TWENTY-FIFTH ANNUAL MATHEMATICS CONTEST
Sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

GEOMETRY TEST, 1981

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Horace E. Williams

Scoring Formula: 4R - W + 40

This test was prepared from a list of Geometry questions submitted by Austin Peay State University.

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem there are listed 5 possible answers; one and only one is correct. You are to work each problem, determine the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers, so it is advisable not to do much wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

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1. In $\triangle ABC$, AB = 6, BC = 8, and AC = 9. Which of the following is true?

   (a) $\triangle ABC$ is an obtuse triangle.

   (b) $\triangle ABC$ is an acute triangle.

   (c) $\triangle ABC$ is a right triangle.

   (d) $m(\angle C) > m(\angle A)$

   (e) $\triangle ABC$ is an isosceles triangle.

2. Which of the following triplets do <u>not</u> represent the lengths of the sides of a triangle?

   (a) 3, 4, 5

   (b) 2, 2, 1

   (c) 1, 2, 3

   (d) 2, 4, 5

   (e) 2, 3, 4

3. Let A, B, and C be three noncollinear points in 3 dimensional space. Then the set of all points equidistant from A, B, and C is

   (a) a line

   (b) a single point

   (c) a sphere

   (d) a plane

   (e) a triangle

4. In the figure below, $\overline{AB}$ is a diameter of the circle, AB = 13, and AC = 12. Then BC =

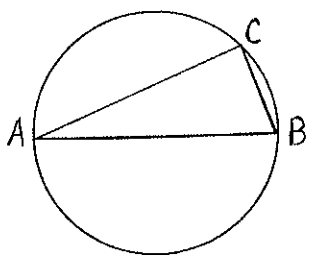
   (a) 3

   (b) 4

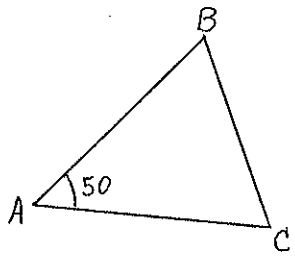
   (c) 5

   (d) $\sqrt{24}$

   (e) $\sqrt{26}$



5. Given that $\triangle ABC$ is a triangle such that $AB = AC$ and $m(\angle A) = 50$. Then $m(\angle B) =$



(a) 50

(b) 55

(c) 60

(d) 65

(e) 70

6. Which of the following is <u>not</u> a property of a rhombus?

(a) Diagonals are congruent.

(b) Opposite sides are congruent.

(c) Diagonals are perpendicular.

(d) All four sides are congruent.

(e) Diagonals bisect each other.

7. A central angle whose chord is equal in length to the radius of a circle measures

(a) 45^0

(b) 60^0

(c) 75^0

(d) 30^0

(e) 1 Radian

8. A man makes 100 moves as follows:

He first steps one step to the right.
He then steps two steps to the left.
He then steps three steps to the right.
He continues in this manner until, finally, he steps 100 steps to the left.

Where is his final position relative to his starting position?

(a) same as starting position

(b) one step to the right

(c) 100 steps to the left

(d) 50 steps to the right

(e) 50 steps to the left

9. In the figure below, $\overline{DB}$ is a diameter of the circle with center A. If $m(\angle CAB) = 45$, then $m\angle CDB =$

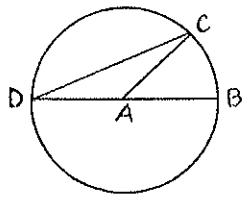
(a) 15

(b) 20

(c) $22\frac{1}{2}$

(d) 25

(e) $27\frac{1}{2}$



10. The equation of the circle in a coordinate plane with (2, 3) and (4, 7) at opposite ends of a diameter is

(a) $(x - 3)^2 + (y - 5)^2 = \sqrt{5}$

(b) $(x + 3)^2 + (y + 5)^2 = \sqrt{5}$

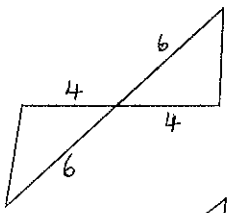
(c) $(x + 3)^2 + (y + 5)^2 = 5$

(d) $(x - 3)^2 + (y - 5)^2 = 20$

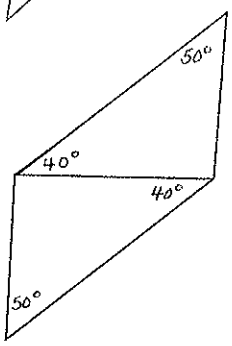
(e) $(x - 3)^2 + (y - 5)^2 = 5$

11. Which one of the following pairs of triangles are not necessarily congruent?

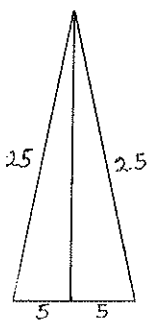
(a)



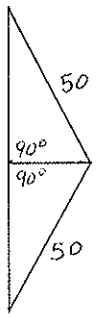
(b)



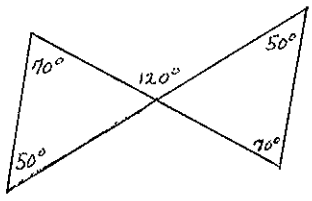
(c)



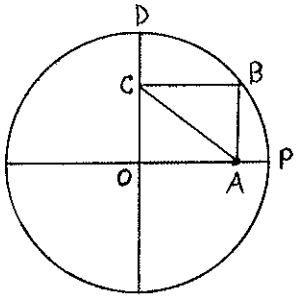
(d)



(e)



12. In the figure below, O is the center of the circle. Quadrilateral OCBA is a rectangle, OA = 5, and AP = 1. Then CA =



(a) 6

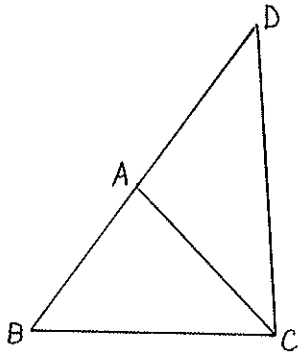
(b) $\sqrt{26}$

(c) 6.5

(d) 5.8

(e) $\sqrt{30}$

13. In the figure below $m(\angle ABC) = 55$, $m(\angle ACB) = 45$, and CA = DA. Then $m(\angle DCB) =$



(a) 100

(b) 95

(c) 90

(d) 85

(e) 80

14. A tractor has wheels with outside diameter of 4 feet. How many revolutions of a wheel would it take for the tractor to travel one mile (in a straight line)?

(a) $\frac{1320}{\pi}$

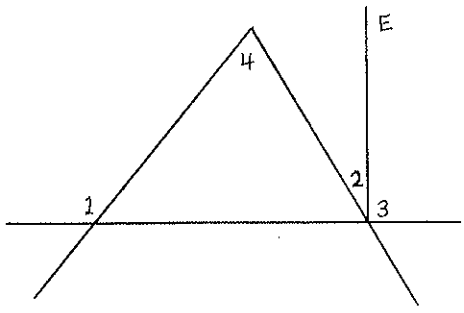
(b) $\frac{660}{\pi}$

(c) 1320

(d) 660π

(e) 1320π

15. If $m(\angle 1) = 130$, $m(\angle 2) = 30$, and $m(\angle 3) = 90$, then $m(\angle 4) =$



(a) 50

(b) 60

(c) 70

(d) 80

(e) 90

16. In the figure below the non-collinear points E, F, G, and H are the midpoints of $\overline{AB}$, $\overline{CB}$, $\overline{AD}$, and $\overline{CD}$, respectively. Which of the following is a true statement?

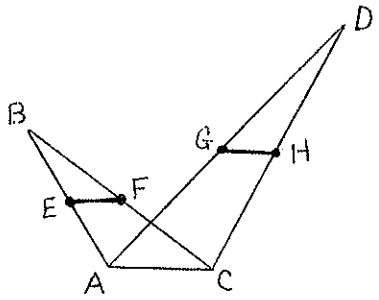
(a) $\overline{EF}$ and $\overline{GH}$ are neither congruent nor parallel.

(b) $\overline{EF}$ and $\overline{GH}$ are both congruent and parallel.

(c) $\overline{EF}$ and $\overline{GH}$ are parallel but not congruent.

(d) $\overline{EF}$ and $\overline{GH}$ are congruent but not parallel.

(e) All of the statements above are false.



17. In the figure below, $\triangle ABC$ is an equilateral triangle with AB = 12. Also $\overrightarrow{CD}$ is the bisector of $\angle C$ and E is the midpoint of $\overline{CD}$. Then BE =

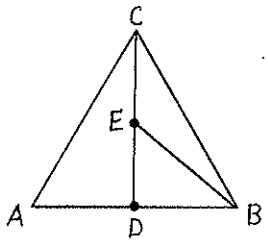
(a) $6\sqrt{3}$

(b) $6\sqrt{2}$

(c) 8

(d) $3\sqrt{7}$

(e) $4\sqrt{3}$



18. Given $\triangle ABC$ with AB = AC, DC = EC = BC, and $m(\angle BAC) = 20$. Then $m(\angle BDE)$ =

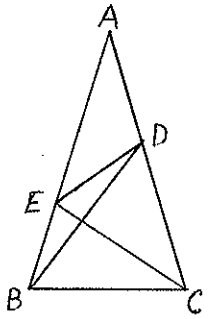
(a) 22.5

(b) 15

(c) 20

(d) 45

(e) 10



19. Let triangle ABC have vertices on a circle. Let $\overline{AD}$ be an altitude of the triangle and let $\overline{AK}$ be a diameter of the circle. If $m(\angle ABC) = 80$ and $m(\angle BCA) = 64$, then $m(\angle DAK)$ =

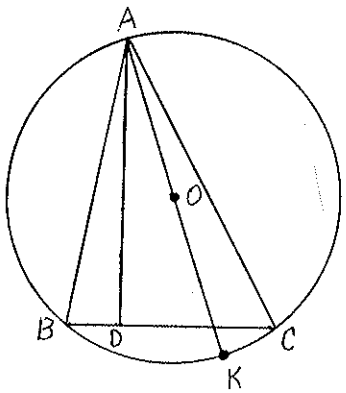
(a) 10

(b) 12

(c) 16

(d) 18

(e) 26



20. If the ratio of the areas of two circles is 1 to 3 and if the radius of the smaller circle is r, then the radius of the larger circle is

(a) $3r$

(b) $2r$

(c) $\frac{3}{2r}$

(d) $\sqrt{3}\, r$

(e) $\frac{r}{\sqrt{3}}$

21. At 11:20 o'clock the hour and minute hands of a clock form an angle of

(a) 150°

(b) 145°

(c) 140°

(d) $137\frac{1}{2}^{\circ}$

(e) 135°

22. A keyhole has the shape pictured in the figure below. $\triangle ADC$ is equilateral, $\overline{AB}$ is a radius of the circle, DC = 14, and BC = 8. The area of the keyhole (shaded area) is:

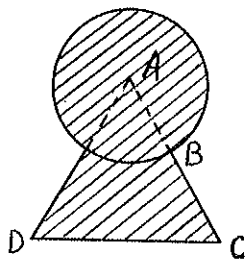
(a) 48π

(b) $36\pi + 45$

(c) $36\pi + 49\sqrt{3}$

(d) $30\pi + 49\sqrt{3}$

(e) $42\pi + 49\sqrt{3}$



23. In $\triangle ABC$, AC = 5, BC = 7, and $\overrightarrow{CD}$ bisects $\angle ACB$. If AD = 4, find AB.

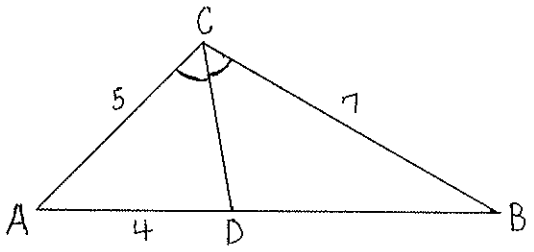
(a) $9\frac{3}{5}$

(b) 10

(c) $10\frac{1}{5}$

(d) $10\frac{1}{2}$

(e) $5\frac{3}{5}$



24. Hexagon ABCDEF is regular with AB = 10. Find AC.

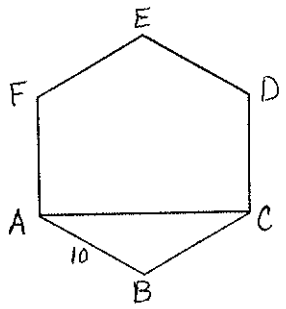
(a) 15

(b) $5\sqrt{2}$

(c) $\frac{5\sqrt{3}}{2}$

(d) $5\sqrt{3}$

(e) $10\sqrt{3}$



25. In the circle below, $\overline{OA}$ is a radius, BC = OA, $m(\angle AOD) = x$, and $m(\angle ACO) = y$. Which of the following is true?

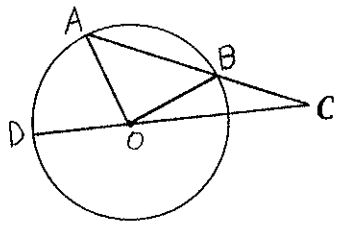
(a) $x = 2y$

(b) $x = \frac{5}{2}y$

(c) $x = y$

(d) $x = 5y$

(e) $x = 3y$



26. Suppose quadrilateral ABCD is a trapezoid, $\overline{EF} \parallel \overline{AD}$, trapezoid AEFD is similar to trapezoid EBCF, BC = a and AD = b. Then EF =

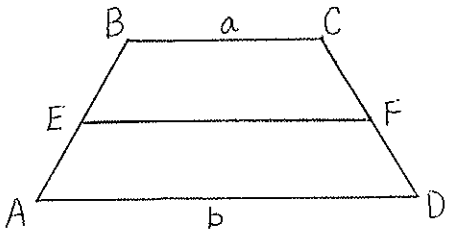
(a) $\frac{2ab}{a + b}$

(b) $\sqrt{ab}$

(c) $\frac{a + b}{2}$

(d) $\sqrt{\frac{a^2 + b^2}{2}}$

(e) $\frac{\sqrt{a} + \sqrt{b}}{2}$



27. Let quadrilateral ABCD be a square with AB = 2. Find the area of the smaller square produced by drawing lines from each of the vertices to the midpoint of an opposite side as indicated in diagram.

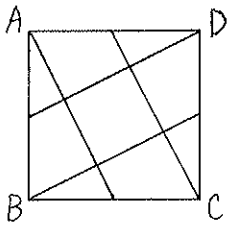
(a) 16/9

(b) 4/5

(c) 1/4

(d) 9/16

(e) 1



28. Suppose that $\triangle ABC$ is a scalene triangle such that AB, BC and AC are all whole numbers and the perimeter of $\triangle ABC$ is less than or equal to 12. How many such noncongruent triangles exist?

(a) 1

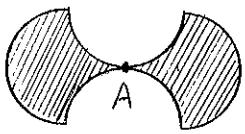
(b) 2

(c) 3

(d) 4

(e) 10

29. Each of the four arcs pictured below is a semicircle with radius 2, and A is the midpoint of each of the two arcs on which it lies. Which of the following is a true statement?



(a) The area of the shaded region is 4π.

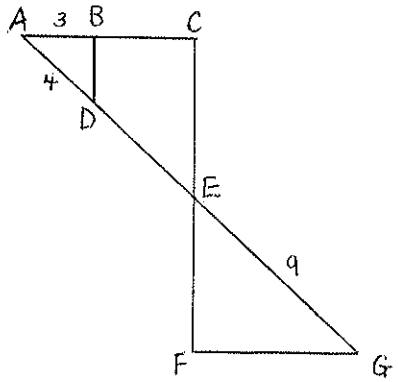
(b) The area of the shaded region is $4(7 - \pi)$.

(c) The area of the shaded region is 14.

(d) The area of the shaded region is 16.

(e) There is not enough information given to determine the area of the shaded region.

30. $\overline{AC} \parallel \overline{FG}$, $\overline{BD} \parallel \overline{CF}$, AB = 3, AD = 4, and EG = 9. Then FG =



(a) 27/4

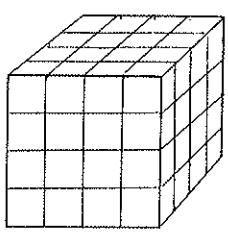
(b) 23/4

(c) 12

(d) 36/5

(e) 6

31. A cubical block of wood is painted black and sliced into n^3 smaller blocks of equal size. The figure shows how this is done for n = 4. Find a general formula for the number of smaller blocks with exactly one black face.



(a) $(n - 1)^3$

(b) $12(n - 2)$

(c) $6n$

(d) $6(n - 2)^2$

(e) $6(n^2 - 4)$

32. The sides of a right triangle have lengths $x - y$, x, and $x + y$ where $x > y > 0$. The ratio of x to y is

(a) 3:2

(b) 2:1

(c) 3:1

(d) 4:1

(e) 4:3

33. A right circular cylinder of radius r and altitude h is inscribed in a cone of radius 4 and altitude 12. The altitude h is equal to

(a) 12 - 2r

(b) 12 - 3r

(c) 12 - 4r

(d) 12 - 5r

(e) 12 - 6r

34. The centers, O and P, of two circles are 17 inches apart. The larger circle has a radius of 5 inches and the smaller one a radius of 3 inches. The length of the common internal tangent $\overline{AB}$ is

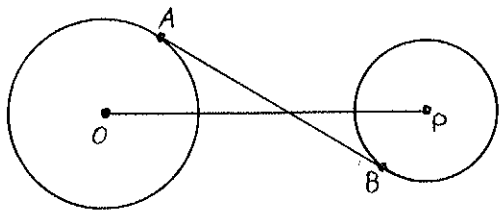
(a) 19 inches

(b) 17 inches

(c) 16 inches

(d) 15 inches

(e) 13 inches



35. If the measure, in degrees, of arc $\overset{\frown}{AC}$ and $\overset{\frown}{BD}$ is 70 and 20, respectively, then $m(\angle APC) =$

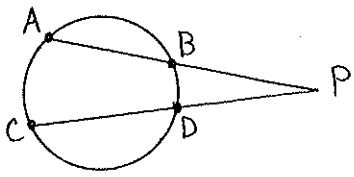
(a) 20

(b) 70

(c) 45

(d) 50

(e) 25



36. In the figure below $\triangle ABC$ is an equilateral triangle, D is the midpoint of $\overline{AC}$, $\overline{FI} \perp \overline{AB}$, $\overline{FG} \perp \overline{BC}$ and $\overline{FH} \perp \overline{AC}$. Which of the following is true?

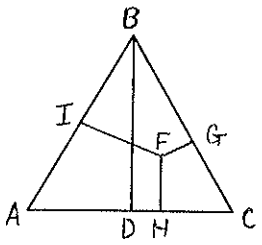
(a) FG + FH + FI = BD

(b) FG + FH + FI < BD

(c) FG + FH + FI > BD

(d) FG + FH + FI = AC

(e) All of the above are false.



37. In the figure below, quadrilateral ABCD is a square and M is the midpoint of $\overline{DC}$. The ratio of the area of $\triangle CEB$ to the area of quadrilateral AEMD is

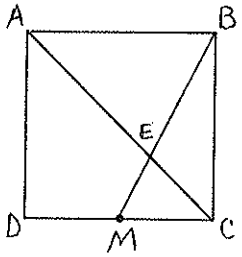
(a) 1:2

(b) 2:5

(c) 3:7

(d) 4:9

(e) 3:8



38. In the right triangle ABC, with right angle at A, D is the midpoint of $\overline{BC}$. If a circle with diameter $\overline{AD}$ intersects $\overline{BC}$ again at E, BC = 50 and DE = 7, then AB =

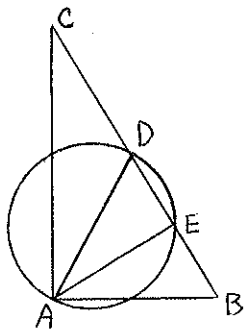
(a) 20

(b) 24

(c) 30

(d) 36

(e) 42



39. Given quadrilateral ABCD with $\overline{AD} \perp \overline{AB}$, $\overline{AB} \perp \overline{BC}$, AD = 6 and BC = 3. Let O be the point of intersection of the diagonals and let M be the point on $\overline{DC}$ such that $\overline{MO} \perp \overline{AB}$. Then OM =

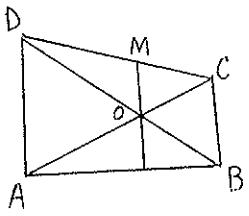
(a) 3/2

(b) 2

(c) 5/2

(d) 3

(e) 7/2



40. Triangle ABC is inscribed in the circle with center O and radius $\overline{OA}$. Also (AB)(AC) = 24 and altitude $\overline{AD}$ has length 3. What is the length of the radius of the circle?

(a) 3

(b) 4

(c) 5

(d) 6

(e) 8