt{2}$
- b)  $2 + 2\sqrt{2}$
- c) 4
- d)  $4 + \sqrt{2}$
- e)  $4 + 2\sqrt{2}$

6. The equation of the circle with center at  $(-3,4)$  and tangent to the  $y$  - axis would be

Choose one of the following:

- a)  $(x-3)^2 + (y-4)^2 = 9$
- b)  $(x+3)^2 + (y-4)^2 = 16$
- c)  $(x-3)^2 + (y+4)^2 = 16$
- d)  $(x+3)^2 + (y-4)^2 = 9$
- e)  $(x+3)^2 + (y+4)^2 = 16$

7. In convex quadrilateral ABCD, the diagonals AC and BD are perpendicular,  $\overline{AC} = 3$ , and  $\overline{BD} = 4$ . The area of the quadrilateral ABCD is

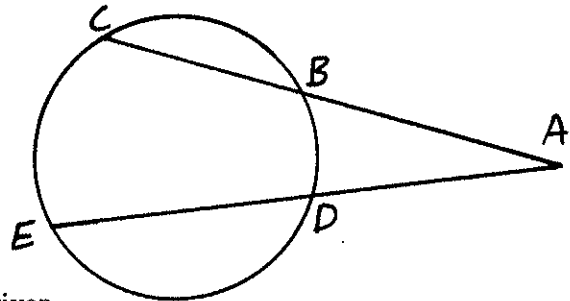
Choose one of the following:

- a) 3.5
- b) 6
- c) 7
- d) 12
- e) cannot be determined from the information given.

8. In the given figure, AC and AE are secants to the given circle. If  $\overline{AB} = 20$ ,  $\overline{AC} = 28$ , and  $\overline{AE} = 32$ , then  $\overline{AD} =$

Choose one of the following:

- a) 15
- b) 17.5
- c) 20
- d) 22.5
- e) cannot be determined from the information given.



9. If A, B, C, and D are distinct points such that  $\overleftrightarrow{AC}$  contains B and  $\overleftrightarrow{BD}$  contains C, which of these statements must be true?

Choose one of the following:

- a) B is between A and C.
- b)  $\overleftrightarrow{BC}$  contains A.
- c)  $\overleftrightarrow{AC}$  and  $\overleftrightarrow{BD}$  intersect at B and C only.
- d)  $\overleftrightarrow{AD}$  and  $\overleftrightarrow{BC}$  do not intersect.
- e)  $\overleftrightarrow{AC}$  is opposite to  $\overleftrightarrow{DB}$ .

10. A triangular prism has \_\_\_\_\_ diagonals.

Choose one of the following:

- a) 6
- b) 3
- c) 2
- d) 1
- e) 0

11. If the measures of two angles of a triangle are  $(45 + x)^\circ$  and  $(45 - x)^\circ$ , what is the measure of the third angle?

Choose one of the following:

- a)  $45^\circ$
- b)  $60^\circ$
- c)  $90^\circ$
- d)  $180^\circ$
- e)  $135^\circ$

12. The inverse of the statement "If (not A), then B", is the statement

Choose one of the following:

- a) If B, then A.
- b) If B, then (not A).
- c) If (not B), then A.
- d) If A, then (not B).
- e) If (not A), then (not B).

13. The radius of the base of a regular cone is 3, and the height is 9. The lateral area is

Choose one of the following:

- a)  $\sqrt{\pi}$
  - b)  $9\pi$
  - c)  $9\sqrt{10}$
  - d)  $18\pi\sqrt{5}$
  - e)  $9\pi\sqrt{10}$
14. A median of a triangle divides the triangle into two triangles which are always
- Choose one of the following:
- a) similar
  - b) congruent
  - c) equal in area
  - d) equal in perimeter
  - e) equal in height
15. Which of the following sets of points is convex?
- Choose one of the following:
- a) A line
  - b) A circle
  - c) A sphere
  - d) A line with a single point removed
  - e) A set consisting of exactly two distinct points

16. The area of a right triangle is six square units. If the length of one leg is 4, what is the length of the altitude to the hypotenuse?

Choose one of the following:

- a)  $\frac{6}{5}$
- b)  $\frac{4}{3}$
- c)  $\frac{3}{2}$
- d)  $\frac{12}{5}$
- e)  $\frac{15}{4}$

17. Assume that a chord of a circle is parallel to a tangent of that circle and bisects the radius that contains the point of tangency. If the measure of the chord is 12, find the measure of the radius.

Choose one of the following:

- a)  $2\sqrt{3}$
- b) 3
- c)  $4\sqrt{2}$
- d)  $4\sqrt{3}$
- e) 6



18. Of the following descriptions of triangles, which allow for two different triangles to be constructed?

Choose one of the following:

- a)  $m(\sphericalangle \text{ at } A) = 30^\circ$ ,  $\overline{AB} = 2$ ,  $m(\sphericalangle \text{ at } B) = 90^\circ$
- b)  $m(\sphericalangle \text{ at } A) = 55^\circ$ ,  $\overline{AB} = 5$ ,  $\overline{AC} = 3$
- c)  $m(\sphericalangle \text{ at } A) = 35^\circ$ ,  $\overline{AB} = 6$ ,  $\overline{BC} = 4$
- d)  $m(\sphericalangle \text{ at } A) = 70^\circ$ ,  $m(\sphericalangle \text{ at } B) = 60^\circ$ ,  $m(\sphericalangle \text{ at } C) = 50^\circ$
- e)  $\overline{AB} = 5$ ,  $\overline{BC} = 3$ ,  $\overline{AC} = 4$

19. Two secants drawn to a circle from an external point P form an angle of  $30^\circ$ . The larger intercepted arc is  $80^\circ$ . The smaller arc is

Choose one of the following:

- a)  $20^\circ$
- b)  $25^\circ$
- c)  $40^\circ$
- d)  $50^\circ$
- e)  $30^\circ$

20. Two chords,  $\overline{AB}$  and  $\overline{CD}$ , of a circle intersect at P. If  $\overline{AP} = 2$ ,  $\overline{BP} = 8$ , and  $\overline{CP} = 4$ , find  $\overline{PD}$ .

Choose one of the following:

- a) 2
- b) 4
- c) 6
- d) 8
- e) 10

21. If the measure of chord  $\overline{AB}$  of circle C is 8, and the radius is 5, find the shortest distance from the center to the chord  $\overline{AB}$ .

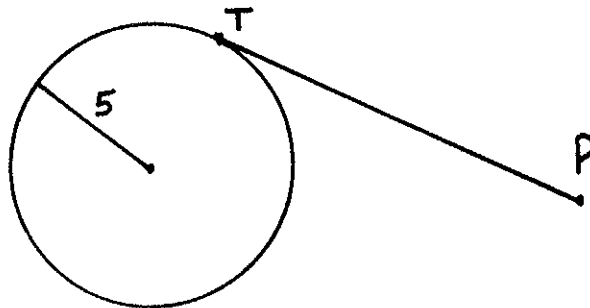
Choose one of the following:

- a) 2
- b) 3
- c) 4
- d) 6
- e) 10

22. In the given figure, the radius of the circle is 5 and the length of the tangent from external point P to the circle is 12. What is the shortest distance from P to the circle?

Choose one of the following:

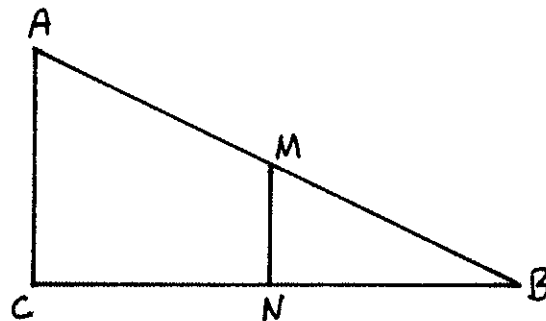
- a) 5
- b)  $2\sqrt{5}$
- c) 8
- d) 10
- e)  $2\sqrt{2}$



23. The hypotenuse of the right triangle ABC is 6 units in length, M is the midpoint of the hypotenuse,  $\overline{MN} = 2$ , and MN is perpendicular to BC. What is the length  $\overline{BC}$ ?

Choose one of the following:

- a)  $\sqrt{5}$
- b)  $\sqrt{13}$
- c) 4
- d)  $2\sqrt{13}$
- e)  $2\sqrt{5}$



24. Two polygons are similar, and the area of one is 9 times the area of the other. If a side of the smaller polygon is 5, what is the corresponding side of the larger polygon?

Choose one of the following:

- a)  $9\sqrt{5}$
- b) 45
- c) 15
- d) 14
- e)  $3\sqrt{5}$

25. The total surface area of a regular triangular pyramid all of whose edges are 6 is

Choose one of the following:

- a)  $18\sqrt{3}$
- b)  $27\sqrt{3}$
- c)  $36\sqrt{3}$
- d)  $45\sqrt{3}$
- e)  $27\sqrt{3} + 36$

26. A square is inscribed in a circle whose radius is 4. Find the area of the square.

Choose one of the following:

- a) 16
- b) 32
- c)  $16\sqrt{27}$
- d) 64
- e)  $32\sqrt{2}$

27. If the line segment joining the midpoints of two sides of an equilateral triangle measures 12, find the measure of the perimeter of the triangle.

Choose one of the following:

- a) 24
- b) 36
- c) 48
- d) 72
- e) 144

28. Two tangents to a circle from a point outside the circle intercept an arc of  $140^\circ$ . How many degrees are there in the angle formed by the two tangents?

Choose one of the following:

- a)  $40^\circ$
- b)  $50^\circ$
- c)  $55^\circ$
- d)  $70^\circ$
- e)  $30^\circ$

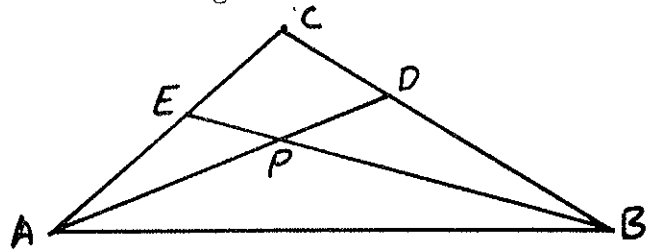
29. Describe what is represented by the equation  $x^2 + y^2 + z^2 - 6x + 8y + 4z + 29 = 0$ .

Choose one of the following:

- a) a point
- b) a line
- c) a circle
- d) a sphere
- e) no points satisfy this equation

30. In triangle ABC, AD and BE bisect angles A and B respectively and intersect in point P. The measure of angle ACB is  $70^\circ$ . The measure of angle APE is
- Choose one of the following:

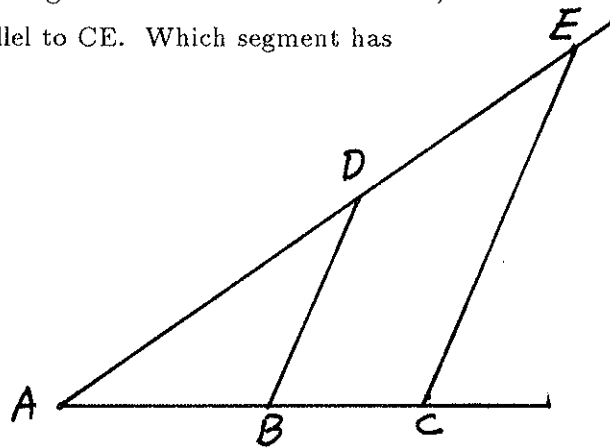
- a)  $50^\circ$
- b)  $55^\circ$
- c)  $60^\circ$
- d)  $67^\circ$
- e) Cannot be determined from the information given.



31. In the given diagram, the lengths of segments are as follows:  $\overline{AB} = 1$ ,  $\overline{BC} = x$ , and  $\overline{AD} = y$ . BD is parallel to CE. Which segment has a length equal to the product  $xy$ ?

Choose one of the following:

- a)  $\overline{AE}$
- b)  $\overline{BD}$
- c)  $\overline{AC}$
- d)  $\overline{CE}$
- e)  $\overline{DE}$



32. In the diagram in the preceding problem, suppose the lengths of the segments are as follows:  $\overline{AB} = x$ ,  $\overline{BC} = 1$ ,  $\overline{AD} = y$ . Which segment has a length equal to the quotient  $y \div x$ ?

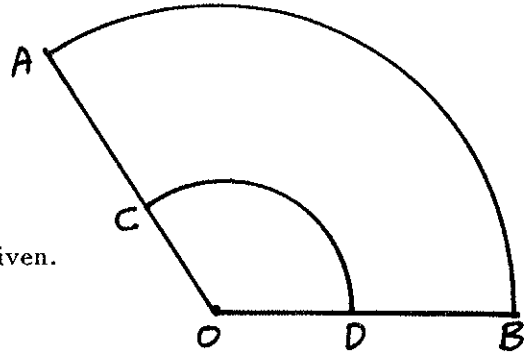
Choose one of the following:

- a)  $\overline{AE}$
- b)  $\overline{BD}$
- c)  $\overline{AC}$
- d)  $\overline{CE}$
- e)  $\overline{DE}$

33. In the given figure, the area of circular sector OAB is 20 and  $\overline{OD} = \overline{DB}$ . The area of the circular sector OCD is

Choose one of the following:

- a) 5
- b) 7
- c) 10
- d)  $3\pi$
- e) Cannot be determined from the information given.



34. In the figure for the preceding problem, suppose arc length  $\widehat{CD} = 5$ . Then arc length  $\widehat{AB}$  would be

Choose one of the following:

- a)  $2\pi$
- b) 7
- c) 8
- d) 10
- e) Cannot be determined from the information given.

35. The locus of points equidistant from the vertices of a regular quadrangular prism is

Choose one of the following:

- a) a point
- b) a line
- c) a plane
- d) a sphere
- e) a circle

36. The inside dimensions of a closed box (rectangular parallelepiped) are 3, 4, and 5 feet. What is the length of the longest straight rod that could be placed in the box?

Choose one of the following:

- a) 5 feet
- b)  $\sqrt{34}$  feet
- c)  $\sqrt{41}$  feet
- d)  $\sqrt{50}$  feet
- e) 8 feet

37. Given:  $\overleftrightarrow{AB}$ ,  $\overleftrightarrow{EC} \perp \overleftrightarrow{AB}$ ,  $\overleftrightarrow{DC} \perp \overleftrightarrow{AB}$ , E not equal to D, and D not in  $\overleftrightarrow{EC}$

Which of the following is NOT true:

Choose one of the following:

- a) When F is in  $\overleftrightarrow{ED}$ ,  $\overleftrightarrow{FC} \perp \overleftrightarrow{AB}$
- b) Plane EDC  $\perp$   $\overleftrightarrow{AB}$
- c) Plane DCB contains A
- d) When G is not in plane EDC,  $\overleftrightarrow{GC} \perp \overleftrightarrow{AB}$
- e) All of these.

38. The center of a sphere inscribed in a tetrahedron is always

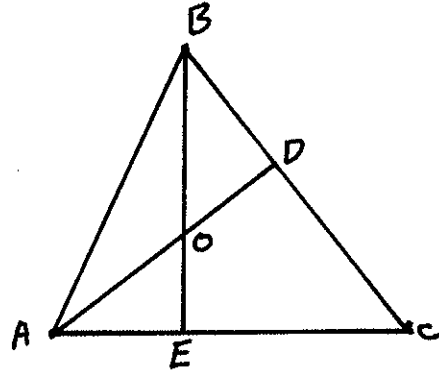
Choose one of the following:

- a) equidistant from the vertices
- b) equidistant from the faces
- c) on all of the altitudes
- d) all of a, b, c
- e) both a and c, but not b.

39. In triangle ABC, AD and BE are altitudes meeting in point O. If  $\overline{AO} = 4$ ,  $\overline{DO} = 6$ , and  $\overline{BE} = 11$ , then the length of BO is

Choose one of the following:

- a) 4
- b) 5
- c) 6
- d) 7
- e) 8



40. In the given figure, the circles of radii  $r$  and  $s$ , respectively, are externally tangent and the two common external tangents meet at P. What is the shortest distance from P to the smaller circle?

Choose one of the following:

- a)  $r + s$
- b)  $\frac{2rs}{r-s}$
- c)  $\frac{2rs}{r+s}$
- d)  $\frac{2s^2}{r-s}$
- e)  $\frac{2r^2}{r-s}$

