TWENTY-EIGHTH ANNUAL MATHEMATICS CONTEST
Sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

GEOMETRY TEST 1984

Scoring Formula: 4R − W + 40

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DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem there are listed 5 possible answers; one and only one is correct. You are to work each problem, determine the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers, so it is advisable not to do much wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.
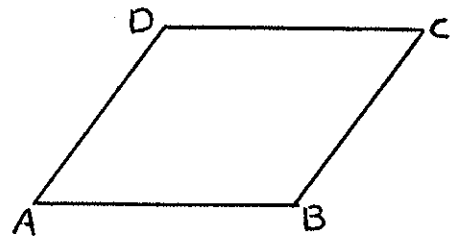

Contributors to TMTA for Annual Mathematics Contest:

1. The figure ABCD is a parallelogram.

    If m($\angle$A) = 35°. what is m($\angle$ B)?

    a) 55°

    b) 65°

    c) 145°

    d) 155°

    e) 325°

2. In the circle m($\overset{\frown}{AB}$) = 90°. What is m($\angle$ACB)?

    a) 30°

    b) 45°

    c) 60°

    d) 90°

    e) Not enough information
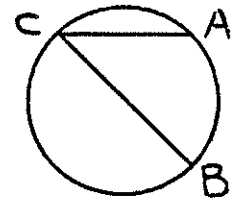
3. In the circle m($\overset{\frown}{AB}$) = 90° and m($\overset{\frown}{CD}$) = 40°. What is m($\angle$AEB)?

    a) 20°

    b) 25°

    c) 40°

    d) 50°

    e) 65°
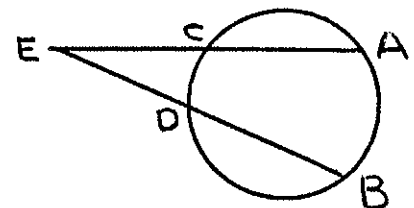
4. The sum of the interior angles of a hexagon is

    a) 360°   d) 900°

    b) 540°   e) 1080°

    c) 720°

5. The points of intersection of the altitudes of an obtuse triangle determine a(n).

    a) Point   d) Right triangle

    b) Equilateral triangle   e) Obtuse triangle

    c) Acute triangle   2

6. Segments AB and DE are chords of a circle; the segments intersect in a point C which is <u>exterior</u> to the circle. If AC = 4, BC = 5 and CD = 2, what is CE?

a) 3

b) 5

c) 7

d) 9

e) 10

7. Segments AB and DE are chords of a circle; the segments intersect in a point C which is <u>interior</u> to the circle. If AC = 4, BC = 5 and CD = 2, what is CE?

a) 3

b) 5

c) 7

d) 9

e) 10

8. Let A and B be distinct points. The locus of points P such that PA = PB is

a) A straight line

b) A single point

c) A circle

d) An ellipse

e) A hyperbola

9. Let A and B be points. The locus of points P such that ∠APB is a right angle is

a) A straight line

b) A single point

c) A circle

d) An ellipse

e) A hyperbola

10. Let A and B be points. The locus of points P such that PA = 2PB is

a) A straight line

b) A single point

c) A circle

d) An ellipse

e) A hyperbola

3

11. In the figure the lines AA^1, BB^1, CC^1 are medians of the triangle ABC. ABC. If $AA^1 = 12$, $BB^1 = 11$, $CC^1 = 10$, what is the length of AO?

    a) 4

    b) 6

    c) 8

    d) 10.5

    e) $\frac{2}{3}\sqrt[3]{1320}$
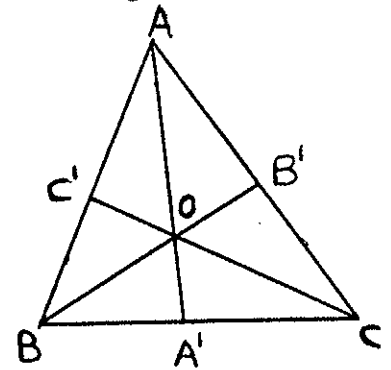
12. In the figure circle 0 is inscribed in triangle ABC, meeting the triangle at E, F and G. Then necessarily

    a) G is the midpoint of AC

    b) OG is an altitude of the triangle ABC

    c) OC bisects angle C

    d) The area of circle 0 is $1/2$ (area ABC)

    e) The area of circle 0 is $1/\pi$ (area ABC)
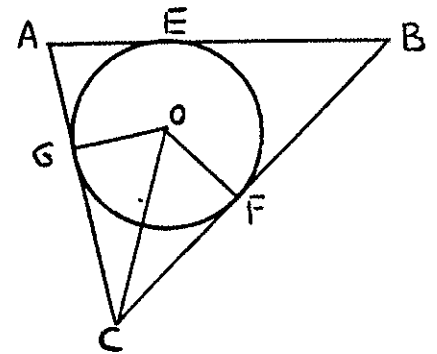
13. The diameter of a circle is increased by 2 cm. The area is increased by

    a) $4\pi\,cm^2$

    b) $8\pi\,cm^2$

    c) $12\pi\,cm^2$

    d) $16\pi\,cm^2$

    e) Cannot be determined

14. Find the height of a circular arch having a radius of 5 feet if the height of the arch is one fourth of its span.

    a) 1

    b) 2

    c) 3

    d) 4

    e) 5

4

15. The triangle ABC is a right triangle
    with AB = 3 inches and AC = 4 inches.
    Segment AD is perpendicular to BC.
    What is the length of AD?

    a) 6/5   d) 24/5

    b) 12/5   e) 5

    c) 16/5

16. In the figure, the circle is circumscribed
    about triangle ABC, BD is an altitude of
    the triangle, and BK is a diameter of the
    circle. If m($\angle$ BAC) = 70° and m($\angle$ BCA) = 50°,
    then m($\angle$ DBK) =

    a) 10° d) 40°

    b) 20° e) 60°

    c) 30°

17. In the figure ABC is an equilateral triangle
    with AB = 12. The distance from point P to AB
    is $\sqrt{3}$ and from P to BC is $2\sqrt{3}$. What is the
    distance from P to side AC?

    a) $\sqrt{3}$

    b) $2\sqrt{3}$

    c) $3\sqrt{3}$

    d) $4\sqrt{3}$

    e) $5\sqrt{3}$

18. The sides of a triangle are 3,6 and 7 inches. What is the area?

    a) 9   d) $\sqrt{20}$

    b) 10.5   e) $\sqrt{80}$

    c) 21

5

19. The sides of a triangle are 3, 4, and 6 inches. The largest angle
    of the triangle is

    a) Acute

    b) Right

    c) Obtuse

20. In triangle ABC, $m(\angle A) = 30^\circ$ and $m(\angle C) = 135^\circ$. If AB = 12 inches,
    what is BC?

    a) 4   d) $6\sqrt{3}$

    b) 6   e) 10

    c) $6\sqrt{2}$

21. A square is inscribed in a circle whose radius is 8 inches. What
    is the area of the square?

    a) 64π d) 92

    b) 16π e) 128

    c) 64

22. An equilateral triangle is inscribed in a circle whose radius is 8
    inches. What is the area of the triangle?

    a) 8π d) $32\sqrt{3}$

    b) $8\sqrt{3}$ e) $48\sqrt{3}$

    c) $16\sqrt{3}$

23. Find the area of a rhombus whose shorter diagonal and whose sides
    are each 2 inches

    a) $2\sqrt{3}$ d) $5\sqrt{3}$

    b) $3\sqrt{3}$ e) $6\sqrt{3}$

    c) $4\sqrt{3}$

24. Given the triangle ABC, if CD = DA and CE = EB, what is the ratio of AF/FE?

    a) 3

    b) 2

    c) 1

    d) 1/2

    e) $\sqrt{2}$
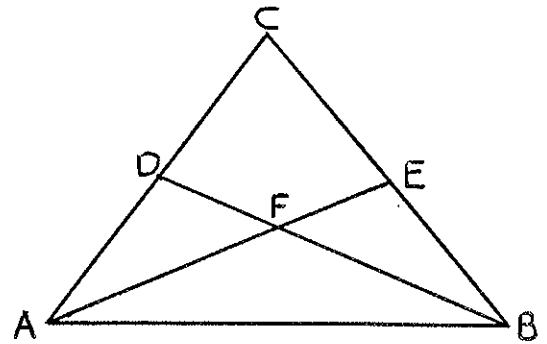
25. Let l and m be two lines in <u>space</u>. Which of the following must be true?

    a) l and m intersect in a point.

    b) l and m either intersect or are parallel.

    c) There is a plane which contains both l and m.

    d) There is a plane which is perpendicular to both l and m.

   e) There is a line which is perpendicular to both l and m.

26. A rectangular room is 12 foot long, 10 foot wide, and 8 foot high. What is the distance from a corner of the ceiling to the opposite corner on the floor?

    a) 16   d) $\sqrt{244}$

    b) 18   e) $\sqrt{308}$

    c) 20

27. In the triangle ABC, the bisector of angle A meets side BC in its midpoint. The triangle must be

    a) Isosceles   d) Obtuse

    b) Equilateral   e) None of the above

    c) Acute

28. A man 6 ft tall casts a shadow 12 feet in length when he is 20 feet from the base of a lamp post. How long will the shadow be when the man is only 10 feet from the lamp post?

a) 2 feet

b) 4 feet

c) 6 feet

d) 8 feet

e) 10 feet

29. In the triangle ABC, AB has length 5, BC has length 12 and AC has length 13. If BD is the altitude to side AC, what is the length of BD?

a) 65/17

b) $12-\sqrt{5}$

c) 13/5

d) 25/13

e) 60/13

30. In the figure angle 0 is 30°, OB is 4 inches and AB is perpendicular to OA. What is the length of AB?

a) $\sqrt{2}$

b) $\sqrt{3}$

c) 1.5

d) 2

e) 3

31. In the figure angle 0 is 15° and. angle B is 120°, OB is 4 inches. What is the length of BC?

a) 1

b) $\sqrt{2}$

c) 2

d) $2(\sqrt{3} - 1)$

e) $2\sqrt{3}$

8

32. In the figure, O is the center of the circle and ABCD is a square. If the radius of the circle is 1, what is the area of the square?
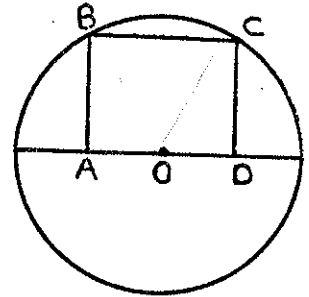
a) 1/4

b) 1/2

c) 4/5

d) 1

e) 4/3

33. In the figure, the sum of the angles A, B, C, D, E is
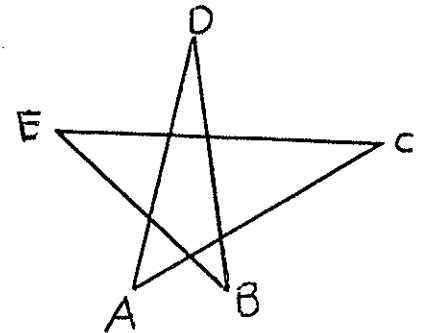
a) Less than 180°

b) 180°

c) Greater than 180° and less than 360°

d) 360°

e) Cannot be determined

34. Given the statement if p, then not q, which of the following statements follows logically?

a) If p, then q

b) If q, then p

c) If q, then not p

d) If not q, then p

e) If not p, then q

35. In triangle ABC side AB = 6, side AC = 3 and side BC = 4. What is the length of the median to side AB?

a) $\sqrt{3}$

b) $\sqrt{3.5}$

c) 2

d) $\sqrt{4.5}$

e) $\sqrt{5}$

36. In the figure ABCD is a circumscribed quadrilateral with AB = 4, BC = 5, CD = 3. What is AD?
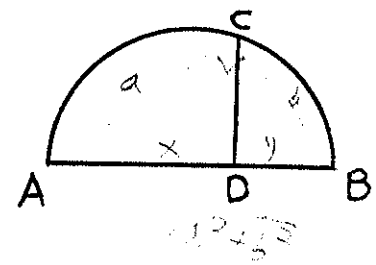
    a) 1

    b) 2

    c) 2.4

    d) 3

    e) 3.75
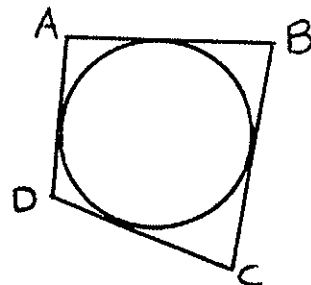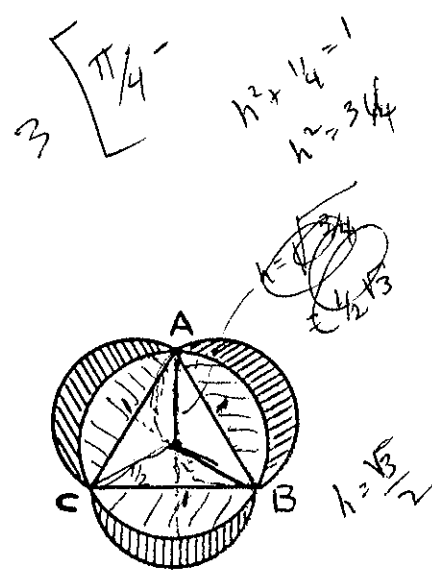
37. A semicircle is constructed on diameter AB, and CD is perpendicular to AB. If AC = a and BC = b, then CD =

    a) ab

    b) $\sqrt{ab}$

    c) a + b

    d) (a + b)/2

    e) 2ab/(a + b)

38. In the figure, ABC is an equilateral triangle, 1 inch on a side. The shaded regions are bounded by the circle which is circumscribed about the triangle and the semicircle constructed with the sides of the triangle as diameters. What is the area of the shaded region?

    a) $(6\pi - 4\sqrt{3})/27$

    b) $(\pi + 4\sqrt{3})/18$

    c) $(2\pi + \sqrt{3})/12$

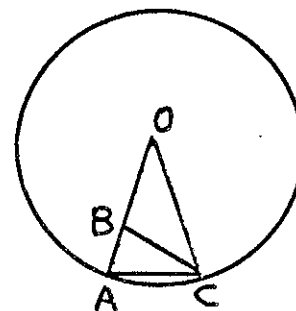   d) $(\pi + 6\sqrt{3})/24$

    e) $(3\pi + 18\sqrt{3})/2$

10

39. In the figure 0 is the center of the circle,

    $\dfrac{OA}{OB} = \dfrac{OB}{AB}$ and AC = BO. What is m($\angle$ AOC)?

    a) 18°

    b) 24°

    c) 30°

    d) 36°

    e) 42°

40. In the figure AB = 2 and BC = 6. The
    point P lies on the line AD which is
    perpendicular to the line ABC. What
    is the length of AP if the angle BPC
    is to be a maximum?

    a) $\sqrt{6}$

    b) 4

    c) $\sqrt{24}$

    d) 5

    e) 8