THIRTY-THIRD ANNUAL MATHEMATICS CONTEST
sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

GEOMETRY 1989

Scoring formula: 4R - W + 40

Prepared by:
Mathematics Department
Motlow State Community College
Tullahoma, TN
Co-ordinated by: Iris McMurtry and
Bob Crosslin
Edited by: Larry Bouldin, Roane State
Community College

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem there are listed 5 possible answers. You are to work each problem, determine the best answer, and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers, so it is advisable not to do wild guessing.

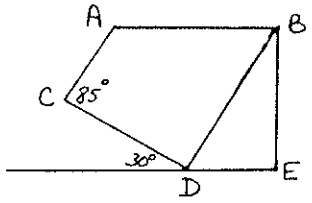
If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet and begin. The working time for the entire test is 80 minutes.

Contributors to TMTA for Annual Mathematics Contest:

Dr. Hal Ramer, President, Volunteer State Community College, Gallatin, Tennessee
Donnelley Printing Company, Gallatin, Tennessee
Sears, Madison, Tennessee
TRW, Ross Gear Division, Lebanon, Tennessee
IBM, Nashville, Tennessee

1. In the given figure $\overleftrightarrow{AB} \parallel \overleftrightarrow{DE}$, $\overleftrightarrow{AC} \parallel \overleftrightarrow{BD}$, and $\overleftrightarrow{BE} \perp \overleftrightarrow{DE}$. Find $m\angle A$.



a. 55°

b. 100°

c. 115°

d. 135°

e. 125°

2. The sum of the measures of the interior angles of a regular polygon is 2520°. How many sides has the polygon?

a. 7

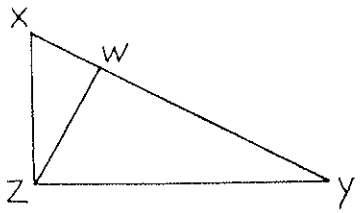
b. 12

c. 14

d. 16

e. 9

3. In right triangle XYZ, $\overline{ZW}$ is perpendicular to hypotenuse $\overline{XY}$, $\overline{XZ} = 4$, and $\overline{ZY} = 6$. Find WY.



a. $\frac{18}{\sqrt{13}}$

b. $\sqrt{13}$

c. $\frac{36}{\sqrt{13}}$

d. $\sqrt{26}$

e. $\frac{18}{\sqrt{26}}$

4. In a rhombus, one angle is 120°, and the shorter diagonal is 12 inches. Find the area of the rhombus.

   a. $144\sqrt{3}$ in^2

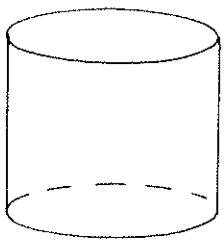
   b. $36\sqrt{3}$ in^2

   c. 144 in^2

   d. $72\sqrt{3}$ in^2

   e. $48\sqrt{3}$ in^2

5. A right circular cylinder has altitude 8 inches and radius 4 inches. Find the total surface area.



   a. 80π in^2

   b. 160π in^2

   c. 96π in^2

   d. 48π in^2

   e. $(32 + 32\pi)$ in^2

6. A, B, C, and D are distinct coplanar points, no three of which are collinear. If E is a point not in the plane of A, B, C, and D, how many distinct planes are determined by the five points?

   a. 4

   b. 5

   c. 6

   d. 7

   e. 8

7. Given $\overleftrightarrow{QR}$ perpendicular to plane T, what is the locus of points 3 inches from $\overleftrightarrow{QR}$ and 3 inches from T?

   a. two planes

   b. two lines

   c. two circles

   d. four points

   e. two cylinders

8. Jim left home on his bike and rode 5 miles north, then 4 miles east, and then 3 miles north. How far from home was he at the end of this trip?

   a. $5\sqrt{41}$ miles

   b. $4\sqrt{5}$ miles

   c. $\sqrt{66}$ miles

   d. $\sqrt{42}$ miles

   e. 12 miles

9. A large storage tank has the shape of a right circular cylinder surmounted by a dome in the shape of a hemisphere. If the cylinder has height 60 feet and radius 15 feet, find the volume of the tank.

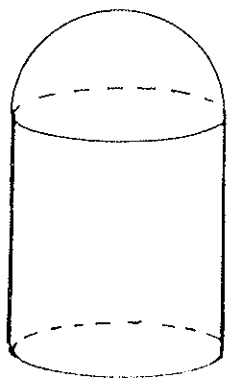
   a. $18{,}000\pi$ ft^3

   b. $15{,}750\pi$ ft^3

   c. $14{,}625\pi$ ft^3

   d. 4050π ft^3

   e. 8100π ft^3



10. Identify the results for the final column of the given truth table.

p	q	$p \rightarrow q$
T	T	
T	F	
F	T	
F	F	

a. T, F, F, T

b. T, T, F, T

c. T, F, T, T

d. T, F, F, F

e. T, T, T, F

11. In a trapezoid the bases are 5 inches and 8 inches, and the legs are 4 inches and 6 inches. If the legs are extended to meet in a point, how much must the shorter leg be extended?

a. 10 inches

b. $5\frac{1}{3}$ inches

c. $8\frac{1}{2}$ inches

d. 4 inches

e. $6\frac{2}{3}$ inches

12. The area of a regular polygon is 36 in^2. Another regular polygon with the same number of sides has half the perimeter of the first polygon. Find the area of the second polygon.

a. 4 in^2

b. 9 in^2

c. 12 in^2

d. 18 in^2

e. 20 in^2

13. Quadrilateral EFGH is inscribed in a circle. If $m\measuredangle E = 4x + 40$, $m\measuredangle G = 5x + 5$, and $m\measuredangle H = 3x + 10$, then $m\measuredangle F =$

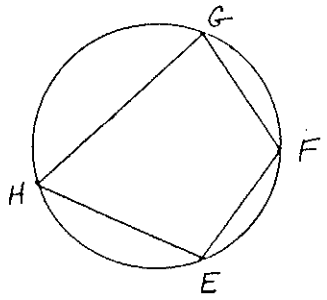
a. 90°

b. 115°

c. 75°

d. 135°

e. 125°



14. Find the ratio of the area of a 45°-45°-90° triangle inscribed in a circle to the area of the part of the circle remaining when the triangle is removed?

a. $1 : \sqrt{2}$

b. $1 : \sqrt{2}\,\pi$

c. $\pi : (\sqrt{2}\,\pi - 1)$

d. $1 : (\pi - 1)$

e. $1 : (\pi - \sqrt{2})$

15. In right triangle ABC, $\overline{CM}$ is the median to hypotenuse $\overline{AB}$, and $\overline{CD}$ is the altitude to hypotenuse $\overline{AB}$. If $m\measuredangle MCD = \frac{1}{3}\, m\measuredangle A$ and $m\measuredangle A > m\measuredangle B$, then $m\measuredangle MCD =$

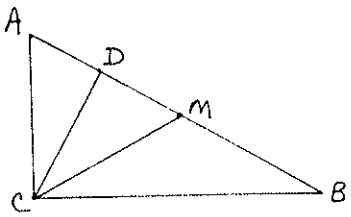
a. 30°

b. 45°

c. 18°

d. 15°

e. 10°



16. The medians of a triangle have lengths 15 cm, 15 cm, and 18 cm. Find the area of the triangle.

a. 72 cm^2

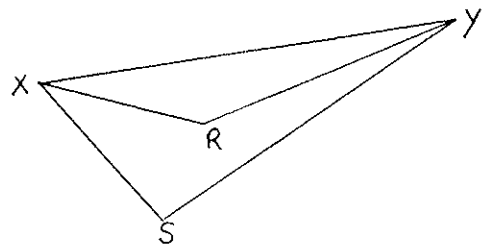
b. 288 cm^2

c. 135 cm^2

d. 144 cm^2

e. 225 cm^2

17. In the given figure $\overrightarrow{XR}$ bisects $\measuredangle YXS$, $\overrightarrow{YR}$ bisects $\measuredangle XYS$, and $m\measuredangle S = a$. Express the measure of $\measuredangle R$ in terms of a.



a. $90 + \frac{a}{2}$

b. $\frac{180 - a}{3}$

c. $\frac{2a + 90}{3}$

d. $180 + \frac{a}{2}$

e. $\frac{a - 90}{3}$

18. One of two pieces of wire having equal lengths is bent to form a square, and the other is bent to form an equilateral triangle. If A is the area of the triangle and B is the area of the square, how are A and B related?

a. $A = \frac{\sqrt{3}}{9} B$

b. $A = \frac{4\sqrt{3}}{9} B$

c. $A = \frac{9\sqrt{3}}{4} B$

d. $A = \frac{9}{4\sqrt{3}} B$

e. $A = \frac{3\sqrt{3}}{2} B$

19. Two parallel chords on the same side of the center of a circle are 12 inches and 20 inches long and 2 inches apart. Find the radius of the circle.

a. 8 in.

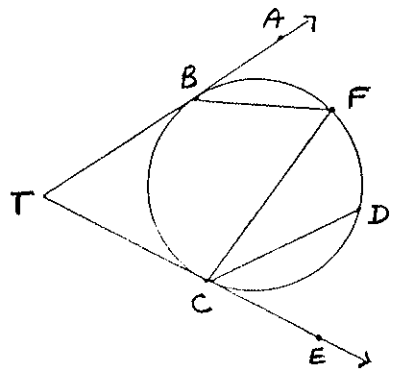
b. 15 in.

c. $2\sqrt{34}$ in.

d. $5\sqrt{13}$ in.

e. $16\sqrt{2}$ in.

20. In the given figure $\overrightarrow{TA}$ and $\overrightarrow{TE}$ are tangent to the circle at B and C respectively, with $m\measuredangle ABF = 30°$, $m\measuredangle T = 70°$, and $m\measuredangle FCD = 20°$. Find the measure of minor arc $\overset{\frown}{BC}$.



a. 140°

b. 100°

c. 80°

d. 120°

e. 110°

21. In $\triangle ABC$, $\overline{AC} > \overline{AB}$. Point D is chosen on $\overline{AC}$ making $\overline{AD} = \overline{AB}$. If $m\measuredangle ABC$ is 20° more than $m\measuredangle C$, find $m\measuredangle CBD$.

a. 30°

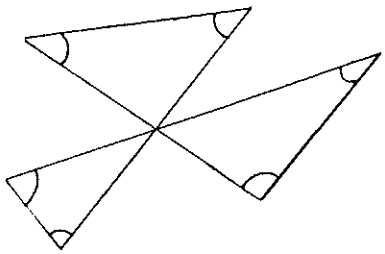
b. 20°

c. 15°

d. 10°

e. 5°

22. In the given figure the sum of the marked angles is



a. 180°

b. 360°

c. 540°

d. 270°

e. cannot be determined without more information

23. If we assume the truth of the statement, "No gog is a jeb," which of the following is true?

a. Sahop must be a jeb since he is not a gog.

b. If Sahop is not a gog, then he is a jeb.

c. If Sahop is a jeb, then he is not a gog.

d. If Sahop is not a jeb, then he is a gog.

e. Sahop is a jeb if and only if he is a gog.

24. In a right circular cone the radius is halved and the height is doubled. The ratio of the volume of the original cone to that of the new cone is

a. 4 : 1

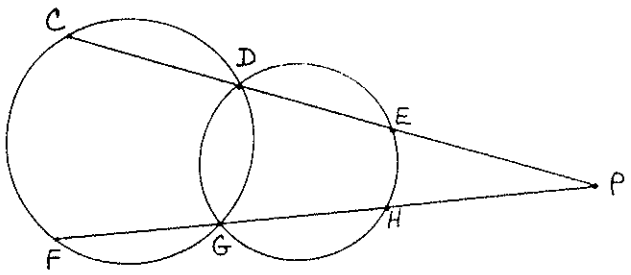
b. 1 : 1

c. 2 : 1

d. 1 : 2

e. 1 : 4

25. In the given figure, $\overline{PE} = 9$, $\overline{DE} = 3$, $\overline{DC} = 6$, and $\overline{PH} = 8$. Find $\overline{FG}$.



a. $\frac{16}{3}$

b. $\frac{5}{2}$

c. $\frac{11}{2}$

d. $\frac{8}{3}$

e. 5

26. Find the area of a semicircular region which requires 40 rods of fence to enclose it.

a. $\frac{800}{\pi}$ rd^2

b. $\frac{1600}{\pi}$ rd^2

c. $\frac{1600\pi}{\pi + 2}$ rd^2

d. $\frac{800\pi}{(\pi + 2)^2}$ rd^2

e. $\frac{1600\pi}{(\pi + 2)^2}$ rd^2

27. Which of the following is not a method for proving triangles congruent?

a. side, side, side

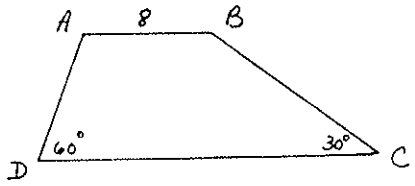
b. side, angle, side

c. angle, angle, side

d. side, side, angle

e. angle, side, angle

28. In the given figure, trapezoid ABCD has perimeter 40. Find the length of an altitude of the trapezoid.



a. $8\sqrt{3} - 4$

b. $18 - 6\sqrt{3}$

c. $4 + \sqrt{3}$

d. $6\sqrt{3} - 6$

e. $4 - \sqrt{3}$

29. It can be proven that there are only five types of regular polyhedrons: tetrahedron, cube, octahedron, dodecahedron, and icosahedron. Identify the three of these that have faces with the same number of edges.

a. cube, octahedron, and icosahedron

b. octahedron, dodecahedron, and icosahedron

c. tetrahedron, cube, and octahedron

d. cube, dodecahedron, and icosahedron

e. tetrahedron, octahedron, and icosahedron

30. The diagonals of a parallelogram are 16 cm and 20 cm, and one side is 12 cm. The area of the parallelogram is

a. $18\sqrt{3}$ cm^2

b. $12\sqrt{3}$ cm^2

c. $36\sqrt{6}$ cm^2

d. $4\sqrt{82}$ cm^2

e. $60\sqrt{7}$ cm^2

31. Identify all of the following statements which are true:

(I) Any three distinct noncollinear points lie in exactly one circle.
(II) A circle may be inscribed in any triangle.
(III) The center for the circumscribed circle of any triangle is the point where the angle bisectors meet.
(IV) The center for the circumscribed circle of a scalene triangle is located outside the triangle.
(V) In a right triangle the orthocenter is at the vertex of the right angle.

a. I, II, IV, V

b. I, II, III

c. I, IV, V

d. I, III, IV

e. II, IV, V

32. A circle is inscribed in a right triangle with hypotenuse 10 and one leg 8. The area of the triangle exceeds the area of the circle by

a. $24 - 4\pi$

b. $24 - 3\pi$

c. $24 - 6\pi$

d. $48 - 4\pi$

e. $48 - 9\pi$

33. Find the area of the triangle with vertices (-2, 1), (2, 4), and (3, 1).

a. 7.5

b. $\frac{10 + \sqrt{10}}{2}$

c. 6.4

d. $10 + \sqrt{10}$

e. 12.5

34. In $\triangle ABC$ with side lengths 6, 7, and 8, the angle bisector of the largest angle divides the opposite side into two segments. Find the length of the shorter segment.

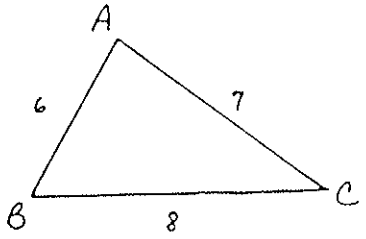
a. $\frac{48}{13}$

b. $\frac{12}{5}$

c. $\frac{16}{5}$

d. $\frac{21}{5}$

e. $\frac{42}{13}$



35. A rectangular tank with base dimensions of 120 cm and 80 cm contains water to a depth of 50 cm. The water rises $6\frac{2}{3}$ cm when a solid metal cube is submerged in the tank. The length of an edge of the cube is

a. 16 cm

b. 32 cm

c. 64 cm

d. 48 cm

e. 40 cm

36. Points P and Q are the midpoints of diagonals $\overline{AC}$ and $\overline{BD}$, respectively, of trapezoid ABCD. If $\overline{PQ} = 30$ and the longer base $\overline{AB} = 100$, then $\overline{DC} =$

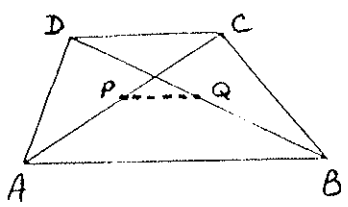
a. 30

b. 35

c. 40

d. 45

e. 50



37. A right prism with regular hexagons as bases has volume $96\sqrt{3}$ cm^3. If all eighteen edges are congruent, then the length of an edge is

a. 6 cm

b. 2 cm

c. 5 cm

d. 4 cm

e. 3 cm

38. Right triangles ABC and ADC are each inscribed within the circle, as shown. $m\measuredangle CAD = 45°$, $m\measuredangle BAC = 30°$, $\overline{AB} = 10$. Find $\overline{DC}$.

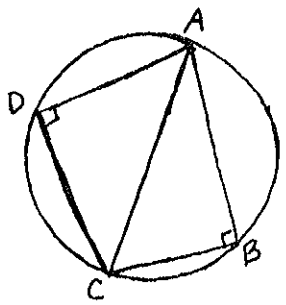
a. $\frac{20\sqrt{6}}{3}$

b. 10

c. 20

d. $\frac{10\sqrt{2}}{3}$

e. $\frac{10\sqrt{6}}{3}$



39. A point of light is 12 feet from a wall. How far from the wall, and parallel to it, should a rectangular piece of cardboard be held such that the shadow on the wall is nine times as great as the area of the cardboard?

a. 8 ft.

b. 4 ft.

c. 6 ft.

d. 9 ft.

e. $10\frac{2}{3}$ ft.

40. In $\triangle ABC$, CA = 10 and CB = 15. Point D is taken on side $\overline{CA}$ such that CD = 6. When side $\overline{CB}$ is extended to point P, then the area of $\triangle CDP$ is $\frac{3}{4}$ the area of $\triangle ABC$. Find CP.

a. 25

b. 22.25

c. 18.75

d. 16

e. 26.25