# THIRTIETH ANNUAL MATHEMATICS CONTEST
Sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

GEOMETRY 1986

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Scoring formula: 4R - W + 40

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DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem there are listed 5 possible answers. You are to work each problem, determine the best answer, and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers, so it is advisable not to do wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

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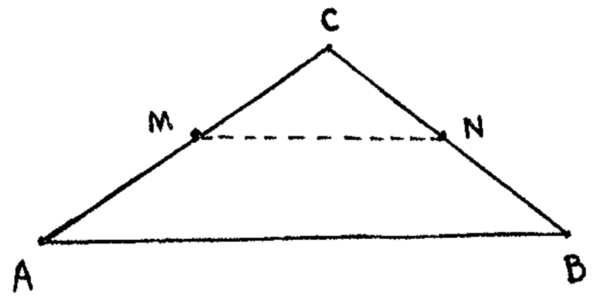
1. If M is the midpoint of $\overline{AC}$ and N is the midpoint of $\overline{BC}$, then $\overline{MN}$ =




   a. 2AB
   b. NB
   c. 1/2 AC
   d. 1/2 AB
   e. 2NC

2. If the circumference of a circle is 4cm, then the area of the circle is

   a. $4/\pi$ cm$^2$
   b. $8\pi$ cm$^2$
   c. $4\pi$ cm$^2$
   d. 16 cm$^2$
   e. $4\pi^2$ cm$^2$

3. A tetrahedron is a special kind of

   a. pyramid
   b. triangle
   c. polygon
   d. prism
   e. quadrilateral

4. One angle of a rhombus has measure 120°. If the shorter diagonal has measure 12cm, then the longer diagonal has measure

   a. 16cm
   b. $8\sqrt{3}$ cm
   c. $12\sqrt{3}$ cm
   d. 12 cm
   e. $24\sqrt{3}$ cm

5. The sum of the measures of the interior angles of a regular heptagon is

   a. 360°

   b. 700°

   c. 900°

   d. 1080°

   e. 514°

6. If the circumference of a circle is 100cm then the measure of one side of a square inscribed in the circle is:

   a. $\frac{25\sqrt{2}}{\pi}$ cm

   b. $\frac{50\sqrt{2}}{\pi}$ cm

   c. $\frac{100}{\pi}$ cm

   d $\frac{100\sqrt{2}}{\pi}$ cm

   e. $\frac{50}{\pi}$ cm

7. If an angle of a triangle remains unchanged but each of its two including sides is doubled, then the area is multiplied by

   a. 2

   b. 3

   c. 4

   d. 6

   e. 8

8. Let $\overline{AB}$ be a diameter and $\overline{MN} \perp \overline{AB}$ at N. If the circle has radius 9 and $\overline{NB} = 8$, then $\overline{MB}$ equals:

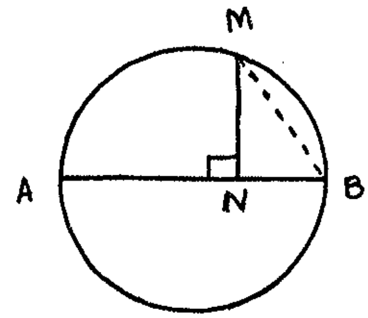
a. 12

b. 10

c. $5\sqrt{3}$

d. $6\sqrt{2}$

e. 14




9. If $\overline{AB}$ and $\overline{AC}$ are tangents to the circle with center O, $m\angle BAC = 60°$ and $\overline{AB} = 4$, then AD =

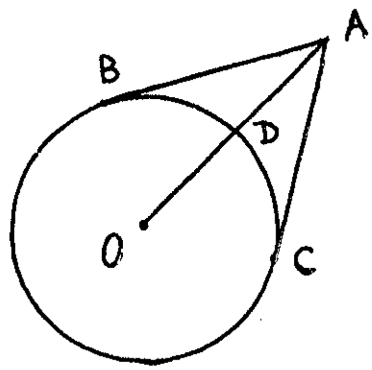
a. $\frac{\sqrt{3}}{3}$

b. $\sqrt{3}$

c. $\frac{2\sqrt{3}}{3}$

d. $\frac{4\sqrt{3}}{3}$

e. 3




10. In the figure, the smaller circle is tangent to the larger circle at B and passes through the center, A, of the larger circle. If the area of the larger circle is 12, find the area of the smaller circle.

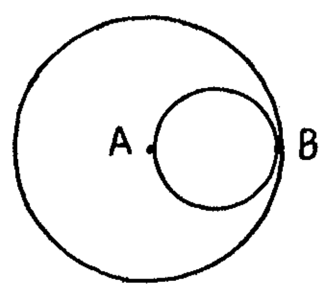
a. 3

b. 2

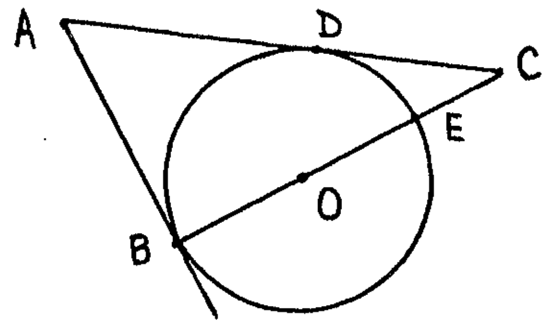
c. 4

d. 6

e. 8



11. In the diagram $\overline{AC}$ and $\overline{AB}$ are tangent to the circle with center O. If $\overline{AD} = 15$ and $\overline{CE} = 5$ and $\overline{BC} = 20$, then $\overline{DC} =$




a. 15

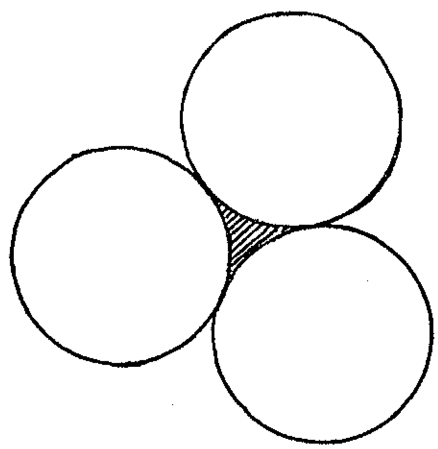
b. 10

c. 7.5

d. 5

e. 4

12. Three circles each with area $81\pi$ square feet are tangent to each other as shown. Find the area of the shaded region



a. $81\sqrt{3}$ sq ft

b. $81(\sqrt{3} - \pi/2)$ sq ft

c. $\frac{27\pi}{2}$ sq ft

d. $81(\pi - \sqrt{3})$ sq ft

e. $\frac{81(\sqrt{3} - \pi)}{2}$ sq ft

13. Find the area of a regular hexagon with sides of measure 2 ft.

a. $3\pi$ sq ft

b. $4\pi$ sq ft

c. $3\sqrt{3}$ sq ft

d. $4\sqrt{3}$ sq ft

e. $6\sqrt{3}$ sq ft

14. The length of the longer diagonal of a rhombus is 40cm. The altitude shown is 24 cm. Find the length of the side of the rhombus.

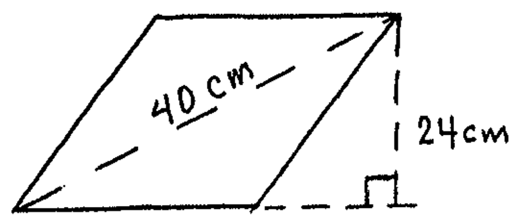
a. 18cm

b. 20cm

c. 25cm

d. 30cm

e. 36cm




15. If O is the midpoint of AB , m∢AOD = m∢COB and OC = OD , then it follows that

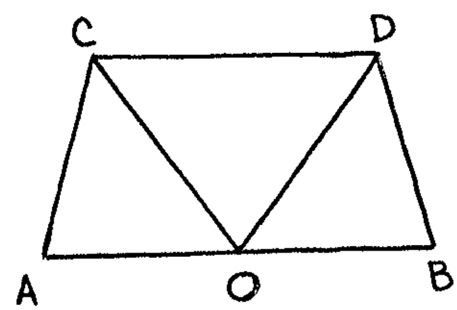
a. $\Delta COD \cong \Delta BOD$

b. $\Delta CAO \cong \Delta DBO$

c. $\Delta COD$ is equilateral

d. $\Delta CAO$ is isosceles

e. $\Delta DBO$ is a right triangle




16. The composition of two reflections in intersecting lines is a

a. reflection

b. translation

c. rotation

d. glide reflection

e. stretch reflection

17. If $\Delta ABC \cong \Delta DEC$ then it follows that

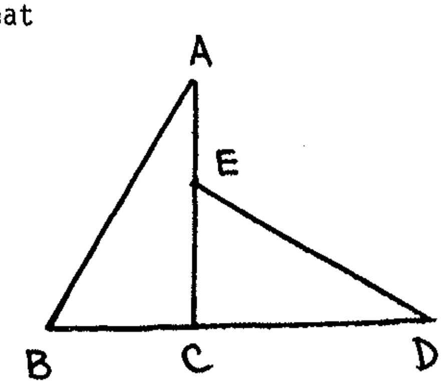
a. $BD = BA$

b. $BD < BA$

c. $BD > BA$

d. $BD = AC$

e. $BD < AC$



18. The area of the triangle shown below is:

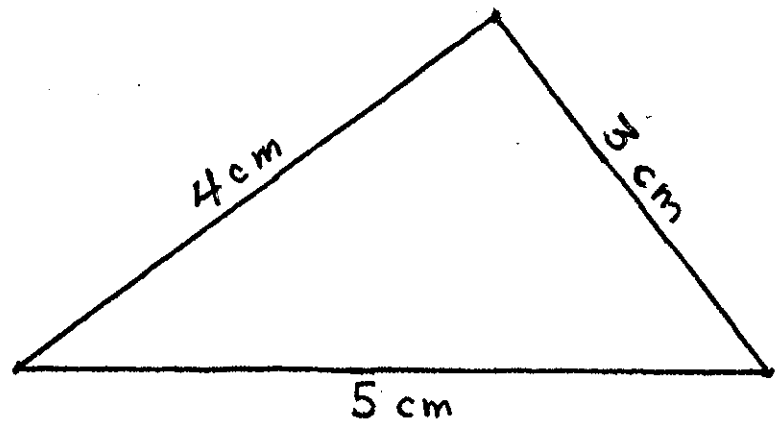
a. $\frac{5\sqrt{3}}{2}$ cm$^2$

b. $2\sqrt{3}$ cm$^2$

c. $5/2$ cm$^2$

d. 6 cm$^2$

e. $3\sqrt{5}$ cm$^2$




19. If m∡ADC = 90°, then the area of quadrilateral ABCD shown below is:

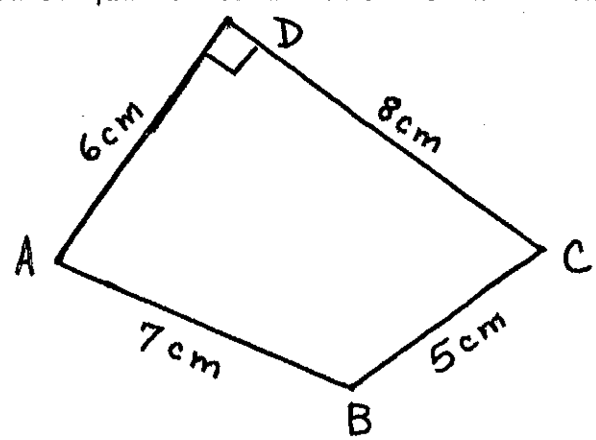
a. $165/4$ cm$^2$

b. 48 cm$^2$

c. 35 cm$^2$

d. $83/2$ cm$^2$

e. $24 + 2\sqrt{66}$ cm$^2$




20. If D is any point on side $\overline{AB}$ and $\overline{DE}$ is parallel to $\overline{BC}$, then

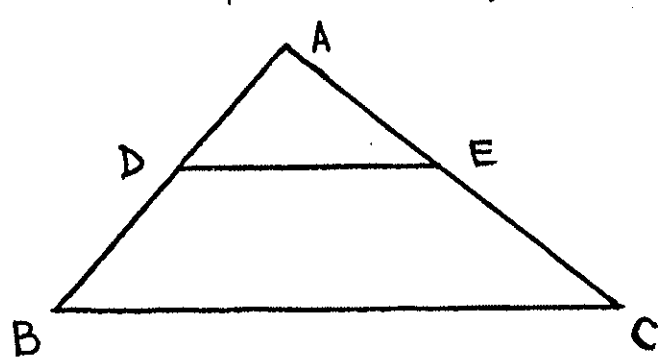
a. DE = 1/2 BC

b. DB/AB = EC/AC

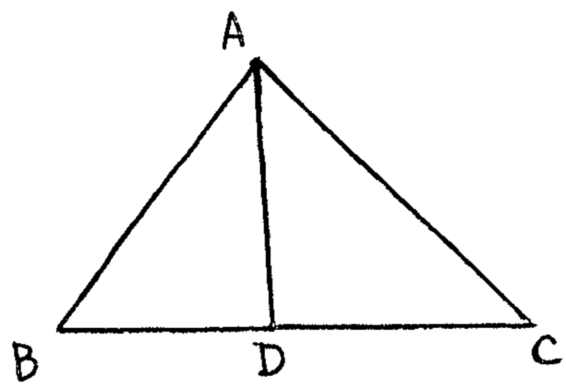
c. AD/DB = AC/EC

d. AD/AE = AD/DB

e. DB/EC = DE/BC



21. If AD is the bisector of ∢BAC , AB = 6 , AC = 8 and BC = 4 then DC =




a. 12/7

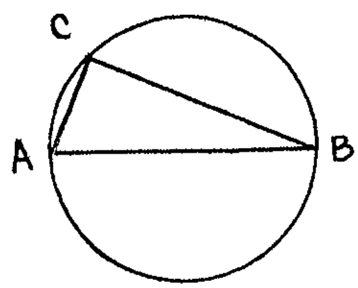
b. 16/3

c. 16/7

d. 3

e. 2

22. If AB is a diameter of the circle and m∢BAC = 72° , then the measure of the minor arc AC is:




a. 36°

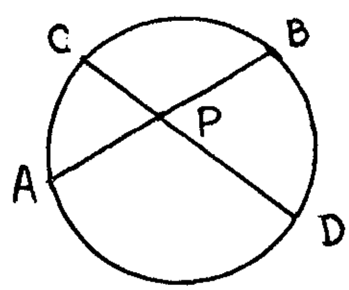
b. 72°

c. 90°

d. 18°

e. 9°

23. If $\overline{AB}$ and $\overline{CD}$ are any two chords of a circle which intersect at point P, which is the midpoint of $\overline{AB}$ , and $\overline{CP}$ = 2, and $\overline{PD}$ = 18, then $\overline{AB}$ =




a. 20

b. 18

c. $3\sqrt{5}$

d. 12

e. $5\sqrt{3}$

24. The orthocenter of a triangle is the point of intersection of the

a. Angle bisectors

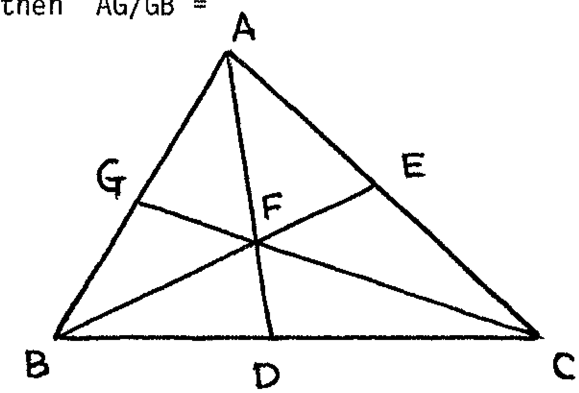
b. Medians

c. Cevians

d. Altitudes

e. Orthogons

25. If in triangle ABC, BD/DC = 1/2, CE/EA = 1/2, $\overline{BE}$ intersects $\overline{AD}$ at F, and $\overline{CF}$ intersects side $\overline{AB}$ at G, then AG/GB =




a. 1/2

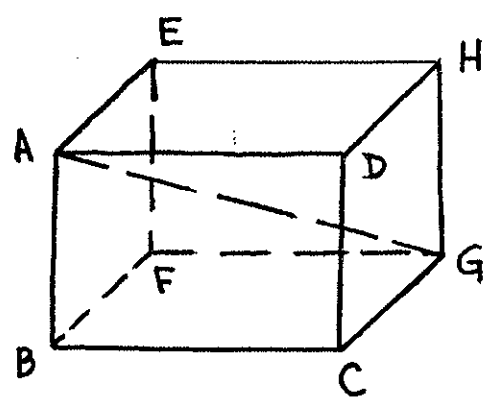
b. 1

c. 2

d. 4

e. 8

26. A rectangular solid is a polyhedron with six rectangular faces. If the edges of a rectangular solid have measure 3, 4, and 5, the length of the diagonal is:




a. 6

b. 7

c. $5\sqrt{2}$

d. 12

e. $2\sqrt{5}$

27. If a swimming pool, built in the shape of a right trapezoidal prism with dimensions shown below were filled with water until the surface was 1 ft. from the top, then the volume of the water in the pool would be:

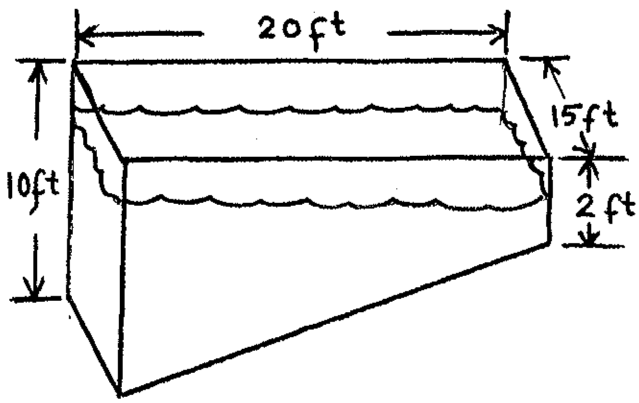
a. 1800 cu.ft.

b. 1700 cu.ft.

c. 1600 cu.ft.

d. 1500 cu.ft.

e. 1400 cu.ft.




28. A regular pyramid has a hexagonal base. Each edge of the base is 1 ft. long. If the altitude of the pyramid is 6 ft., then its volume is:

a. $3\sqrt{3}$ cu.ft.

b. $9\sqrt{3}$ cu.ft.

c. $1/2\sqrt{3}$ cu.ft.

d. $4\sqrt{3}$ cu.ft.

e. $8\sqrt{3}$ cu.ft.

29. A tennis ball is made in the shape of a sphere. The outside diameter is 8cm, the material is 1/2 cm thick. What is the volume of the material in the ball?

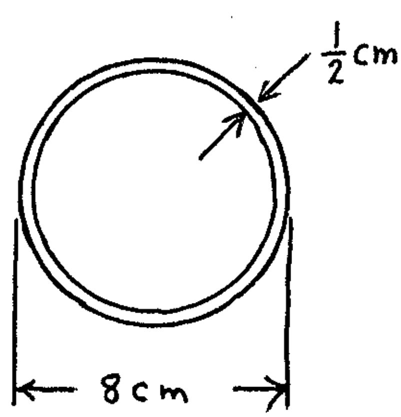
a. $(84)\pi$

b. $(355/2)\pi$

c. $(169)\pi$

d. $(134)\pi$

e. $(169/6)\pi$



30. Which of the following is <u>not</u> a regular polyhedron?

a. icosahedron

b. cube

c. tetrahedron

d. dodecahedron

e. cuboctahedron

31. In the figure, $\overline{CE}$ and $\overline{DE}$ are equal chords of a circle with diameter $\overline{CD}$ and center O . If arc AB is a quarter-circle, then the ratio of the area of $\triangle CED$ to the area of $\triangle AOB$ is:

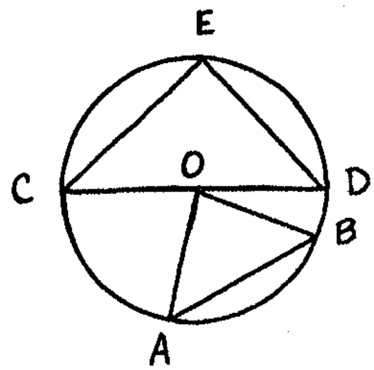
a. $\sqrt{2} : 1$

b. $\sqrt{3} : 1$

c. $2 : 1$

d. $3 : 1$

e. $4 : 1$




32. In the diagram below, $\triangle ADE$ is any triangle and B and C are points in the segment $\overline{AD}$ . Which of the following angle relationships is true?

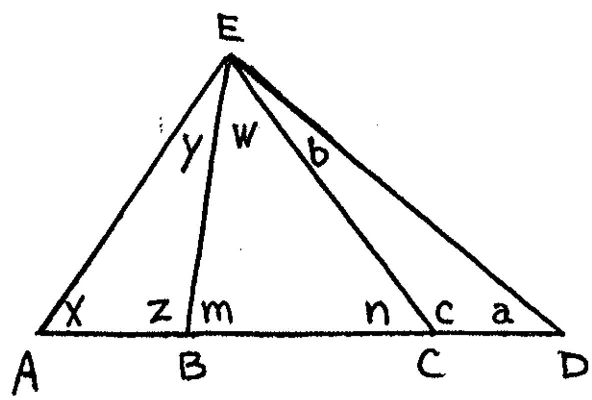
a. $x + z = a + b$

b. $y + z = a + b$

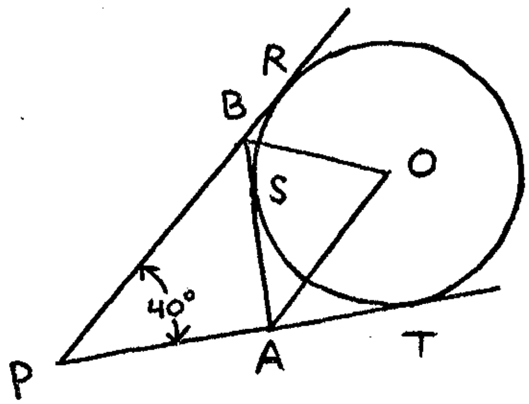
c. $m + x = w + n$

d. $x + z + n = w + c + m$

e. $x + y + n = a + b + m$



33. If triangle PAB is formed by three tangents to circle O , and m∡APB = 40°, then m∡AOB =




a. 45°

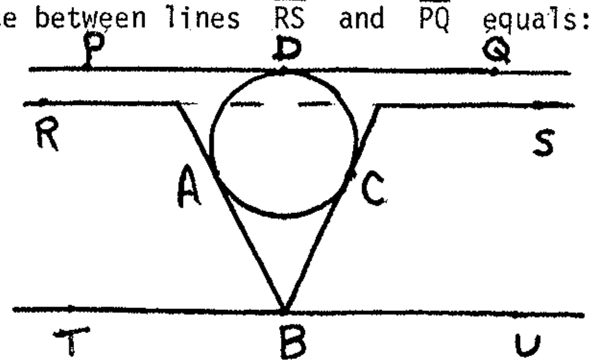
b. 50°

c. 55°

d. 60°

e. 70°

34. In the diagram the circle has diameter 3/8" . $\overline{PQ} \parallel \overline{RS} \parallel \overline{TU}$ . $\overline{AB}$ is tangent to the circle at A , $\overline{BC}$ is tangent to the circle at C , $\overline{PQ}$ is tangent to the circle at D and m∡ABC = 60° . If the distance between lines $\overline{RS}$ and $\overline{TU}$ is 1/2 inch, then the distance between lines $\overline{RS}$ and $\overline{PQ}$ equals:




a. 1/32 in.

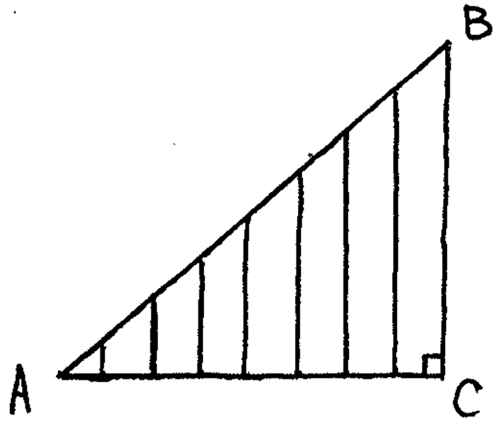
b. 1/16 in.

c. 3/32 in.

d. 1/8 in.

e. 3/16 in.

35. Side $\overline{AC}$ of right triangle ABC is divided into 8 equal parts. Seven line segments, parallel to BC are drawn to AB from the points of division. If AC=8 and BC = 10 , then the sum of the lengths of the seven line segments is:




a. 32

b. 33

c. 34

d. 35

e. 36

36. If in $\triangle ABC$ , AC = CD and $(m\angle CAB - m\angle ABC) = 30°$ , then $m\angle BAD$ is:

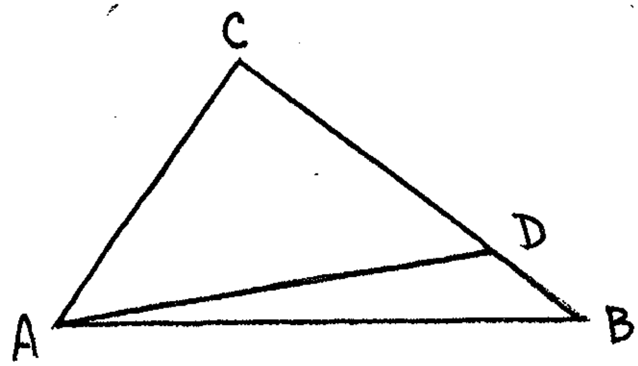
a. 10°

b. 15°

c. 20°

d. 22 1/2°

e. 30°




37. If circle O has diameters AB and CD perpendicular to each other, and AM is any chord intersecting CD at P , then AP · AM =

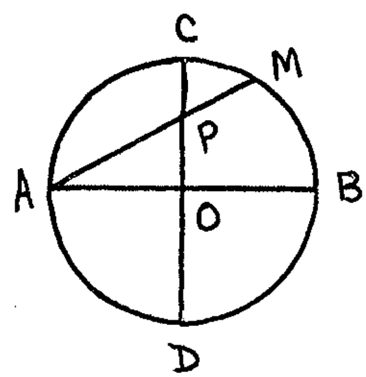
a. AO · OB

b. AO · AB

c. CP · CD

d. CP · PD

e. CO · OP




38. In a circle with center O , chord $\overline{AB} \cong$ chord $\overline{AC}$ . Chord $\overline{AD}$ cuts $\overline{BC}$ at E . If AC = 12 and AE = 8 then AD

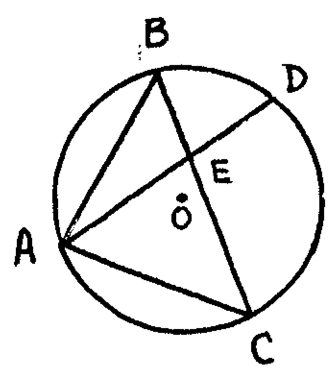
a. 14

b. 16

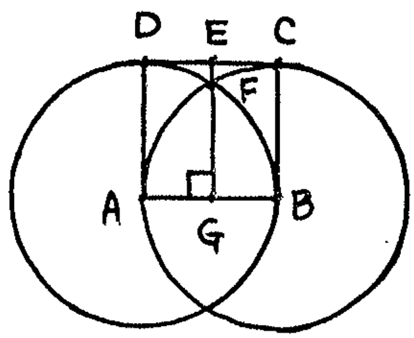
c. 18

d. 20

e. 21



39. In the figure below, A and B are centers of circles; quadrilaterial ABCD is a square; and $EG \perp AB$ . Find FE




a. $\frac{1}{2}(2 - \sqrt{3})AB$

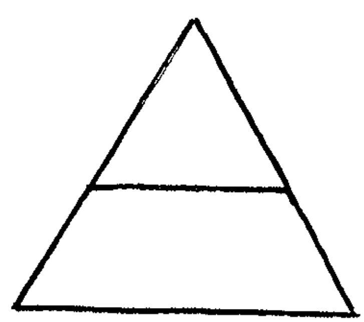
b. $\frac{1}{2}(\sqrt{3} - 1)AB$

c. $\frac{1}{2}(3 - \sqrt{3})AB$

d. $\frac{1}{2}(\sqrt{5} - 2)AB$

e. $\frac{1}{2}(\sqrt{5} - \sqrt{3})AB$

40. A line intersects two sides of an equilateral triangle and is parallel to the third side. If this line divides the triangular region into a trapzoid and a smaller triangle having equal perimeters, then the ratio of the area of the trapezoid to the area of the smaller triangle is:



a. 9/7

b. 7/4

c. 7/9

d. 4/7

e. 9/11