

TWENTY-SECOND ANNUAL MATHEMATICS CONTEST
Sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

GEOMETRY TEST

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Scoring Formula: $4R - W$

This test was prepared from a list of Geometry questions submitted by Vanderbilt University.

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school geometry. For each problem there are listed 5 possible answers; one and only one is correct. You are to work each problem, determine the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. You must use a pencil with soft lead (No. 2 lead or softer). A sample problem follows:

1. If $2x = 3$, then x equals

- (a) $2/3$. (b) 3. (c) 6.
(d) $3/2$. (e) none of these

1. A B C D E

The correct answer for the sample problem is $3/2$, which is answer (d); so you would answer this problem by making a heavy black mark under space D as indicated above.

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers so it is advisable not to do much wild guessing.

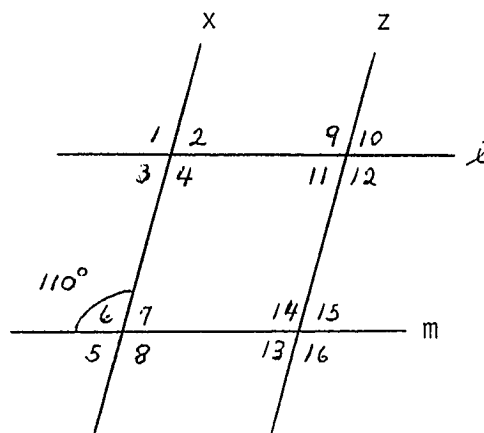
If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet.

The answer sheets will be used for a statistical compilation and will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet to page 1 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

1. From the figure: Line ℓ is parallel to line m . Line x is parallel to line z . The measure of angle 6 is 110° . The measure of angles 15, 16, and 3 are respectively

- (a) $110^\circ, 70^\circ, 110^\circ$
 (b) $40^\circ, 70^\circ, 110^\circ$
 (c) $70^\circ, 110^\circ, 70^\circ$
 (d) $70^\circ, 40^\circ, 70^\circ$
 (e) $80^\circ, 120^\circ, 80^\circ$



2. Given the proposition "If two distinct lines in a plane are parallel and if the lines are both intersected by a third line, the alternate interior angles are congruent.", which one of the following statements is true?
- (a) The proposition does not hold if the third line is perpendicular to the first two lines.
 (b) The proposition holds only if the third line is perpendicular to the first two lines.
 (c) The proposition always holds.
 (d) The proposition holds only if the third line is not perpendicular to the first two lines.
 (e) The proposition holds only if the third line is not in the same plane as the first two lines.
3. Which one of the following is not true?
- (a) The medians of a triangle are concurrent.
 (b) The altitudes of a triangle are concurrent.
 (c) The bisectors of the exterior angles of a triangle are concurrent.
 (d) The bisectors of the interior angles of a triangle are concurrent.
 (e) The perpendicular bisectors of the sides of a triangle are concurrent.

4. The perimeter of a square inscribed in a circle of radius 4 units is
- (a) $16\sqrt{2}$
 - (b) $8\sqrt{2}$
 - (c) 64
 - (d) 16
 - (e) $8\sqrt{3}$

5. Given the statements:

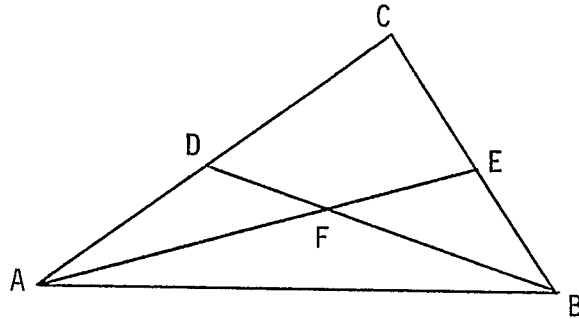
- (p) If an animal is a dog, it is a four-legged animal.
- (q) If an animal is not four legged, it is not a dog.

Which one of the following is true?

- (a) (q) is the inverse of (p).
 - (b) (q) is the converse of (p).
 - (c) (p) and (q) are equivalent statements.
 - (d) Whenever (p) is true it automatically follows that (q) is false.
 - (e) If (p) is true, no conclusion can be made about the truth or falsity of (q).
6. Which one of the following statements is true concerning the construction of angle bisectors using a straight-edge and a compass?
- (a) Only for acute angles may the bisector be constructed.
 - (b) The bisector can not be constructed for a right angle.
 - (c) Sometimes the bisector may be constructed but other times not.
 - (d) The bisector may be constructed for any angle.
 - (e) This is an unsolved problem.

7. Given the triangle ABC, as shown in the figure, such that $CD = DA$ and $CE = EB$, which one of the following is true?

- (a) $AF = FE$
 (b) $AF = 2FE$
 (c) $AF = 3FE$
 (d) $AF = \frac{1}{2}FE$
 (e) $AF = \sqrt{2}FE$



8. Given a right triangle the lengths of whose sides are 13, 12, 5, the circle inscribed in this triangle has an area of

- (a) 36π
 (b) $\frac{169\pi}{4}$
 (c) $\frac{25\pi}{4}$
 (d) 25π
 (e) 4π

9. The circumference of a circle inscribed in a square, each of whose sides is 4 units in length, is

- (a) 2π units
 (b) 16π units
 (c) 8π units
 (d) $8\sqrt{2}$ units
 (e) 4π units

10. Which one of the following statements is true for two distinct lines in space?

- (a) They intersect in one and only one point.
 (b) They either intersect or are parallel.
 (c) They neither intersect nor are parallel.
 (d) They are either parallel or non-intersecting.
 (e) None of the above.

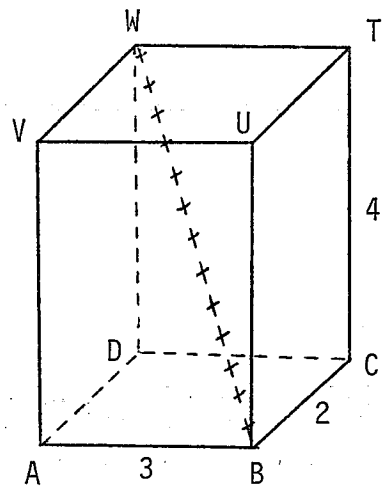
11. PQ and P'Q' are chords in circle O intersecting at A. Let $x = AQ$, $y = AQ'$, $w = AP$, and $z = AP'$. Which one of the following statements is true?
- (a) $wx = yz$
 - (b) $x^2 = y^2$
 - (c) $zw = yx$
 - (d) $wy = zx$
 - (e) $z^2 + w^2 = y^2$
12. Two right circular cones have a common vertex and concentric circles in the same plane for their bases. The area of the larger circle is 56π . The larger circle has twice the area of the smaller circle. If the length of a slant height of the larger cone is $\sqrt{88}$, what is the volume of the smaller cone?
- (a) $\frac{\sqrt{28}\pi}{3}$
 - (b) 144π
 - (c) 112π
 - (d) $84\pi^2$
 - (e) $\frac{112\sqrt{2}}{3}\pi$
13. A leg of a right triangle is 6 inches in length and one of the acute angles has measure 30° . The set of all possible lengths for the hypotenuse of this triangle is
- (a) $\{12\}$
 - (b) $\{4\sqrt{3}\}$
 - (c) $\{6\sqrt{2}\}$
 - (d) $\{12, 4\sqrt{3}\}$
 - (e) $\{12, 6\sqrt{2}\}$

14. Given a right triangle the lengths of whose sides are 13, 12, 5, the interior of the circumscribed circle has an area of
- (a) 36π
 - (b) $\frac{169\pi}{4}$
 - (c) $\frac{25\pi}{4}$
 - (d) 25π
 - (e) 4π
15. If triangle ABC is equilateral and M is the midpoint of \overline{AB} , which of the following assertions is not true?
- (a) All three sides are equal in length.
 - (b) All three vertex angles are equal in measure.
 - (c) $\overline{CM} \perp \overline{AB}$
 - (d) $(BM)^2 + (MA)^2 = (BC)^2$
 - (e) $4(CM)^2 = 3(AB)^2$
16. Each side of an equilateral triangle has length k inches. The area of the triangle in square inches is
- (a) $k^2/2$
 - (b) $\sqrt{3} k^2/2$
 - (c) $3k^2/4$
 - (d) $\sqrt{3} k^2/4$
 - (e) $k^2/\sqrt{2}$
17. Given a circle C of radius 4 inches. The locus of points from which tangents of length 3 inches may be drawn to the circle is
- (a) a straight line 5 inches from the center of C.
 - (b) non-existent.
 - (c) a circle of radius 5 inches concentric to C.
 - (d) a circle of radius 5 inches which passes through the center of C.
 - (e) a point 5 inches from the center of C.

18. A point moves so that the difference of the squares of its distances from two given fixed points is always the same positive number. The locus of this moving point is
- (a) a straight line.
 - (b) a circle.
 - (c) a square.
 - (d) two straight lines.
 - (e) a hyperbola.

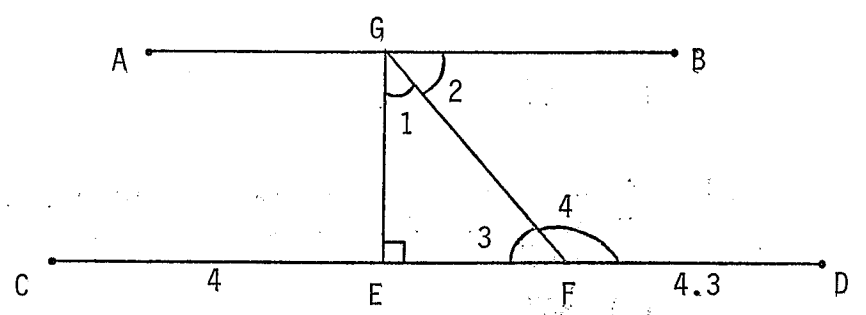
19. In the given rectangular solid, $AB = 3$, $BC = 2$, and $TC = 4$. Find the length of BW .

- (a) 5
- (b) 29
- (c) $6\sqrt{2}$
- (d) $\sqrt{29}$
- (e) 4.5



20. In the figure below, \overline{AB} is parallel to \overline{CD} , angle GEF is a right angle, the measure of angle 4 is 124° , $CE = 4$, $CD = 11.8$, $FD = 4.3$. The distance EF , and the measures of angles 1, 2, 3 are respectively,

- (a) 3.5, 30° , 56° , 65°
- (b) 3.5, 34° , 56° , 56°
- (c) 3.8, 60° , 54° , 56°
- (d) 5, 30° , 56° , 30°
- (e) 4, 34° , 40° , 60°



21. An equilateral triangle is inscribed in a circle whose area is 24 square feet. A second circle is inscribed in the equilateral triangle. The area of the second circle is

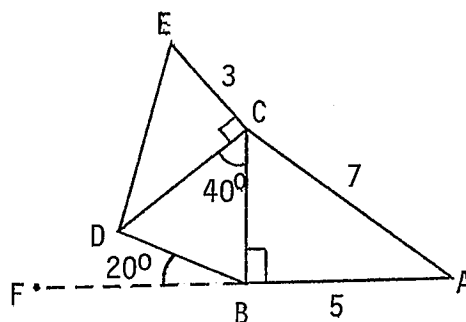
- (a) $\frac{\sqrt{3}}{2}\pi$ square feet.
 (b) $6\sqrt{3}$ square feet.
 (c) 12 square feet.
 (d) 12π square feet.
 (e) 6 square feet.

22. The volume in cubic inches of a prism whose height is 20 inches and whose base is an equilateral triangle with 4-inch sides is

- (a) $80\sqrt{3}$.
 (b) $40\sqrt{3}$.
 (c) 160.
 (d) 80.
 (e) 320.

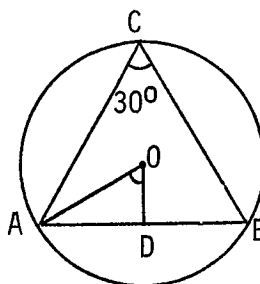
23. Given the figure below with $AB = 5$, $AC = 7$, $CE = 3$, measure of angle DBF is 20° , measure of angle DCB is 40° , angle CBA and angle ECD are right angles. Also \overline{AF} is a line segment which contains B . The length of side \overline{DE} is

- (a) $\sqrt{15}$
 (b) $11\sqrt{3}$
 (c) $2\sqrt{6}$
 (d) $3\sqrt{2}$
 (e) $\sqrt{33}$



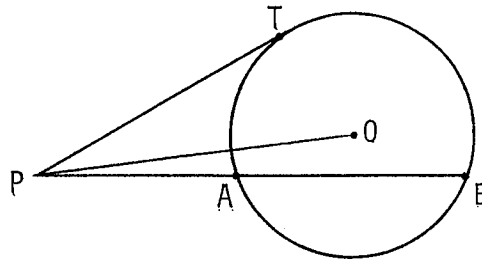
24. Given a circle with center at O with triangle ABC inscribed, \overline{OD} is perpendicular to \overline{AB} . The measure of an angle ACB is 30° . The measure of angle AOD is

- (a) 60°
 (b) 75°
 (c) 30°
 (d) 10°
 (e) None of the above



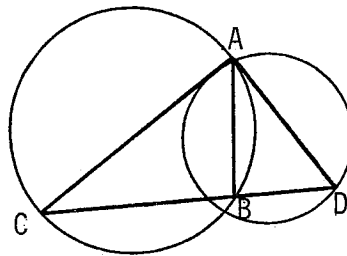
25. In circle O , \overline{PT} is a tangent. $PA = 4$, $AB = 5$, $OP = \sqrt{45}$. The radius of O and the length of segment \overline{PT} are respectively,

- (a) 4, $\sqrt{20}$
 (b) 3, 6
 (c) $3\frac{1}{2}$, 6
 (d) 5, $\sqrt{20}$
 (e) 4, $9/2$



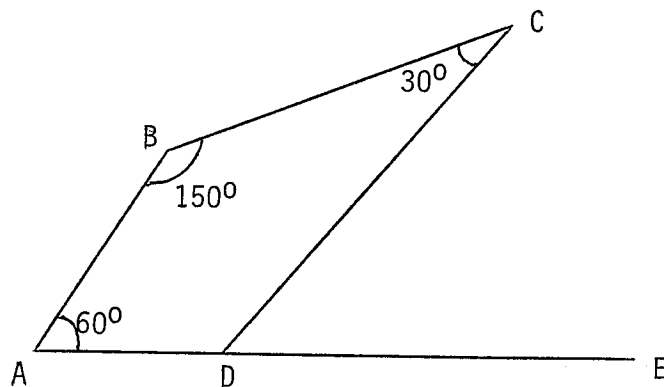
26. \overline{AC} and \overline{AD} are tangents to the smaller and larger circles respectively. Angle DAC is a right angle. $CB = 9$ in., $BD = 4$ in. Then AB is

- (a) 5 inches
 (b) $4\sqrt{3}$ inches
 (c) 6 inches
 (d) $4\sqrt{2}$ inches
 (e) Cannot be determined.



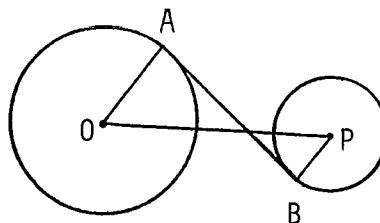
27. Given the quadrilateral $ABCD$ with the measures of the angles as given in the figure, the measure of angle EDC is

- (a) 50°
 (b) 60°
 (c) 120°
 (d) 45°
 (e) 30°



28. In the figure below \overline{AB} is tangent to both circle O and circle P at points A and B respectively. $OA = 9$, $PB = 4$, $AB = 13$. Find OP .

- (a) $9\sqrt{2}$
 (b) $13\sqrt{2}$
 (c) 13
 (d) $5\sqrt{10}$
 (e) $4\sqrt{13}$



29. The number of distinct planes determined by 4 non-coplanar points in space is

- (a) 4
 (b) 5
 (c) 1 and only 1
 (d) 3
 (e) cannot be determined

30. The area of a triangle whose sides have lengths 3, 6, 7, is

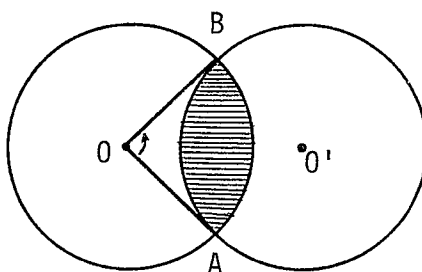
- (a) $4\sqrt{5}$ square units.
 (b) 9 square units.
 (c) $9/2\sqrt{2}$ square units.
 (d) 21 square units.
 (e) $3/2\sqrt{5}$ square units.

31. A cone, a hemisphere, and a cylinder have the same base radius R . The ratios of the volumes are 1:2:3 respectively. The altitudes of the cone and cylinder are respectively,

- (a) R , $2R$
 (b) \sqrt{R} , $3\sqrt{R}$
 (c) $2R^2$, R^2
 (d) R , R
 (e) $2R$, $2R$

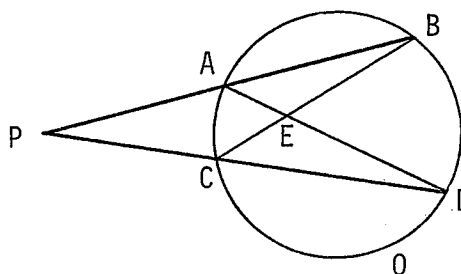
32. In the figure below, the measure of angle AOB is 90° , O is the center of the circle on the left, and O' is the center of the circle on the right. Both circles are of radius 2 inches. The shaded area is in square inches equal to

- (a) $4 - \pi$
 (b) $2\sqrt{2}\pi$
 (c) $4\pi - 8$
 (d) $8 - 2\pi$
 (e) $2\pi - 4$



33. In circle O , \overline{PB} and \overline{PD} are secants. \overline{AD} and \overline{BC} are chords intersecting at E . The measure of angle AEB is 120° . The measure of arc \widehat{BD} is twice the measure of arc \widehat{AC} . The measure of angle APC is

- (a) 60°
 (b) 80°
 (c) 40°
 (d) 20°
 (e) none of the above



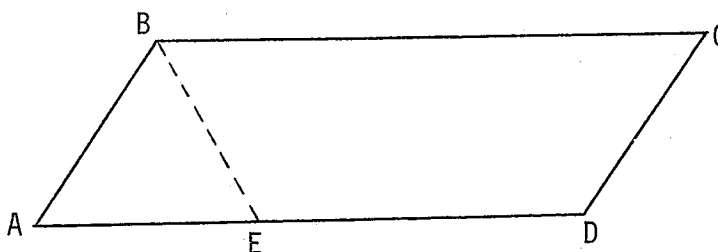
34. The sum of the lengths of the legs of a right triangle is 24 inches. A perpendicular is constructed from the right angle to the hypotenuse. The length of the hypotenuse is 4 times the length of the perpendicular. The area of the triangle is

- (a) 72 square inches
 (b) 48 square inches
 (c) 70 square inches
 (d) 288 square inches
 (e) cannot be found

35. Given: The area of parallelogram $ABCD$ is 44 sq. in.; $CD = 4\sqrt{2}$ in.;
 E is midpoint of \overline{AD} . The measure of angle $BCD = 45^\circ$.

The area of triangle ABE is

- (a) 11 sq. in.
 (b) 5.5 sq. in.
 (c) 14 sq. in.
 (d) 7.5 sq. in.
 (e) $11\sqrt{2}$ sq. in.



36. One diagonal of a rhombus has length 3 inches. A side of the rhombus has length 2 inches. The length of the second diagonal is
- (a) 5 inches (c) $3\sqrt{2}$ inches (e) $\sqrt{7}$ inches
 (b) 3 inches (d) $\sqrt{13}$ inches
37. The number of distinct planes determined by five parallel lines in space, no three of which are coplanar, is
- (a) 5 (c) 6 (e) cannot be determined
 (b) 10 (d) 1 and only 1
38. \overline{AB} and \overline{CD} are chords of a circle intersecting in a right angle at a point P. $AP = 3$, $PB = 4$, and $CP = 2$. The radius of the circle is
- (a) $\frac{\sqrt{65}}{2}$ (c) 5 (e) 4
 (b) $\sqrt{12}$ (d) $\sqrt{20}$

39. Given the following statements:

- I. The statement "p is true and q is false" is false.
 II. It is false that q and r are both true.
 III. If p is false then q and r are both true.

Which one of the following is a logical consequence of these statements?

- (a) p is true and q and r are false.
 (b) p is false.
 (c) p and q are true and r is false.
 (d) q is false.
 (e) p and r are true and q is false.
40. In the figure below, you are given the circle with center O and \overline{AB} tangent to the circle at D. Also \overline{CD} and \overline{FE} are diameters of the circle. If $AD = 1/3$ and $DB = 3$, then the circle is of radius

- (a) $2/3$
 (b) $\frac{\sqrt{2}}{3}$
 (c) $\frac{\sqrt{3}}{2}$
 (d) $1/2$
 (e) $\frac{\sqrt{2}}{2}$

