THIRTY-SECOND ANNUAL MATHEMATICS CONTEST
sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

GEOMETRY 1988

Scoring formula: 4R - W + 40

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DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem there are listed 5 possible answers. You are to work each problem, determine the best answer, and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers, so it is advisable not to do wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

Contributors to TMTA for Annual Mathematics Contest:

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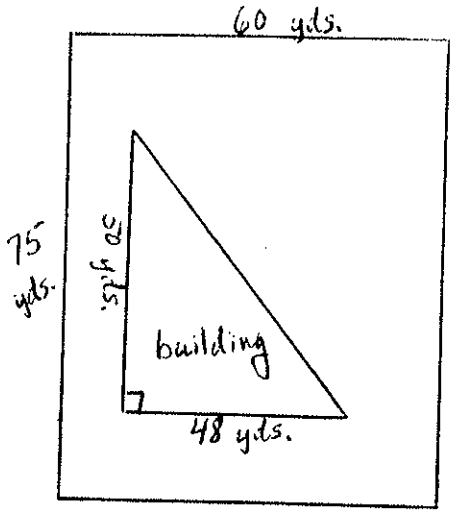
GEOMETRY

1. An office building in a rectangular parking lot is being planned as shown in the sketch. Find the cost of the parking lot if the contractor charges $22 per square yard.



(a) $46,200
(b) $72,600
(c) $3300
(d) $99,000
(e) $66,000

2. Given two solid objects, A and B, of the same shape where object A is larger than object B, how do the ratios of the surface area of an object to its volume compare?

(a) ratio for A < ratio for B
(b) ratio for B < ratio for A
(c) ratio for A = ratio for B
(d) ratio for A + ratio for B = 1
(e) cannot be determined

3. The most that can be said for the figure formed when the midpoints of sides of a quadrilateral are joined consecutively is that:

(a) it is a quadrilateral
(b) it is always a trapezoid
(c) it is always a parallelogram
(d) it is always a rectangle
(e) it is always a rhombus

4. For a particular polyhedron, the sum of the number of vertices and the number of faces is fourteen. The number of edges minus the number of faces is six. For which polyhedron would these two statements be true?

(a) tetrahedron
(b) cube
(c) octahedron
(d) dodecahedron
(e) pyramid with a hexagonal base

5. One side of a triangle is nine cm long. Another side is less than three cm long. The third side must be:

(a) 6 cm
(b) 12 cm
(c) less than 6 cm
(d) less than 12 cm
(e) more than 8 cm

6. In order to determine the number of diagonals in a convex n-gon a student calculates the combination of n things (vertices) taken two at a time. She finds, however, that this calculation exceeds the actual number of diagonals in all cases by:

(a) 2 (b) n! (c) 2n (d) n (e) $\frac{n(n-3)}{2}$

7. Using statement variables and connectives from the "algebra" of symbolic logic, what is the identity element for $\wedge$?

(a) $\sim$ (b) F (c) $\vee$ (d) $\emptyset$ (e) T

8. $(p \wedge q) \longrightarrow r$ is logically equivalent to:

(a) $r \longrightarrow (p \wedge q)$

(b) $\sim r \longrightarrow (\sim p \wedge \sim q)$

(c) $(\sim r \wedge p) \longrightarrow \sim q$

(d) $(\sim r \wedge \sim p) \longrightarrow q$

(e) $\sim (p \wedge q) \longrightarrow \sim r$

9. Examine the figure at the right carefully. C is the center of the circ
What is the measure of angle BCA ?

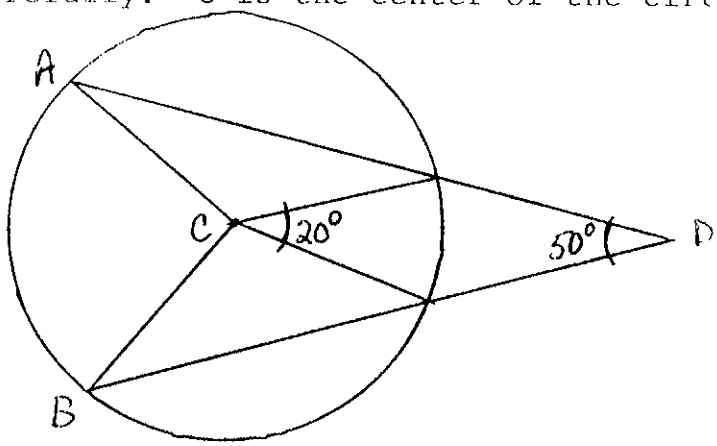
(a) 80°

(b) 70°

(c) 90°

(d) 100°

(e) 120°



10. The figure at the right features two circles, one larger than the other. The smaller circle's diameter equals the larger circle's radius. The ratio of the larger circle's circumference to that of the smaller is:

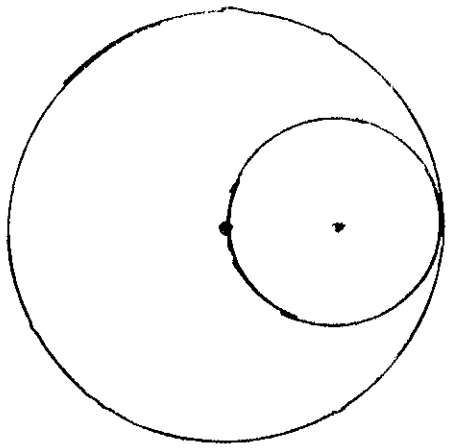
(a) 2

(b) 4

(c) π

(d) 2π

(e) 4π



11. In the figure immediately above, the ratio of the large circle's area to that of the small is:

(a) 2

(b) 4

(c) π

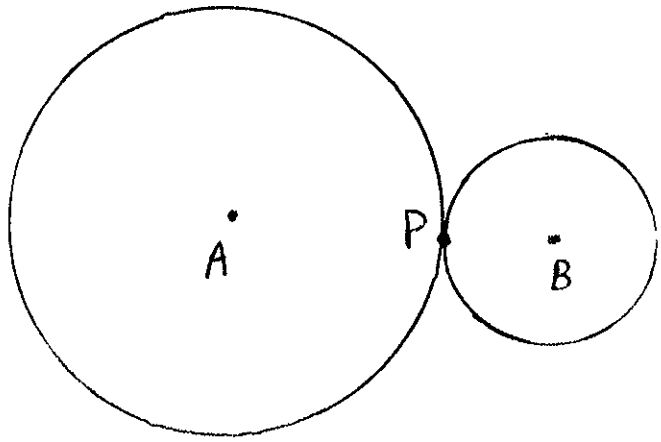
(d) 2π

(e) 4π

12. A solid sphere weighs 64 pounds. A second solid sphere made of the same material has a diameter half that of the former. How much does the second sphere weigh?

(a) 32 lbs.
(b) 16 lbs.
(c) 8 lbs.
(d) 4 lbs.
(e) 4π lbs.

13. In the figure at the right assume that circle A is immovable and that circle B can roll around the circumference of A. If the radius of circle B is half that of A and it makes one trip around circle A returning to its original position, how many rotations will it make?



(a) 1

(b) 2

(c) 3

(d) 4

(e) $\frac{3\pi\sqrt{2}}{2}$

14. Let point P divide the line segment $\overline{AB}$, seen below, in the ratio m/n. Then let A and B have coordinates x_1 and x_2, respectively. What is the coordinate of P ?

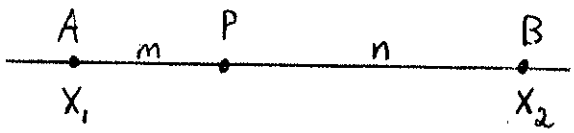
(a) $\frac{m}{m+n}x_1 + \frac{n}{m+n}x_2$

(b) $\frac{n}{m+n}x_1 + \frac{m}{m+n}x_2$

(c) $\frac{1}{n}x_1 + \frac{1}{m}x_2$

(d) $\frac{1}{m}x_1 + \frac{1}{n}x_2$

(e) $nx_1 - mx_2$



15. In the figure at the right $m(\angle\alpha)=150^\circ$ and $m(\angle\beta)=45^\circ$. What is $m(\angle\theta)$?

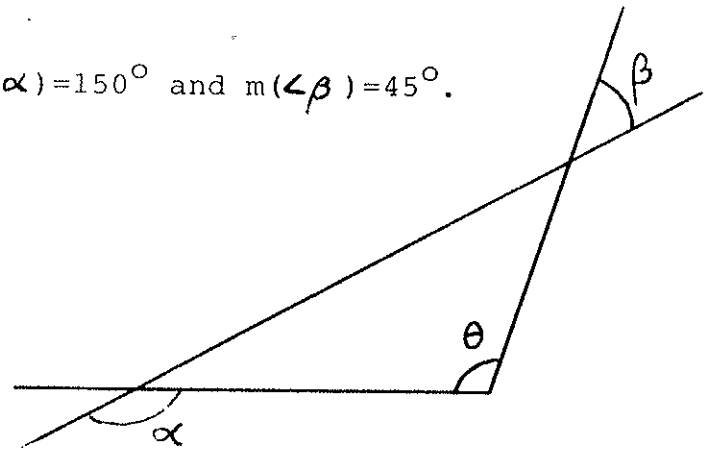
(a) 90°

(b) 110°

(c) 95°

(d) 105°

(e) 100°



16. If the center of a circle is the point (2,-3) and the point (5,7) is on the circle, then what is the area of the circle?

(a) 109π

(b) 88π

(c) 56π

(d) 123π

(e) $\frac{5\pi\sqrt{5}}{2}$

17. A rectangle has a side of length 6 and a diagonal of length 12. The area of the rectangle is:

(a) 84

(b) 72

(c) $18\sqrt{3}$

(d) $36\sqrt{3}$

(e) 36

18. In the figure, $\overline{AC}$ is perpendicular to $\overline{CB}$ and $\overline{CD}$ is perpendicular to $\overline{AB}$. Suppose that the length of $\overline{CD}$ is the square root of the length of $\overline{DB}$. Which of the following segments has unit length?

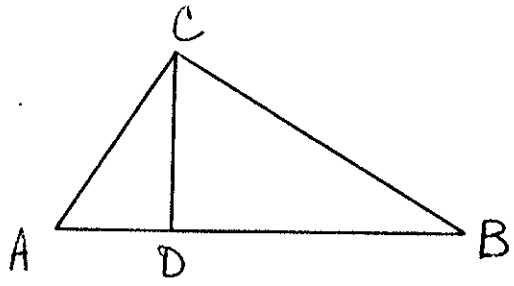
(a) $\overline{AC}$

(b) $\overline{CB}$

(c) $\overline{DB}$

(d) $\overline{AD}$

(e) $\overline{AB}$



19. What is the locus in a plane of all points that are equidistant from two fixed points A and B ?

(a) A circle having $\overline{AB}$ as a diameter.

(b) A circle through A and B and a third arbitrary point C.

(c) The intersection of a line through A and a circle having diameter $\overline{AB}$.

(d) The perpendicular bisector of segment $\overline{AB}$.

(e) A perpendicular bisector of any chord of a circle having diameter $\overline{AB}$.

20. If a triangle is scalene, then it cannot be:

(a) right

(b) obtuse

(c) isosceles

(d) acute

(e) congruent to a right triangle

21. In the figure, O is the center of the circle, $\overline{OA}$ has length 10, and $\overline{OA}$ is perpendicular to $\overline{OB}$. The area of the shaded region is:

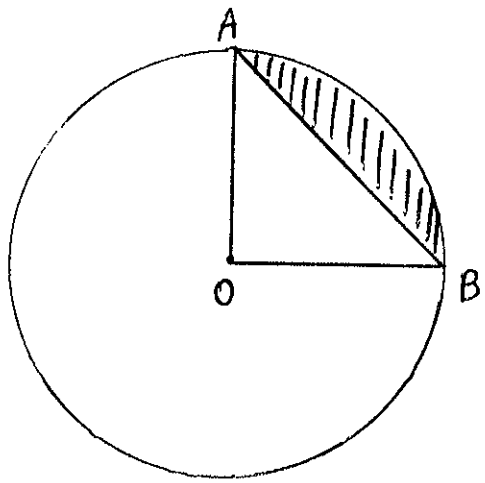
(a) $25\pi - 50$

(b) $50\pi - 50$

(c) $5\pi - 20$

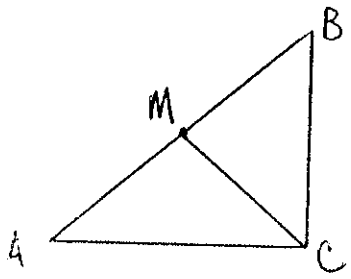
(d) $100\pi - 50$

(e) $50 - 50\pi$



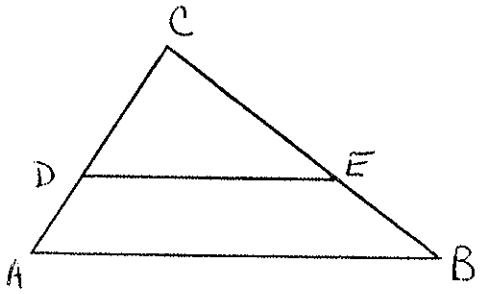
22. If $\angle BCA$ is a right angle, and M is the midpoint of side $\overline{AB}$, and the measure of $\angle CBA$ is 21°, then the measure of $\angle BMC$ is:

(a) 90°
(b) 69°
(c) 103°
(d) 138°
(e) 42°



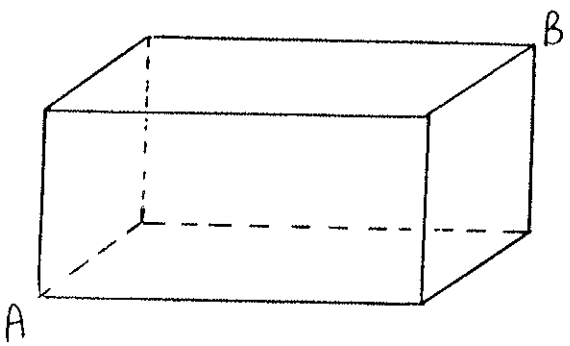
23. If $\overline{DE}$ is parallel to $\overline{AB}$, and if the length of $\overline{AD}$ is 2, the length of $\overline{DC}$ is 3, the length of $\overline{CB}$ is 11, then the length of $\overline{EB}$ is:

(a) 4
(b) 22/5
(c) 2
(d) 11/5
(e) 22/3

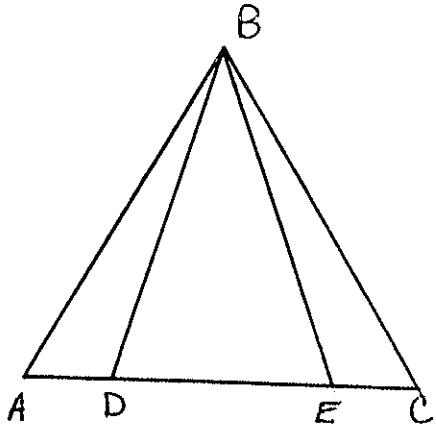


24. The rectangular solid pictured has dimensions s, 2s, and s. The diagonal $\overline{AB}$ of the solid has length:

(a) $s\sqrt{5}$
(b) $s\sqrt{3}$
(c) $s\sqrt{6}$
(d) 4s
(e) 6s

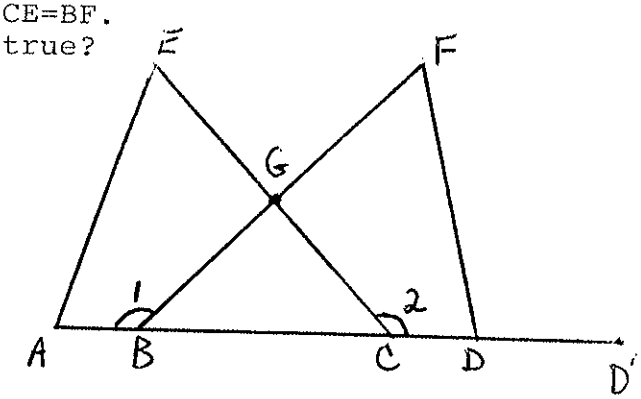


25. In the figure AB=CB and $m(\angle ABE)=m(\angle CBD)$. Which one of the following is always true?



(a) Triangle ABC is equilateral.
(b) Triangle ABE is isosceles.
(c) Triangle BDE is isosceles.
(d) Triangle CBD is isosceles.
(e) Triangle CEB is isosceles.

26. In the figure AB=CD, $m(\angle 1)=m(\angle 2)$ and CE=BF. Which one of the following is always true?



(a) $m(\angle EGF) = m(\angle FDC)$
(b) $m(\angle AEG) = m(\angle DFG)$
(c) $m(\angle 1) = m(\angle FDE)$
(d) $m(\angle CGB) = m(\angle ABE)$
(e) $m(\angle FAB) = m(\angle FDD')$

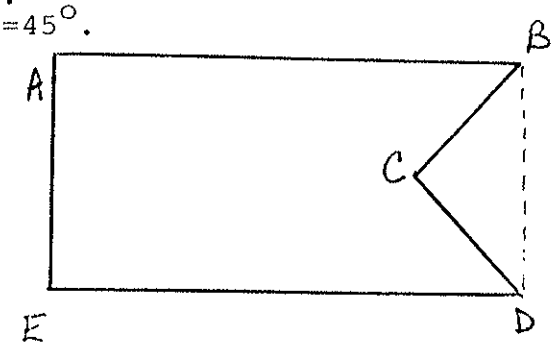
27. Points A and B are fixed in space. C is a point in space such that $m(\angle ACB)=90^{\circ}$. What is the locus of the point C in space $\cup \{A,B\}$

(a) a sphere
(b) a paraboloid
(c) an hyperboloid
(d) a cone
(e) a plane

28. The areas of two similar triangles are in the ratio 16:25. What is the ratio of their corresponding sides?

(a) 16:25
(b) 4:5
(c) 9:41
(d) 16:9
(e) 3:4

29. In the figure $m(\angle A)=m(\angle E)=m(\angle DCB)=90^{\circ}$. AE = 6", AB = 12" and $m(\angle CBD)=m(\angle CDE)=45^{\circ}$. What is the area of the figure ABCDE?



(a) 72 square inches
(b) 36 square inches
(c) 63 square inches
(d) 18 square inches
(e) 64 square inches

30. If P=(-2,-2) and Q=(4,10), which one of the following points lies on the perpendicular bisector of $\overline{PQ}$?

(a) (1,4)
(b) (-3,5)
(c) (9,3)
(d) (15,2)
(e) (7,3)

31. What is the graph of $9x^2 + 9y^2 - 6x + 18y + 1 = 0$?

(a) a parabola
(b) an ellipse
(c) a circle
(d) a hyperbola
(e) a pair of straight lines

32. Which one of the following is a focus of the ellipse

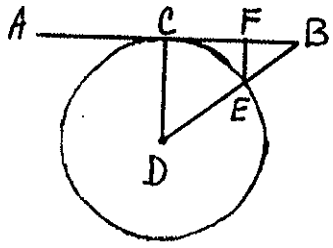
$$\frac{x^2}{16} + \frac{(y+2)^2}{36} = 1 \quad ?$$

(a) $\left(2(1+\sqrt{5}),0\right)$
(b) $\left(0,-2(1+\sqrt{5})\right)$
(c) $\left(2(1+\sqrt{5}),-2(1+\sqrt{5})\right)$
(d) $(2\sqrt{5},-2\sqrt{5})$
(e) $(0,-2\sqrt{5})$

33. The contrapositive of the statement "If not A, then B" is:

(a) If B, then not A
(b) If A, then not B
(c) If not B, then A
(d) If not A, then not B
(e) If B, then A

34. Line segment $\overline{AB}$ has length 16 inches and is tangent to the circle centered at D whose diameter is 12 inches. The point of tangency C is the midpoint of $\overline{AB}$, and $\overline{FE}$ is parallel $\overline{CD}$. Find the length of $\overline{FB}$.

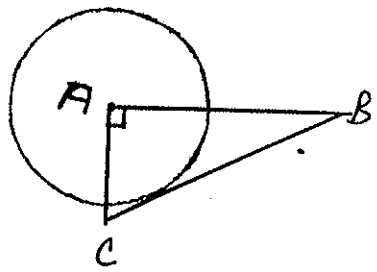


(a) 16/3
(b) 24/5
(c) 12/5
(d) 4
(e) 16/5

35. Three of four vertices of a parallelogram are at (-1,1), (2,-3/2) and (4,3) and the fourth is diagonally opposite (2,-3/2). Find the coordinates of the fourth vertex.

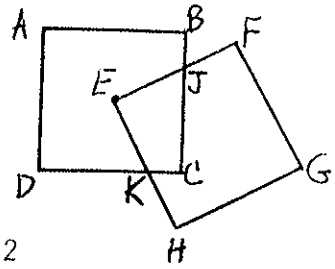
(a) (1,11/12)
(b) (1,11/2)
(c) (2,3/2)
(d) (1/2,-39/8)
(e) (1/2,35/8)

36. In the figure at the right, ABC is a right triangle and the circle centered at A is tangent to the hypotenuse $\overline{BC}$. Find the radius of the circle if $\overline{AB}$ = 2 and $\overline{AC}$ = 1.

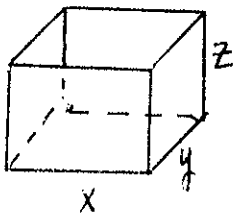


(a) $2/\sqrt{5}$
(b) $\sqrt{2}$
(c) 1
(d) $\sqrt{2}/5$
(e) $\sqrt{5}$

37. The two squares in the figure at the right each have side length 12 and are placed so that a corner of one lies at the center of the other. Find the area of quadrilateral EJCK if the length of BJ = 3.



(a) 32 (b) 48 (c) 36 (d) 24 (e) 42

38. The surface area of the box  with open top is

(a) xyz
(b) y(x + 2z) + 2xz
(c) y(x + 4z)
(d) 5xy
(e) y(x + 3z)

39. The area of a circle written as a function of the circumference C is:

(a) $A = \sqrt{C/4\pi}$

(b) $A = C/\sqrt{4\pi}$

(c) $A = C/2\pi$

(d) $A = \pi r^2$

(e) $A = C^2/4\pi$

40. The (perpendicular) distance between the lines y=x+1 and y=x is:

(a) 1

(b) $\sqrt{2}$

(c) 1/2

(d) $\sqrt{2}/2$

(e) $\sqrt{3}/2$