THIRTY-FIRST ANNUAL MATHEMATICS CONTEST
sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

GEOMETRY 1987

Scoring formula: 4R - W + 40

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DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem there are listed 5 possible answers. You are to work each problem, determine the best answer, and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers, so it is advisable not to do wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

Contributors to TMTA for Annual Mathematics Contest:

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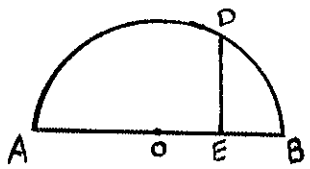
1. The arc $\overset{\frown}{AB}$ is a semicircle of radius 20 and DE is perpendicular to the diameter AB. If the length of DE is 16, then the length of AE is



(a) 24

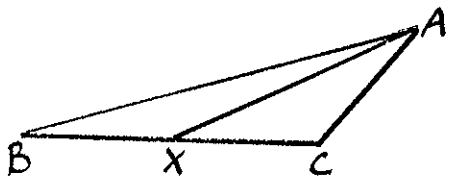
(b) $8\sqrt{5}$

(c) 18

(d) 32

(e) 36

2. In the figure below, the triangle ABC has area 360, BX = 5, and XC = 15. The area of triangle ABX is



(a) 120

(b) 90

(c) 75

(d) $10\sqrt{15}$

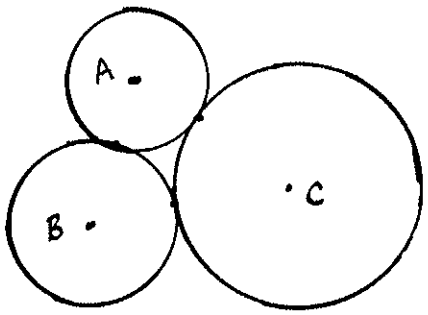
(e) $20\sqrt{5}$

3. A boat sails 60 km. east, then 10 km. south, and then 20 km. west. The distance, in km., of the boat from its starting point is

(a) 90 (b) $20\sqrt{2}$ (c) 50

(d) 70 (e) $10\sqrt{17}$

4. Each of the circles with centers shown is tangent to the other two circles. If AB = 10, AC = 15, and BC = 20, then the radius of the circle with center C is



(a) 12

(b) 12.5

(c) 15

(d) 17.5

(e) $3\sqrt{15}$

5. If it is given that AB is a straight line and m $\measuredangle$ 1 is less than m $\measuredangle$ 2, then which must be true?

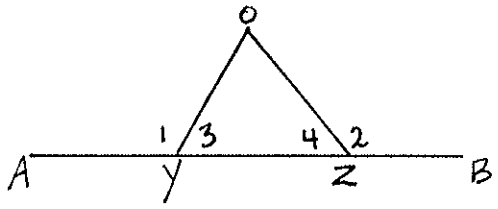
   (a) AY < BZ

   (b) m $\measuredangle$ 3 > m $\measuredangle$ 4

   (c) OY > OZ

   (d) m $\measuredangle$ 3 < m $\measuredangle$ 4

   (e) m $\measuredangle$ 1 + m $\measuredangle$ 2 = 180°



6. AB and CD are cut by transversal MN forming interior angles X and Y. If X - Y = 60° and 3Y - X = 20° then

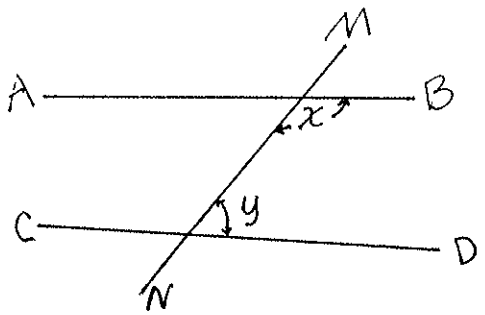
   (a) AB is perpendicular to MN

   (b) AB is not parallel to CD

   (c) CD is perpendicular to MN

   (d) AB is parallel to CD

   (e) X + Y = 180°



7. A, B, C, D are points on a circle and ABCD is an inscribed quadrilateral with sides DC and AB extended. $\overset{\frown}{AB}$ = 40° , m $\measuredangle$ ADC = 60° and m $\measuredangle$ DCB = 90°. Then m $\measuredangle$ 1 is

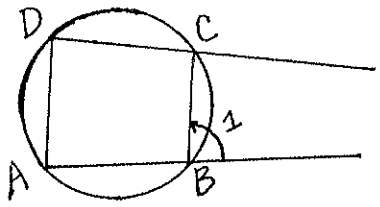
   (a) 30°

   (b) 40°

   (c) 60°

   (d) 120°

   (e) 90°

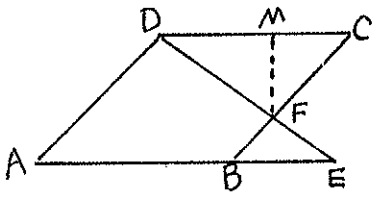


8. If the length of the diagonal of a square is X + Y, then the area of the square is:

   (a) $(X + Y)^2$ (b) $1/2\ (X + Y)^2$ (c) $X^2 + Y^2$

   (d) $1/2\ (X^2 + Y^2)$ (e) $1/2\ (X + Y)$

9. Given, parallelogram ABCD with AB extended so that BE = 1/2 AB. DC = 18 cm and FM = 7 cm. Then the area of triangle BEF is



(a) 15.75 cm^2

(b) 21 cm^2

(c) 63 cm^2

(d) 31.5 cm^2

(e) 13.25 cm^2

10. The areas of the bottom, side, and front of a rectangular box are known. The product of these three areas is equal to:

(a) the volume of the box (b) the square root of the volume

(c) the square of the volume (d) twice the volume

(e) the surface area of the box

11. Right triangle ABC has right angle at B. BD is perpendicular to AC. If CD = 9 and AD = 3, then BC + AB + DB =

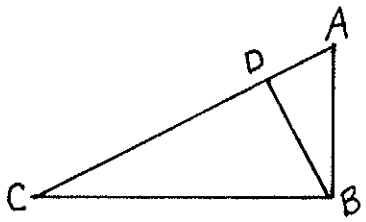
(a) $6\sqrt{3} + 9$

(b) $2\sqrt{2} + 7$

(c) $4 + 3\sqrt{2}$

(d) $6 + 9\sqrt{3}$

(e) $3\sqrt{3} + 14$



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12. The perimeter of a square is the same as the circumference of a circle. Then:

(a) their areas are equal.

(b) the area of the circle is the greater.

(c) the area of the square is the greater.

(d) the area of the circle is π times the area of the square.

(e) the ratio of their areas is π.

13. Increasing the radius of a right circular cylinder by 6 units increases the volume by X cubic units. Leaving the radius unchanged and increasing the height of the cylinder by 6 units also increases the volume by X cubic units. If the original height is 2, then the original radius is:

(a) 2 (b) 4 (c) 6

(d) 8 (e) 6π

14. A regular octagon is to be formed by cutting congruent isosceles right triangles from the corners of a square. If a side of the square has length 1, then a leg of each of the triangles has length:

(a) $\frac{2+\sqrt{2}}{2}$ (b) $\frac{2-\sqrt{2}}{2}$ (c) $\frac{1+\sqrt{2}}{3}$

(d) $\frac{2-\sqrt{2}}{3}$ (e) $\frac{1+\sqrt{2}}{2}$

15. A circular piece of metal of maximum size is cut out of a square piece and then a square piece of maximum size is cut out of the circular piece. The ratio of the area of the second square to the area of the original square is:

(a) $\frac{3}{2}$ (b) $\frac{3}{4}$ (c) $\frac{3}{8}$

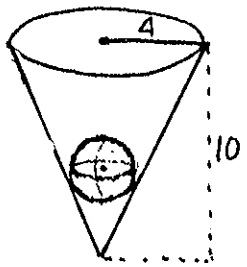
(d) $\frac{1}{2}$ (e) $\frac{2}{3}$

16. Given 15 points in a plane, no three of which are collinear, the number of lines they determine is:

(a) 105 (b) 150 (c) 30

(d) 225 (e) 210

17. A sphere of radius 2 is placed inside a right circular cone of base radius 4 and height 10. The distance from the vertex of the cone to the center of the sphere is



(a) 5

(b) $5\sqrt{2}$

(c) $\sqrt{29}$

(d) $\sqrt{29} - 2$

(e) $\sqrt{29} + 2$

18. The circumferences of two circles have the ratio A:B. Their areas have the ratio:

(a) A:B (b) 2A:2B (c) $A^2:B^2$

(d) $\pi A^2:B^2$ (e) $A^2:\pi B^2$

19. The contrapositive of the statement: "If a quadrilateral is not a rectangle, then it is a square" is:

(a) If a quadrilateral is not a rectangle, then it is not a square.

(b) If a quadrilateral is a square, then it is a rectangle.

(c) If a quadrilateral is a square, then it is not a rectangle.

(d) If a quadrilateral is not a square, then it is a rectangle.

(e) If a quadrilateral is a rectangle, then it is not a square.

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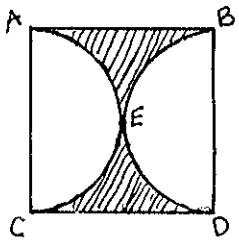
20. If the length and width of a rectangle are each increased by 20%, by what percent is the area increased?

(a) 20% (b) 40% (c) 44%

(d) 50% (e) 22%

21. ABCD is a square of side 10. $\overset{\frown}{AEC}$ and $\overset{\frown}{BED}$ are semicircles. The area of the shaded region is:



(a) $25\pi-100$

(b) 10π

(c) 25π

(d) $100-25\pi$

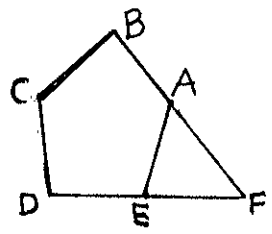
(e) $100-2\pi$

22. The area of a circle inscribed in an equilateral triangle is 27π. The perimeter of the triangle is:

(a) $54\sqrt{3}$ (b) $36\sqrt{3}$ (c) 24

(d) 18 (e) 54

23. If ABCDE is a regular pentagon and BAF and DEF are straight lines intersecting at F, then the degree measure of angle AFE is



(a) 30

(b) 36

(c) 54

(d) 27

(e) 15

24. ABCD is a square of side 4 units. A circle C_1 is inscribed in the square and a circle C_2 is circumscribed about the square. The ratio of the area of C_2 to the area of C_1 is

(a) 4/1 (b) $\sqrt{2}$ (c) 16/1

(d) $\frac{\sqrt{2}}{2}$ (e) 2

25. The measures of the angles of a triangle are in the ratio 3:7:12. The degree measure of the largest angle is closest to

(a) 70 (b) 120 (c) 100

(d) 90 (e) 94

26. The length of a side of an equilateral triangle with area $2\sqrt{3}$ square units is

(a) $2\sqrt{2}$ (b) 4 (c) $2\sqrt{3}$

(d) $\sqrt{3}$ (e) 2

27. The sides of a regular polygon of X sides, X > 4, are extended to form a star. The measure in degrees of the angle at a point of the star is:

(a) $\frac{360}{X}$

(b) $\frac{180\ (X-4)}{X}$

(c) $\frac{180\ (X-2)}{X}$

(d) $180 - \frac{90}{X}$

(e) $\frac{180}{X}$

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28. If AD is the altitude of isosceles triangle ABC with AB = 18 and BC = 12, then the area of triangle ABD is

(a) $36\sqrt{2}$ (b) 108 (c) $72\sqrt{2}$

(d) $12\sqrt{2}$ (e) 54

29. The base of a rectangle is 16 inches. The diagonal is 4 inches longer than the height. The area of the rectangle is:

(a) 544 in^2 (b) 608 in^2 (c) 480 in^2

(d) 1020 in^2 (e) 288 in^2

30. Which one of these statements is <u>not</u> one of Euclid's postulates?

(a) Two points determine a unique line.

(b) If two lines cut by a transversal have equal alternate interior angles, the lines are parallel.

(c) All right angles are equal.

(d) A straight line extends infinitely far in either direction.

(e) It is possible to describe a circle with any point as center and with a radius equal to any finite straight line drawn from the center.

31. Of the following statements, the one that is incorrect is:

(a) Doubling the base of a rectangle doubles the area.

(b) Doubling the altitude of a triangle doubles the area.

(c) Doubling the height of a right circular cylinder doubles its volume.

(d) Doubling the radius of a circle doubles the area.

(e) Doubling the radius of a sphere multiplies the volume by 8.

32. If AB and CD are two intersecting straight lines with m $\measuredangle 1 = (2x-60)^\circ$, m $\measuredangle 2 = (x+y)^\circ$ and m $\measuredangle 3 = (4x-y)^\circ$, then m $\measuredangle 4$ =

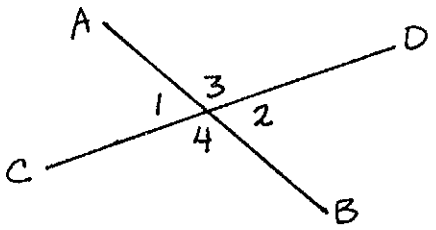
(a) 75°

(b) 12°

(c) 36°

(d) 168°

(e) 155°



33. In the diagram, DE is parallel to AB.
If AC = 20, DC = 12, CE = 15, then BE =

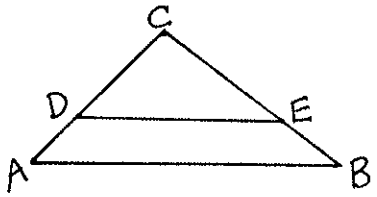
(a) 11

(b) 10

(c) 9

(d) 8.5

(e) 4.5



34. A wire is stretched from point K around a circle of radius 12 in. and back to K, forming an angle of 60° at K. The length of the wire is

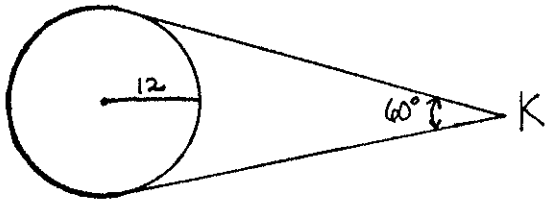
(a) $48 + 8\pi$

(b) $12\sqrt{3} + 16\pi$

(c) $24\sqrt{3} + 16\pi$

(d) $48\sqrt{3} + 8\pi$

(e) $48 + 16\pi$



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35. The volume of the sphere that can be circumscribed about a regular tetrahedron 2 in. on a side is

(a) $\pi\sqrt{6}$ cu. in.

(b) $\frac{2\pi}{3}$ cu. in.

(c) 8π cu. in.

(d) $\frac{4\pi\sqrt{3}}{3}$ cu. in.

(e) $2\pi\sqrt{3}$ cu. in.

36. A point is 4 in. from each face of a dihedral angle of 60°. How far is the point from the edge of the angle?

(a) 8 in.

(b) $4\sqrt{2}$ in.

(c) $4\sqrt{3}$ in.

(d) $\frac{4}{\sqrt{3}}$ in

(e) 2 in.

37. In right triangle ABC, angle A = 30°, angle BDC = 45° and AD = 12. What is the length of BC?

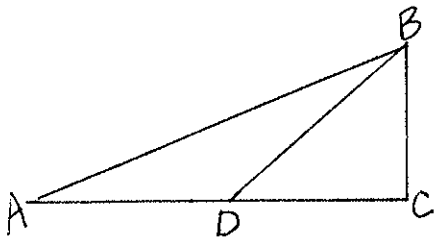
(a) $\frac{6}{\sqrt{3}-1}$

(b) $\frac{12}{\sqrt{3}+1}$

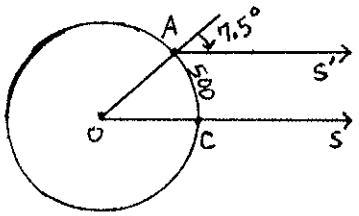
(c) $6\sqrt{2} - 1$

(d) $6(\sqrt{3} + 1)$

(e) $\frac{6}{\sqrt{2}-1}$



38. Eratosthenes (200 B.C.) made a fairly accurate approximation of the circumference of the earth. He observed on the first day of Summer that the sun shone directly into the bottom of a well at Syene, Egypt. This is shown on the diagram as pt. C and means that the sun, S, is directly overhead. At Alexandria, 500 miles north of Syene, at pt. A, a colleague noted that the sun was 7 and 1/2 degrees south of vertical, as shown in the sketch. Eratosthenes assumed rays AS′ and CS were parallel, since the sun is so far away. What amount for the earth's circumference did Eratosthenes calculate?



(a) 24,000

(b) 25,000

(c) 20,000

(d) 23,500

(e) 24,800

39. The development of the non-Euclidean geometry which postulates that through a given point not on a given line can be drawn more than one line parallel to the given line is generally credited to which mathematician?

(a) Riemann

(b) Playfair

(c) Lobachevsky

(d) Bourbaki

(e) Newton

40. Which statement provides a valid conclusion to the following discussion?

All the Eton men in this college play cricket. None of the cricketeers row. All my friends in this college come from Eton. All the scholars are rowing men. All who dine with the Dean are scholars. Therefore,

(a) All my friends dine with the Dean.

(b) None of my friends play cricket.

(c) All the cricket players are from Eton.

(d) None of my friends dine with the Dean.

(e) All the rowing men dine with the Dean.