

TWENTY-FOURTH ANNUAL MATHEMATICS CONTEST
Sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

GEOMETRY TEST 1980

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Austin Peay State University, Clarksville,
Tennessee

Scoring Formula: $4R - W + 40$

This test was prepared from a list of Geometry questions submitted by University of Tennessee at Chattanooga.

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem there are listed 5 possible answers; one and only one is correct. You are to work each problem, determine the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. You must use a pencil with soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers, so it is advisable not to do much wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

Contributors to TMTA for Annual Mathematics Contest:

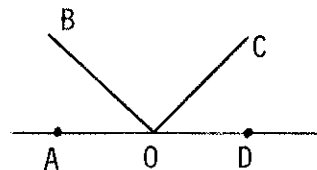
Acme Boot Company, Clarksville, Tennessee
Anderson Ford, Inc., Kingsport, Tennessee
Berkline Company, Morristown, Tennessee
Chattanooga Coca-Cola Bottling Company, Chattanooga, Tennessee
Coca-Cola Bottling Company of Memphis, Memphis, Tennessee
Commercial and Industrial Bank, Memphis, Tennessee
Department of Mathematics, Shelby State Community College, Memphis, TN
Exxon Company, U.S.A., Memphis, Tennessee
First National Bank of Sullivan County, Kingsport, Tennessee
First National Bank, Jefferson City, Tennessee
First People's Bank, Jefferson City, Tennessee
Fulton Siphon Division, Robertshaw Controls, Knoxville, Tennessee
Great Lakes Research Corporation, Elizabethton, Tennessee
Harris Foundation, Johnson City, Tennessee
Holiday Inns, Inc. Memphis, Tennessee
Home Federal Savings and Loan Association, Johnson City, Tennessee
Home Federal Savings and Loan Association, Knoxville, Tennessee
IBM Corporation, Chattanooga, Tennessee
IBM Corporation, Kingsport, Tennessee
IBM Corporation, Nashville, Tennessee
Jefferson County Bank, Dandridge, Tennessee
Johnson City Spring and Bedding Company, Johnson City, Tennessee
Klopman Mills, Incorporated, Johnson City, Tennessee
McDonald's Restaurants, Memphis, Tennessee
Memphis Area Teachers of Mathematics (MAC-O-TOM), Memphis, Tennessee
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Dr. Hal Ramer, President, Volunteer State Community College, Gallatin, TN
Sears, Madison, Tennessee
Shoney's Inc., Nashville, Tennessee
Tennessee Eastman Company, Kingsport, Tennessee
Tennessee Handbag Company, Dandridge, Tennessee
Tri-State Container Corporation, Elizabethton, Tennessee
TRW, Ross Gear Division, Lebanon, Tennessee
Mr. Meeks B. Vaughan, Kingsport, Tennessee

1. Let $\triangle ABC$ be an isosceles triangle with $m(\overline{AB}) = m(\overline{BC}) = 2$ cm. If $m(\sphericalangle BAC) = 30^\circ$, the area of $\triangle ABC$ is
 - (a) 4 cm^2
 - (b) $\sqrt{3} \text{ cm}^2$
 - (c) $2\sqrt{3} \text{ cm}^2$
 - (d) 3 cm^2
 - (e) none of the above

2. If a circle is inscribed in a square 1 cm. on a side, the length of the radius of the circle is
 - (a) 1 cm.
 - (b) $\frac{\sqrt{3}}{2}$ cm.
 - (c) $\frac{1}{2}$ cm.
 - (d) $\frac{\pi}{2}$ cm.
 - (e) none of the above

3. If a man 6 feet tall casts a shadow 12 feet in length when he is 20 feet from the base of a lamppost, the height of the lamppost is
 - (a) 12 feet
 - (b) 14 feet
 - (c) 16 feet
 - (d) 18 feet
 - (e) 20 feet

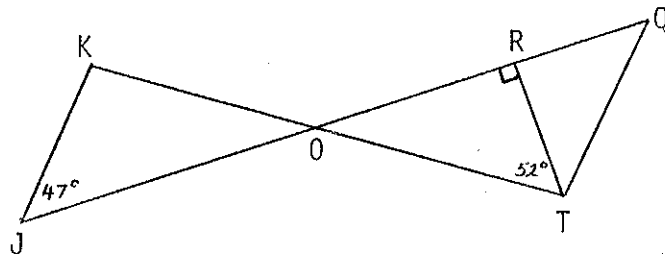
4. In the following diagram, $m(\sphericalangle BOC)$ is a right angle. where O is a point between A and D . Then angles AOB and COD can be said to be
 - (a) complementary
 - (b) supplementary
 - (c) equal
 - (d) adjacent
 - (e) none of the above



5. If statement a implies statement b and statement b implies statement c, and we know that statement c is false, we can conclude that
- (a) statement a is false and b is true.
 - (b) statement b is false and a is true
 - (c) statements a and b are both false
 - (d) statements a and b are both true
 - (e) none of the above

6. In the given diagram, $\overline{JK} \parallel \overline{TQ}$ and $\overline{RT} \perp \overline{JQ}$. Then, $m(\angle JKQ)$ is

- (a) 90°
- (b) 38°
- (c) 105°
- (d) 95°
- (e) 52°



7. If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$, which of the following is true?

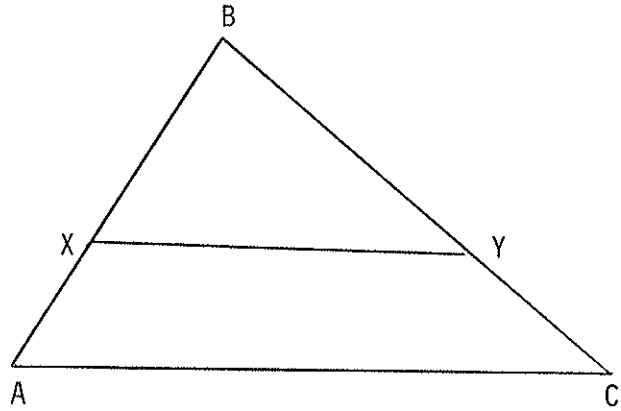
- (a) $ab = cd$
- (b) $ac = ce$
- (c) $\frac{a + c}{b + d} = \frac{e}{f}$
- (d) $a + d = b + c$
- (e) none of the above

8. The volume of a right circular cone of height 10 m. and base diameter 4 m. is

- (a) $4\pi \text{ m}^3$
- (b) $40\pi \text{ m}^3$
- (c) $\frac{40\pi}{3} \text{ m}^3$
- (d) $20\pi \text{ m}^3$
- (e) none of the above

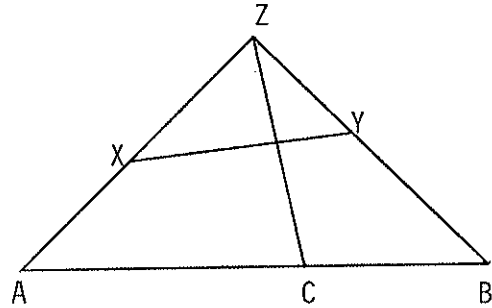
9. If $m(\angle BXY) = m(\angle BAC)$, which of the following is true?

- (a) $\overline{AX} = \overline{YC}$
- (b) $\frac{\overline{BX}}{\overline{BY}} = \frac{\overline{XY}}{\overline{XA}}$
- (c) $m(\angle A) = m(\angle C)$
- (d) $\frac{\overline{AC}}{\overline{XY}} = \frac{\overline{BC}}{\overline{BY}}$
- (e) none of these.



10. In the following figure, $\angle AZB$ is a right angle and $m(\angle ZAB) = m(\angle ZYX)$. Which one of the following statements is true?

- (a) $\triangle AZB \cong \triangle XYZ$
- (b) $\triangle AZC$ is similar to $\triangle XYZ$
- (c) $m(\angle ZAB) = m(\angle ZBA)$
- (d) $\triangle ZAB$ is similar to $\triangle ZYX$
- (e) none of the above



11. The number π is

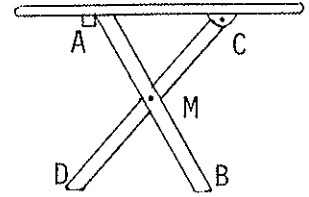
- (a) 3.14
- (b) $\frac{22}{7}$
- (c) 3.1416
- (d) an irrational number
- (e) none of the above

12. The intersection of two distinct non-parallel planes is

- (a) a point
- (b) two points
- (c) the empty set
- (d) a plane
- (e) none of these

13. A collapsible ironing board is constructed so that the supports bisect each other.

The board will always be parallel to the floor because.



- (a) the vertical angles at M are equal.
- (b) if two angles of a triangle are equal, the sides opposite them are equal.
- (c) if the diagonals of a quadrilateral bisect each other, the quadrilateral is a parallelogram.
- (d) If two parallel lines are cut by a transversal, the corresponding angles are equal.
- (e) $m(\angle AMD) + m(\angle AMC) = 180^\circ$
14. In an equilateral triangle inscribed in a circle, the distance measured along a median of the triangle from the intersection of all medians to a vertex angle is 5 cm. The area of the circle is
- (a) $50\pi \text{ cm}^2$
- (b) $10\pi \text{ cm}^2$
- (c) 25 cm^2
- (d) $100\pi \text{ cm}^2$
- (e) $25\pi \text{ cm}^2$
15. The maximum number of straight lines that can be determined by five distinct points is
- (a) 12
- (b) 5
- (c) 15
- (d) 10
- (e) 4

16. Which of the following is an impossible situation for two distinct rays with a common endpoint?
- (a) The union of the rays could form a ray.
 - (b) The union of the rays could form a straight line.
 - (c) The intersection of the rays could form a line segment.
 - (d) The intersection of the rays could be a point.
 - (e) The union of the two rays could form an obtuse angle.
17. An inscribed angle is measured by
- (a) one-half the diameter of the circle.
 - (b) twice its intercepted arc.
 - (c) the lengths of the chords involved.
 - (d) half its intercepted arc.
 - (e) one-third its intercepted arc.
18. In a right circular cylinder, the height measures twice the radius of the base of the cylinder. If the radius is 8 cm, what is the volume of the cylinder?
- (a) $1024\pi \text{ cm}^3$
 - (b) $128/3 \text{ cm}^3$
 - (c) $(2048/3)\pi \text{ cm}^3$
 - (d) $341.\bar{3}\pi \text{ cm}^3$
 - (e) $1024/3 \text{ cm}^3$
19. The locus of a point the sum of whose distance from the two fixed points is a constant describes a figure known as
- (a) a hyperbola.
 - (b) a square.
 - (c) an ellipse.
 - (d) a circle.
 - (e) a straight line.

20. Which of the following may be an impossible compass and straight edge construction?
- (a) A triangle given three line segments to be used as sides of the triangle
 - (b) The bisection of any given angle
 - (c) The bisection of any given line segment
 - (d) The trisection of any given line segment
 - (e) The construction of a line perpendicular to a given line at a given point
21. In an equilateral triangle of sides equal to 10 cm, the area is
- (a) $25\sqrt{3} \text{ cm}^2$
 - (b) 50 cm^2
 - (c) $50\sqrt{3} \text{ cm}^2$
 - (d) $25\sqrt{2} \text{ cm}^2$
 - (e) $50\sqrt{2} \text{ cm}^2$
22. From the top of a cliff 100 m. high, the angle of depression of a boat is 30° . How far is the boat from the foot of the cliff?
- (a) 200 m.
 - (b) 50 m.
 - (c) $50\sqrt{3}$ m.
 - (d) $200\sqrt{3}$ m.
 - (e) $100\sqrt{3}$ m.
23. What is the total surface area of a right circular cylinder which has a base radius of 3 cm. and a height of 10 cm?
- (a) $60\pi \text{ cm}^2$
 - (b) $78\pi \text{ cm}^2$
 - (c) $69\pi \text{ cm}^2$
 - (d) $129\pi \text{ cm}^2$
 - (e) none of the above

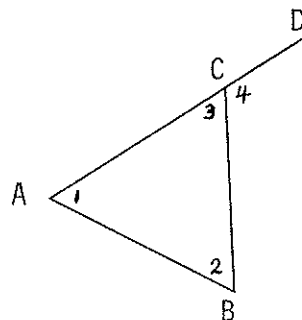
24. In which case below are the two angles described not supplementary?
- (a) Two adjacent angles formed when two straight lines meet.
 - (b) Interior angles on the same side of a transversal intersecting parallel lines.
 - (c) Exterior angles on the same side of a transversal intersecting parallel lines.
 - (d) Alternate exterior angles when a transversal intersects parallel lines.
 - (e) none of the above.

25. Consider the following construction: Given $\triangle ABC$, place compass point at B and swing arcs above \overline{BC} and below \overline{BC} . Using the same radius, place compass point at C and swing arcs above and below \overline{BC} that intersect previous arcs at D and E. Draw \overline{DE} intersecting \overline{BC} at F. Draw \overline{AF} . The lines \overline{DE} and \overline{AF} are, respectively,
- (a) perpendicular bisector and angle bisector.
 - (b) altitude and median.
 - (c) perpendicular bisector and median.
 - (d) median and angle bisector.
 - (e) altitude and angle bisector.

26. Which of the following conditions does not prove congruence of two triangles?
- (a) SAS = SAS
 - (b) SSA = SSA
 - (c) ASA = ASA
 - (d) HS = HS
 - (e) SAA = SAA

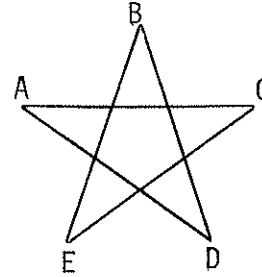
27. In the given diagram, $m(\widehat{AC}) = m(\widehat{BC})$ and D is on the extension of \overline{AC} through C. Which of the following is true?

- (a) $\overline{BC} \perp \overline{AD}$
- (b) $m(\sphericalangle 2) + m(\sphericalangle 3) = m(\sphericalangle 4)$
- (c) $m(\sphericalangle 1) = \frac{1}{2}m(\sphericalangle 4)$
- (d) all of the above
- (e) none of the above



28. The sum of the measures of angles A, B, C, D, and E in the accompanying figure is

- (a) less than 180°
- (b) 180°
- (c) greater than 180° and less than 360°
- (d) 360°
- (e) cannot be determined

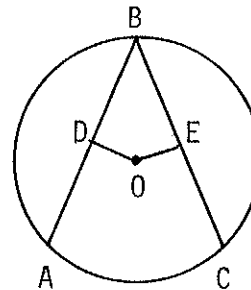


29. Given the statement: "If p, then q," which of the following is not represented by "If not q, then not p?"

- (a) Inverse
- (b) Contrapositive
- (c) Inverse of the converse
- (d) Converse of the inverse
- (e) none of the above

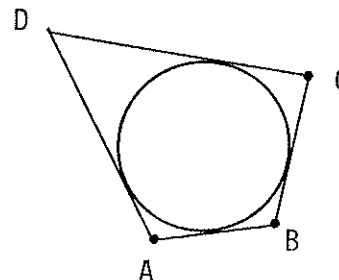
30. In the figure, O is the center of the circle. $\overline{OD} \perp \overline{AB}$, $\overline{OE} \perp \overline{BC}$, $m(\overline{OE}) = m(\overline{OD}) = 5$ cm and $m(\overline{BE}) = 12$ cm. Then $m(\overline{AB})$ is

- (a) 12 cm.
- (b) 13 cm.
- (c) 24 cm.
- (d) 26 cm.
- (e) cannot be determined



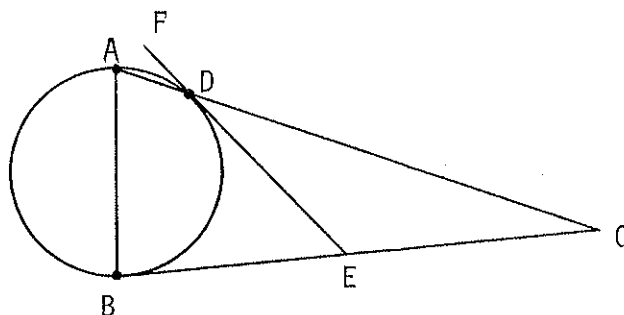
31. In the figure, ABCD is a circumscribed quadrilateral with $m(\overline{DC}) = 6$ cm, $m(\overline{BC}) = 3.5$ cm and $m(\overline{AB}) = 3.4$ cm. Then $m(\overline{AD})$ is

- (a) 6 cm.
- (b) 6.55 cm.
- (c) 12.9 cm.
- (d) 5.9 cm.
- (e) 6.9 cm.



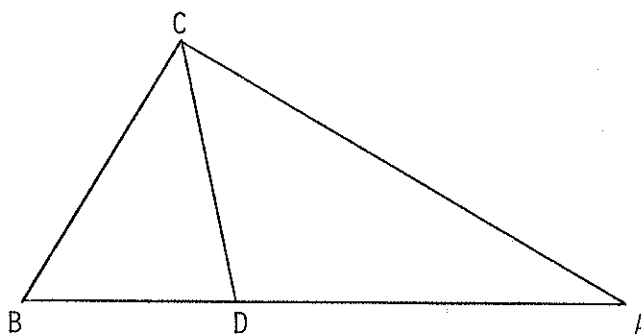
32. In the figure, \overline{AB} is a diameter, \overline{FDE} and \overline{BEC} are tangents and $m\angle C$ is 40° . Then, $m\angle CDE$ is

- (a) 20°
- (b) 25°
- (c) 30°
- (d) 35°
- (e) 40°



33. In the figure, \overline{CD} bisects $\angle C$. If $m(\overline{AC}) = 8$ cm, $m(\overline{BC}) = 5$ cm. and $m(\overline{AB}) = 6.5$ cm., then $m(\overline{AD})$ is

- (a) 4.0625 cm.
- (b) 10.4 cm.
- (c) 2.5 cm.
- (d) 4 cm.
- (e) 6.25 cm.

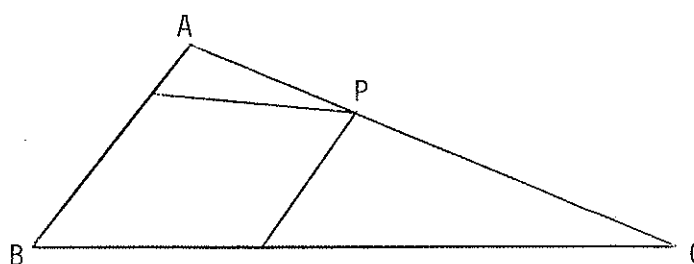


34. Two parallel chords of a circle measure 16 inches and 30 inches, respectively. If the measure of the distance between their midpoints is 23 inches, the radius of the circle measures

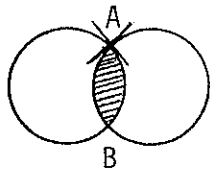
- (a) $\sqrt{644}$ inches
- (b) 15 inches
- (c) $\sqrt{161}$ inches
- (d) 8 inches
- (e) 17 inches

35. In the figure, point P divides the side \overline{AC} of $\triangle ABC$ into segments which have the ratio of 3 to 4. From P, lines are drawn parallel to \overline{AB} and \overline{BC} . The ratio of the area of the parallelogram thus formed to the area of $\triangle ABC$ is

- (a) $\frac{12}{49}$
- (b) $\frac{3}{4}$
- (c) $\frac{9}{16}$
- (d) $\frac{24}{49}$
- (e) $\frac{16}{49}$



36. Three distinct non-collinear points do not determine
- (a) a triangle
 - (b) a plane
 - (c) a circle
 - (d) 3 lines
 - (e) none of the above
37. One vertex of a square is at the origin and the diagonals of the square intersect at the point $(2, 3)$. The area of the square is
- (a) 52 square units
 - (b) $\sqrt{52}$ square units
 - (c) 26 square units
 - (d) $\sqrt{26}$ square units
 - (e) none of the above
38. Two circles, each of which has radius 2, intersect as shown.



If the tangents drawn to the two circles at point A are perpendicular, the area of the shaded region is

- (a) $4 - \pi$
- (b) $2\pi - 4$
- (c) 2π
- (d) $4\pi - 2$
- (e) none of the above

39. A square is inscribed in a circle of radius 3 cm. The area of the 4 regions outside the square and inside the circle is
- (a) $(18 - 9\pi)$ cm²
 - (b) $(9\pi - 18)$ cm²
 - (c) $(36 - 9\pi)$ cm²
 - (d) $(9\pi - 36)$ cm²
 - (e) impossible to determine

40. In the figure, angle B is a right angle, the circle with center at O is inscribed in triangle ABC, $\overline{RC} = 21$, and $\overline{TA} = 4$. Then \overline{BX} is equal to

- (a) $\sqrt{2} - 3$
- (b) $6 - \sqrt{2}$
- (c) $3\sqrt{2} - 3$
- (d) 2
- (e) none of the above

