

TWENTY-SEVENTH ANNUAL MATHEMATICS CONTEST
Sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

GEOMETRY TEST

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East Tennessee State University
Johnson City, Tennessee

Scoring Formula: $4R - W + 40$

This test was prepared from a list of Geometry questions submitted by Vanderbilt University.

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem there are listed 5 possible answers; one and only one is correct. You are to work each problem, determine the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers, so it is advisable not to do much wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

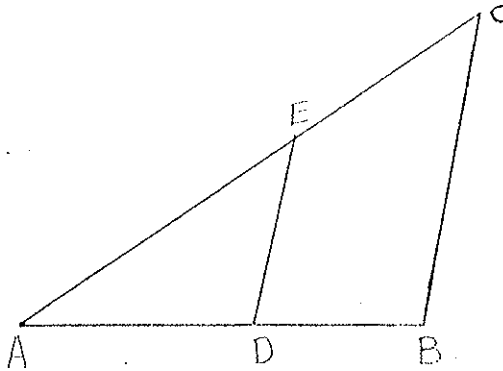
When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

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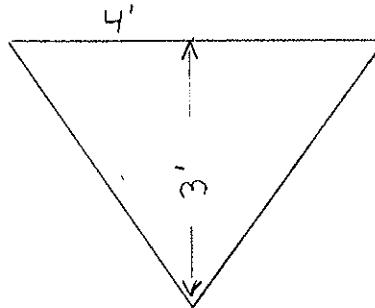
1. Given triangle ABC , $D \in \overline{AB}$ and $E \in \overline{AC}$ such that $\overleftrightarrow{DE} \parallel \overleftrightarrow{BC}$. If $m(\overline{AD}) = 3$, $m(\overline{DB}) = 2$ and $m(\overline{AE}) = 5$, then $m(\overline{EC})$ equals:

- a) $\frac{5}{3}$
- b) $\frac{10}{3}$
- c) $\frac{25}{3}$
- d) $\frac{3}{2}$
- e) 3



2. A trough is 12 ft. long and its ends are in the form of inverted isosceles triangles (see figure), having an altitude of 3 ft. and a base of 4 ft. What is the depth of the water when the trough is filled to one-half of its capacity?

- a) $\frac{2}{3}$ ft.
- b) $\frac{3}{2}$ ft.
- c) $\frac{3}{\sqrt{2}}$ ft.
- d) $\frac{\sqrt{2}}{3}$ ft.
- e) $\frac{2}{\sqrt{3}}$ ft.



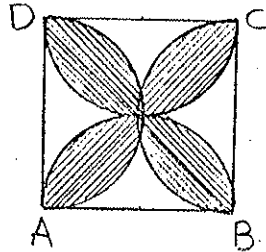
3. The diameter of a circle is increased by 2.5 cm. The circumference of the circle is increased by:

- a) 2.5π cm
- b) 1.25 cm
- c) 5π cm
- d) 6.25π cm
- e) cannot be determined from the given information

4. ABCD is a square of side 2 units. The sides of the square are diameters for 4 semi-circles.

The area of the shaded region is:

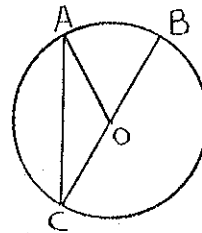
- a) $\pi - 2$
- b) $4 - \pi$
- c) $2\pi - 4$
- d) $8 - 2\pi$
- e) $4\pi - 4$



5. The given circle has its center at O and radius 4.

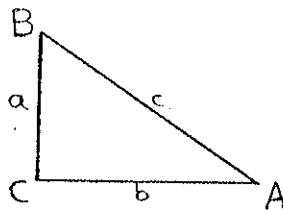
$m(\angle AOB) = 60^\circ$. The area of the triangle is

- a) $\frac{8\pi}{3}$
- b) 8
- c) $\sqrt{3}$
- d) $2\sqrt{3}$
- e) $4\sqrt{3}$



6. In ABC, $\angle C$ is a right angle, so $a^2 + b^2 = c^2$. If a new triangle is formed with the same a and b, but $\angle C$ is an obtuse angle then which of the following is correct for the new triangle:

- a) $a^2 - c^2 > b^2$
- b) $a^2 - c^2 = b^2$
- c) $a^2 + b^2 = c^2$
- d) $a^2 + b^2 < c^2$
- e) $a^2 + b^2 > c^2$



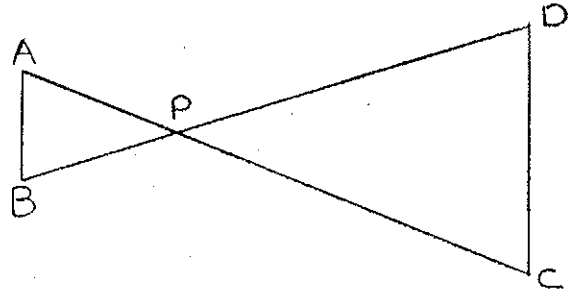
7. The ratio of the surface areas of two spheres is 4:9. The ratio of their volumes is

- a) 2:3
- b) 4:9
- c) 8:27
- d) 16:81
- e) 64:729

8. In the figure $\angle B \cong \angle D$, $m(\overline{CD}) = 20$ and $m(\overline{AB}) = 5$.

Then

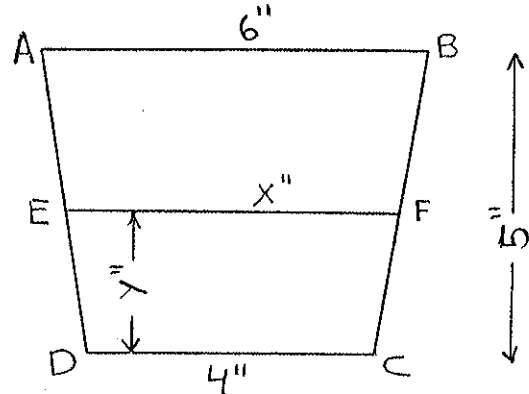
- a) $m(\overline{BP}) = \frac{1}{5} m(\overline{BD})$
- b) $m(\overline{BP}) = \frac{1}{4} m(\overline{BD})$
- c) $m(\overline{BP}) = 5 m(\overline{BD})$
- d) $m(\overline{BP}) = 4 m(\overline{BD})$
- e) $m(\overline{BP}) = .5 m(\overline{BD})$



9. Suppose ABCD is an isosceles trapezoid with 4 inches and 6 inches as the lengths of the parallel sides, and with a height of 5 inches (see figure). Suppose EF is drawn parallel to the parallel sides and has length x inches and is y inches above CD.

Then

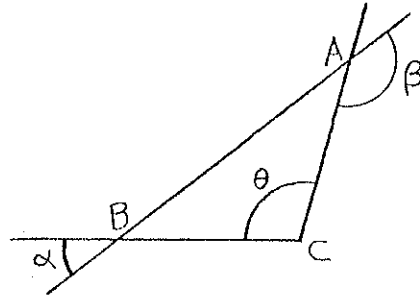
- a) $\frac{x}{y} = \frac{6}{4}$
- b) $\frac{x}{y} = \frac{6}{5}$
- c) $\frac{x}{y} = \frac{5}{4}$
- d) $x = 4 + \frac{2}{5}y$
- e) $x = 4 + \frac{1}{5}y$



10. Indicate which of the four statements about π is not completely correct. If all four statements are correct, then indicate "e" as your answer.
- a) π is the ratio of the circumference of a circle to its diameter.
 - b) π is an irrational number.
 - c) $\pi = 3.1416$
 - d) π is approximately equal to $\frac{22}{7}$.
 - e) All of the above statements are correct.

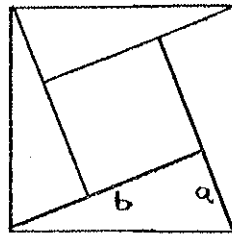
11. In the figure $m(\angle\alpha) = 40^\circ$ and $m(\angle\beta) = 150^\circ$. The $m(\angle\theta)$ equals

- a) 120°
- b) 110°
- c) 100°
- d) 95°
- e) 90°



12. The four right triangles are congruent and $b = 3a$. The area of the rectangle in the interior of the figure is:

- a) b^2
- b) $4b^2$
- c) a^2
- d) $2a^2$
- e) $4a^2$



13. The product of the lengths of the diagonals of a rhombus is 6. The sum of the lengths of the diagonals of the rhombus is 5. The length of a side of the rhombus is

- a) 3
- b) $2\frac{1}{2}$
- c) $\frac{1}{2}\sqrt{5}$
- d) $\sqrt{6}$
- e) $\frac{1}{2}\sqrt{13}$

14. The area of a circle is equal to

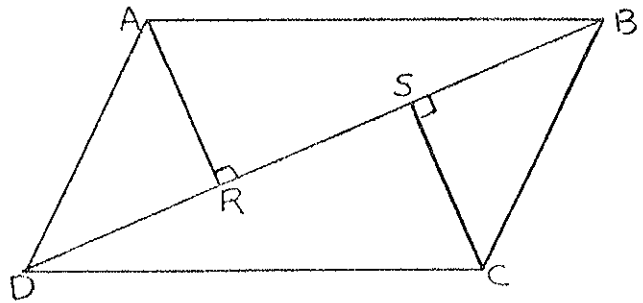
- a) the product of $\frac{\pi}{2}$ and the square of the diameter
- b) the product of $\left(\frac{\pi}{2}\right)^2$ and the square of the diameter
- c) the product of 2π and the radius
- d) the product of the radius and the circumference
- e) the product of one-half the radius and the circumference

15. A right circular cone is inscribed in a sphere of radius r . The diameter of the base of the cone is twice the height of the cone. Which statement below is true?

- a) The diameter of the base of the cone equals the slant height of the cone.
- b) The volume of the sphere is 4 times that of the cone.
- c) The lateral surface area of the cone is $2\pi r^2$.
(Lateral surface area of cone = πrs , where r is radius of base of cone and s is slant height of cone.)
- d) The volume of the sphere is $\frac{1}{3}\pi h^3$, where h is the height of the cone.
- e) The given cone is the inscribed cone of minimum volume.

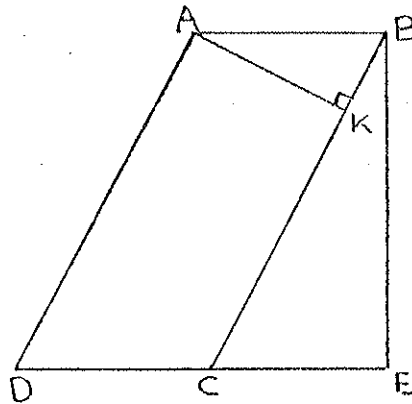
16. In the parallelogram ABCD, $\overline{CS} \perp \overline{BD}$ and $\overline{AR} \perp \overline{BD}$. Then

- a) $\angle ADR \cong \angle BCS$
- b) $\angle ADR \cong \angle CBS$
- c) $\angle SDC \cong \angle SBC$
- d) $\angle ADR \cong \angle DAR$
- e) $\angle DAR \cong \angle RAB$

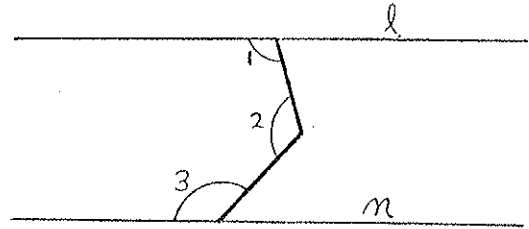


17. In the figure, ABCD is a parallelogram $m(\overline{AB}) = 4$, $m(\overline{AD}) = 8$, C is the midpoint of \overline{DE} , $\overline{AK} \perp \overline{BC}$ and $\overline{BE} \perp \overline{DE}$. The area of the parallelogram is:

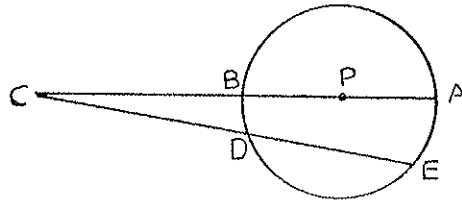
- a) 32
- b) $16\sqrt{5}$
- c) 48
- d) $25\sqrt{3}$
- e) $16\sqrt{3}$



18. In the figure $l \parallel n$, $m(\angle 1) = 100^\circ$ and $m(\angle 2) = 120^\circ$. The $m(\angle 3)$ is
- a) 60°
 - b) 100°
 - c) 120°
 - d) 140°
 - e) 150°

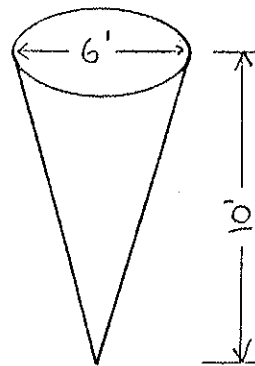


19. In the figure \overline{AC} and \overline{EC} are secant segments of circle P. $m(\overline{BC}) = 3$, $m(\overline{DC}) = 4$, and $m(\overline{ED}) = 2$. The radius \overline{PA} has length
- a) $3/4$
 - b) $3/2$
 - c) 2
 - d) $5/2$
 - e) 5



20. Water is flowing into a tank in the shape of an inverted cone (as shown). The tank measures 10 feet deep, and the diameter of the circular top is 6 feet. What is the volume of water in the tank when the water is 4 feet deep?

- a) $\frac{144}{25} \pi \text{ ft}^3$
- b) $\frac{144}{75} \pi \text{ ft}^3$
- c) $\frac{576}{25} \pi \text{ ft}^3$
- d) $\frac{576}{75} \pi \text{ ft}^3$
- e) $\frac{96}{15} \pi \text{ ft}^3$



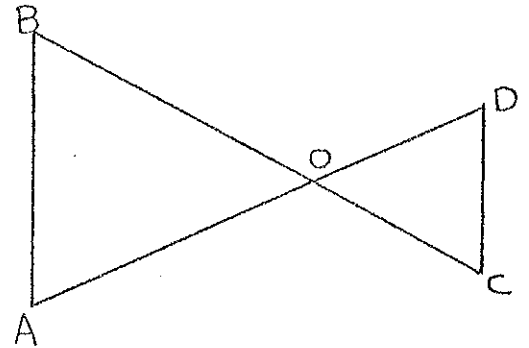
25. A 36 degree angle with vertex at the center of a circle of radius 1 centimeter subtends the arc AB on the circumference. The length of the chord AB is exactly

- a) $\pi/5$ cm.
- b) $2/(1 + \sqrt{5})$ cm.
- c) $3/5$ cm.
- d) $\pi/6$ cm.
- e) $\frac{\sqrt{3}}{2}$ cm.

26. In the figure $\overline{AB} \parallel \overline{CD}$, $m(\angle CDO) = 27^\circ$, and $m(\angle ABO) = 110^\circ$.

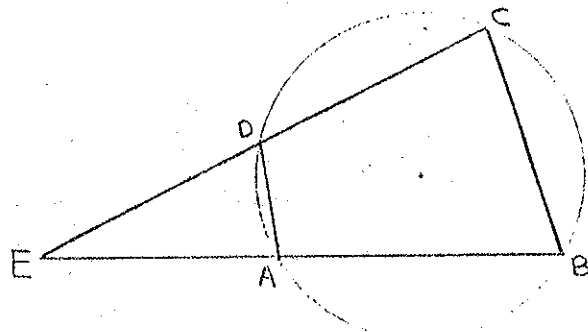
Then $m(\angle BOA)$ is

- a) 43°
- b) 70°
- c) 83°
- d) 110°
- e) 117°



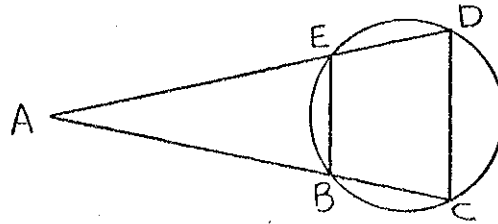
27. The quadrangle ABCD is inscribed in a circle as shown in the figure. If $m(\angle ABC) = 85^\circ$ and $m(\angle AED) = 30^\circ$, then $m(\angle BCD)$ equals

- a) 65 degrees
- b) $66 + \frac{2}{3}$ degrees
- c) 70 degrees
- d) 62.5 degrees
- e) 60 degrees



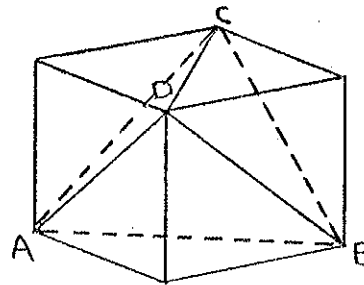
28. In the figure $\overline{EB} \parallel \overline{DC}$, $m(\widehat{BC}) = 120^\circ$, and $m(\widehat{EB}) = 40^\circ$, then $m(\angle EAB)$ is

- a) 15°
- b) 20°
- c) 30°
- d) 40°
- e) 60°



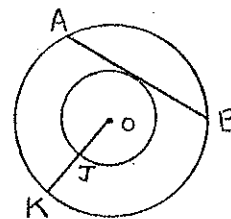
29. A regular tetrahedron is formed by the four vertices A, B, C, D of a cube of edge one centimeter. The volume of the tetrahedron is equal to:

- a) $1/3$ cubic centimeter
- b) $2/5$ cubic centimeter
- c) $2/3$ cubic centimeter
- d) $\frac{1}{2}\sqrt{2}$ cubic centimeter
- e) $\frac{1}{2}\sqrt{3}$ cubic centimeter



30. In the concentric circles with center O, $m(\widehat{JK}) = a$ and J is the midpoint of \overline{OK} . $m(\widehat{AB})$ equals

- a) $2\pi a$
- b) a^2
- c) $3a\sqrt{3}$
- d) $4a$
- e) $2a\sqrt{3}$



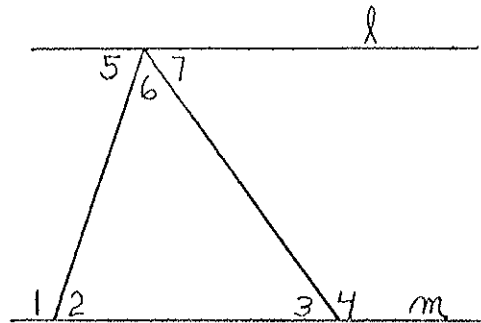
31. The area of a circle is 36π . The area of an equilateral triangle inscribed in the circle is:

- a) $27\sqrt{3}$
- b) $54\sqrt{3}$
- c) $27\sqrt{2}$
- d) 18π
- e) 24π

32. In the figure, $m(\angle 1) = 7x + 10$. $m(\angle 5) = 3x$ and $l \parallel m$.

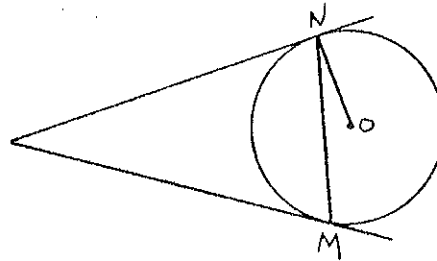
The measure of $\angle 2$ equals

- a) 17
- b) 51
- c) 87
- d) 129
- e) 139



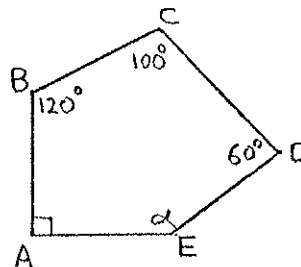
33. In the figure \vec{XN} and \vec{XM} are tangent to Circle O at points N and M respectively. If $m(\angle ONM)$ equals 40° , then $m(\angle NXM)$ equals

- a) 20°
- b) 40°
- c) 60°
- d) 80°
- e) 100°



34. What is the measure of $\angle \alpha$ in the figure ABCDE, where $\angle A$ is a right angle?

- a) 130°
- b) 108°
- c) 150°
- d) 160°
- e) 170°



35. A and B are regular hexagons. Hexagon A has area 48 cm^2 and hexagon B has area 75 cm^2 . The ratio of the sides of hexagon A to hexagon B is

- a) 48 to 75
- b) 16 to 25
- c) 4 to 5
- d) 5 to 4
- e) 25 to 16

36. Two tangents to a circle with center at O meet at an external point P at an angle of 120° . If the distance from P to O is 10 inches, what is the length of each tangent?
- 3 inches
 - 5 inches
 - $5\sqrt{2}$ inches
 - $5\sqrt{3}$ inches
 - 10 inches
37. A ray of light is reflected off a horizontal floor from a point 5 feet away from the point of reflection and 4 feet above the floor. The reflected ray strikes a vertical wall whose base is 5 feet from the point of reflection. How high above the floor does the ray strike the wall?
- 3 feet
 - 4 feet
 - $\frac{28}{5}$ feet
 - 6 feet
 - $\frac{20}{3}$ feet
38. The bases of a trapezoid are 18 and 10 units respectively. The altitude is 4 units. Find the area of the triangle with base 18 units formed when the non-parallel sides are extended.
- 90 square units
 - 81 square units
 - 72 square units
 - 56 square units
 - 45 square units

39. A triangle is inscribed in a circle. The sides of the triangle are of length 10 cm, 24 cm, and 26 cm respectively. The circumference of the circle is
- a) 24π cm
 - b) 26π cm
 - c) 120 cm
 - d) 169π cm
 - e) Cannot be determined from the given information
40. A right circular cylinder of diameter 8 inches is partly filled with water. When a sphere is completely submerged in the water in the cylinder the surface of the water rises 4 inches. The radius of the sphere is:
- a) $2\sqrt{6}$ inches
 - b) $4\sqrt{3}$ inches
 - c) $2\sqrt[3]{6}$ inches
 - d) $4\sqrt[3]{3}$ inches
 - e) $2\sqrt[3]{6\pi}$ inches

