TWENTY-FIRST ANNUAL MATHEMATICS CONTEST
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THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

GEOMETRY TEST

1977

Scoring Formula: 4R - W

EDITED BY:

Billy Edwards
and
James G. Ware
The University of Tennessee
at Chattanooga
Chattanooga, Tennessee

This test was prepared from a list of Geometry questions submitted by The University of Tennessee at Martin.

DIRECTIONS:

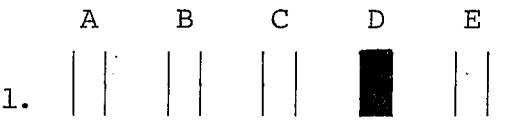
Do not open this booklet until you are told to do so.

This is a test of your competence in high school geometry. For each problem there are listed 5 possible answers; one and only one is correct. You are to work each problem, determine the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. You must use a pencil with soft lead (No. 2 lead or softer). A sample problem follows:

1. If $2x = 3$, then x equals

(a) 2/3. (b) 3. (c) 6.

(d) 3/2. (e) none of these



The correct answer for the sample problem is 3/2, which is answer (d); so you would answer this problem by making a heavy black mark under space D as indicated above.

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers so it is advisable not to do much wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet.

The answer sheets will be used for a statistical compilation and will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet to page 1 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

1. In a right triangle, one leg is 12 cm and the hypotenuse is 13 cm. The other leg will be

   (a) 8 cm

   (b) 6 mm

   (c) $\sqrt{26}$ cm

   (d) 50 mm

   (e) $20\sqrt{6}$ mm

2. In this figure $\overline{AB} \cong \overline{BC}$, $\overline{CE} \cong \overline{CD}$, $\overline{EDF}$ is a straight line and $m\angle E$ is 10°. Then $m\angle B$ is

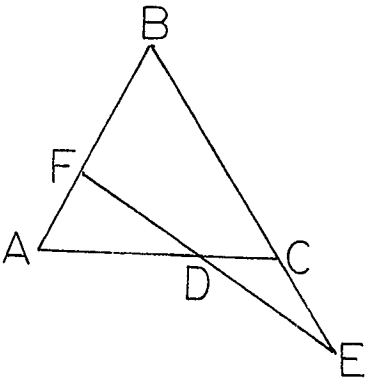
   (a) 35°

   (b) 40°

   (c) 50°

   (d) 70°

   (e) 140°



3. Suppose $m\overline{AO} = 3$, $m\overline{OD} = 1$, and $\overline{BD} \perp \overline{AC}$ in circle O. Then $m\overline{BD}$ is

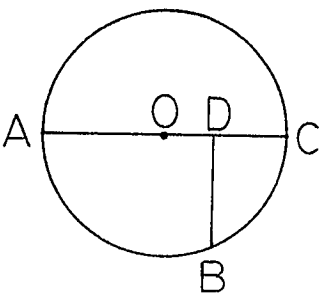
   (a) 3

   (b) $2\sqrt{3}$

   (c) $3\sqrt{2}$

   (d) 4

   (e) $2\sqrt{2}$



4. In the parallelogram ABCD, $m\overline{BO} = 2(x + 5)$ and $m\overline{OD} = x^2 + 5x$. Then $m\overline{BD}$ is

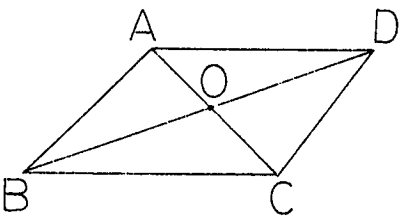
   (a) 4

   (b) 2

   (c) 5

   (d) 10

   (e) 28



5. In the figure if $m\angle CAD = 30°$, $m\angle CBA = 60°$, $\overline{CD} \perp \overline{AB}$ $m\overline{CD} = 3$, then $m\overline{AD}$ is

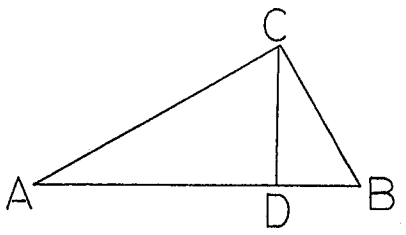
(a) $3\sqrt{5}$

(b) $3\sqrt{3}$

(c) $3\sqrt{2}$

(d) $\frac{3}{\sqrt{3}}$

(e) none of these



6. In the rectangular solid illustrated, $m\overline{AH} = 5$, $m\overline{GH} = 4$ and $m\overline{DH} = 4$. The area of the rectangle ABCD, in square units, is

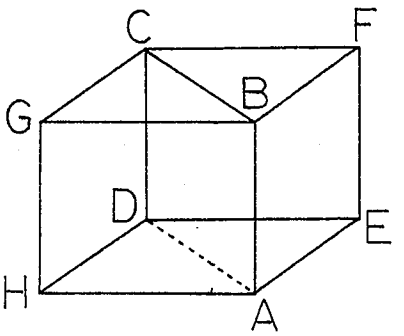
(a) $2\sqrt{41}$

(b) 41

(c) $4\sqrt{41}$

(d) 20

(e) 16



7. Find x such that $\overline{DE} \parallel \overline{AC}$ in the figure.

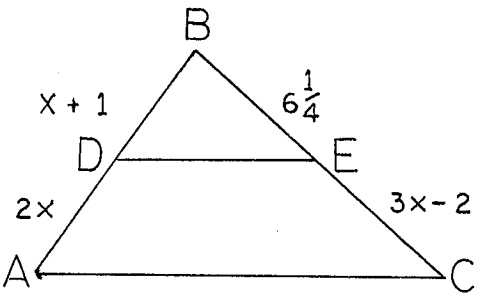
(a) 1

(b) 4

(c) 3

(d) 8

(e) 6



8. A circular cylindrical pipe 12 feet long has an outside diameter of 4 inches and an inside diameter of 3 inches. The volume of metal, in cubic inches, in the pipe is

(a) 84π

(b) $\frac{7\pi}{12}$

(c) 1008π

(d) 288π

(e) none of these

9. Any three distinct points not in a straight line determine

(a) a plane only

(b) a triangle only

(c) a circle only

(d) all of the above

(e) a triangle and a plane only

10. Given

p	q
T	T
T	F
F	T
F	F

the truth values for (p and not q) implies (not q or not p) are

(a) FTTT

(b) TTTT

(c) FTFF

(d) FFFF

(e) FFFT

11. In the diagram, ΔABC is inscribed in a circle and $\overleftrightarrow{CD}$ is tangent to the circle. If $m\angle BCD = 40°$, how many degrees are there in angle A?

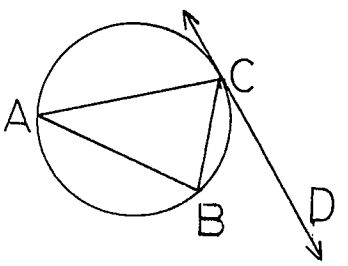
(a) 20

(b) 30

(c) 40

(d) 50

(e) 60



12. If line n is parallel to line w, $m\angle 1 = 25°$ and $m\angle 2 = 140°$, then $m\angle p$ is

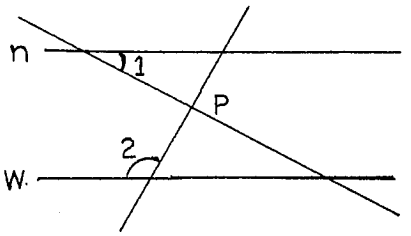
(a) 40°

(b) 25°

(c) 15°

(d) 60°

(e) 65°



13. In ΔABC with right angle at C and $\overline{CE} \perp \overline{AB}$, $\overline{ED} \perp \overline{AC}$ and $m\angle B = 60°$, if $m\overline{AD} = \sqrt{3}$, then $m\overline{EB}$ is

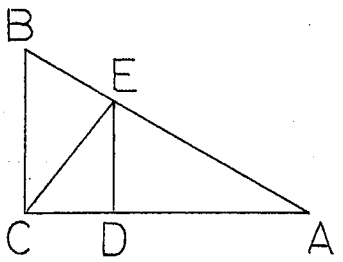
(a) $\frac{\sqrt{6}}{3}$

(b) $\frac{2\sqrt{3}}{3}$

(c) $2\sqrt{3}$

(d) $\frac{2}{3}$

(e) $\sqrt{3}$



14. The height of a parallelogram is 3m less than twice the base. Find the base if the area is $27m^2$.

(a) 9m

(b) 7m

(c) 3m

(d) 4.5m

(e) 5.5m

15. A loop of string is wrapped tightly around the equator of the earth (diameter 8000 miles). Suppose that two yards are added to the length of string thus forming a larger circle going around the equator. Approximately what height above the surface of the earth will the new circle be?

(a) about 1 yard

(b) about one foot

(c) about 1 inch

(d) about 6 inches

(e) no change

16. Triangle ABC has a right angle at C. $\overline{PC}$ is erected at C perpendicular to the plane of $\triangle ABC$. If $\angle PBC \cong \angle ABC$, then $m\angle PAC$ equals

(a) $m\angle PBC$

(b) $m\angle BAC$

(c) $60°$

(d) $45°$

(e) $30°$

17. If $m\overset{\frown}{AB} = 70°$ and $m\angle C = 15°$, then $m\overset{\frown}{DE}$ is

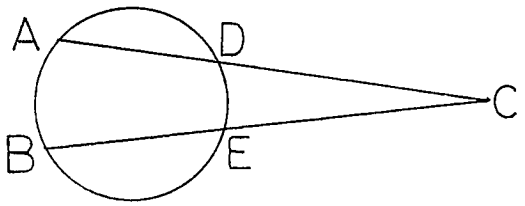
(a) $40°$

(b) $85°$

(c) $55°$

(d) $27\frac{1}{2}°$

(e) $42\frac{1}{2}°$



18. Given that $\overline{PA}$ is tangent to the circle with center at O, which of the following is true?

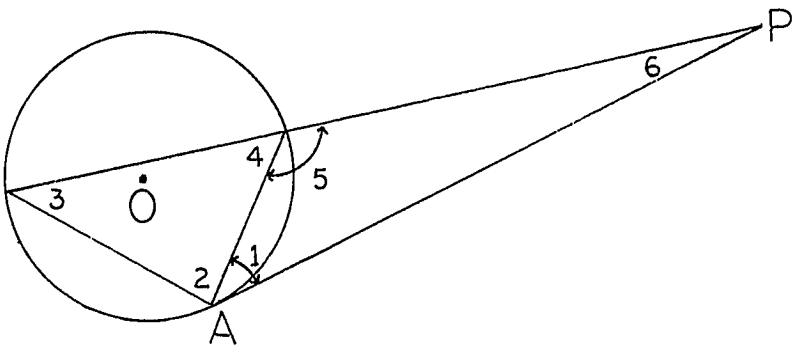
(a) $\angle 5 \cong \angle 2$

(b) $\angle 5 \cong \angle 3$

(c) $\angle 6 \cong \angle 2$

(d) $\angle 3 \cong \angle 1$

(e) $\angle 4 \cong \angle 2$



19. A triangle has sides which are 6 cm, 8 cm and 10 cm long. A rectangle equal in area to the area of the triangle has a width of 3 cm. The perimeter of the rectangle is

(a) 11 cm

(b) 16 cm

(c) 22 cm

(d) 24 cm

(e) 48 cm

20. Given: circle and external secants as shown; $\overline{PA}$ has measure 8. If $\overline{PD}$ measures x, then x has measure

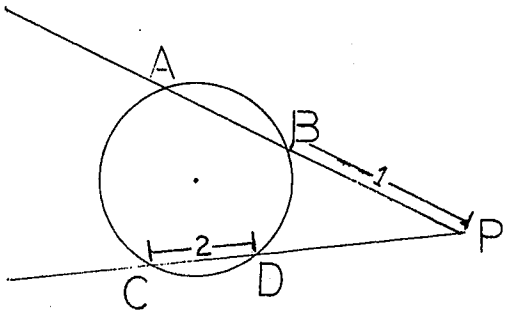
(a) 1

(b) 3.5

(c) 6

(d) 3

(e) 2



21. The sum of the measures of the interior angles of a polygon is 900°. The polygon is

(a) a heptagon

(b) an octagon

(c) a dodecagon

(d) a decagon

(e) a hexagon

22. Given that ABCD is a parallelogram with measures as recorded, the measure of x is

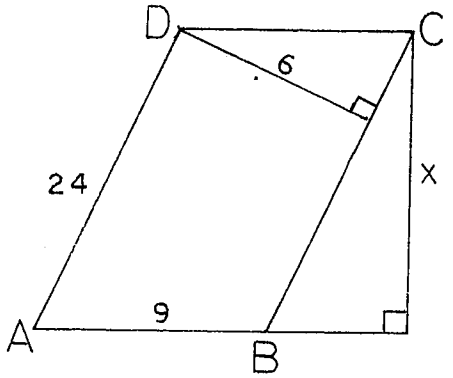
(a) 18

(b) 16

(c) 15

(d) $3\sqrt{55}$

(e) $6\sqrt{15}$



23. The area of the shaded portion, in square units, is

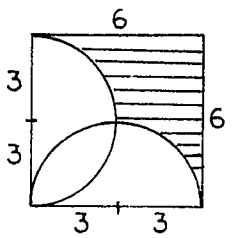
(a) $27 - \frac{9}{2}\pi$

(b) 18

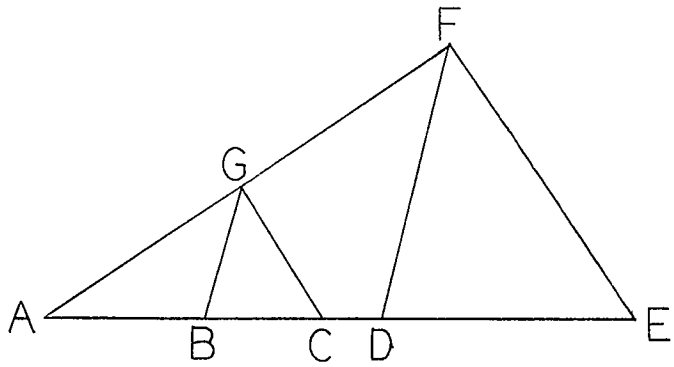
(c) $36 - 9\pi$

(d) $36 - \frac{9}{2}\pi$

(e) 13



24. In the figure, $\overline{BG} \parallel \overline{DF}$ and $\overline{CG} \parallel \overline{EF}$. Which of the following statements is false?



(a) $\overline{AB} : \overline{BD} = \overline{AC} : \overline{CG}$

(b) $\overline{AB} : \overline{AD} = \overline{AG} : \overline{AF}$

(c) $\overline{AG} : \overline{AF} = \overline{BG} : \overline{EF}$

(d) $\overline{BG} : \overline{DF} = \overline{CG} : \overline{EF}$

(e) $\overline{AB} : \overline{AG} = \overline{AD} : \overline{AF}$

25. The area, in square units, of a circle in which a chord 12" long cuts off an arc of 270° is

(a) 36π

(b) 72

(c) 72π

(d) 108π

(e) 144π

26. Consider the three statements:

A. The sum of the angles of a triangle is equal to one straight angle.
B. If two angles of one triangle are equal to two angles of another triangle, the third angles are equal.
C. If two parallel lines are cut by a transversal, the alternate interior angles are equal.

A <u>common</u> sequence for the proofs of these statements is

(a) A, B, C

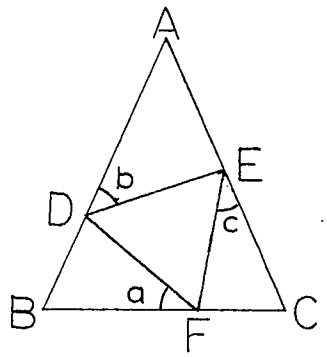
(b) A, C, B

(c) B, A, C

(d) C, A, B

(e) B, C, A

27. In the figure, $m\overline{AB} = m\overline{AC}$ in ΔABC and ΔDEF is equilateral. Let $m\angle BFD = a$, $m\angle ADE = b$ and $m\angle FEC = c$. Then



(a) $b = \frac{a + c}{2}$

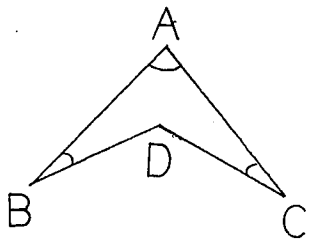
(b) $b = \frac{a - c}{2}$

(c) $a = \frac{b - c}{2}$

(d) $a = \frac{b + c}{2}$

(e) none of these

28. Find the area of the quadrilateral ABCD in the figure if it is known that $m\angle A = 60°$, $m\angle B = m\angle C = 30°$ and $m\overline{AB} = m\overline{AC} = 10$ inches.



(a) 50 square inches

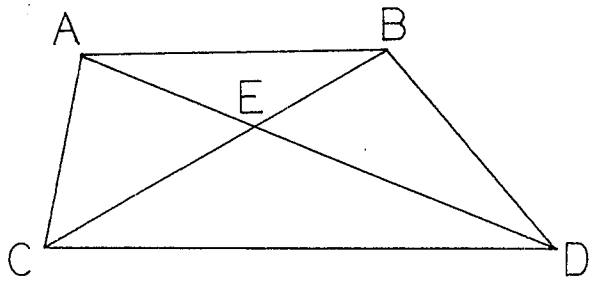
(b) $\frac{50\sqrt{2}}{3}$ square inches

(c) $\frac{50\sqrt{3}}{3}$ square inches

(d) $25\sqrt{3}$ square inches

(e) none of these

29. In the figure, $\overline{AB}$ is parallel to $\overline{CD}$. Which of the following statements is true?



(a) $\Delta EAC \cong \Delta EBD$

(b) $\Delta ECD \sim \Delta EAB$

(c) $\angle ACE = \angle EBD$

(d) $\angle EAC = \angle EBD$

(e) none of these

30. The equation of the locus of points equidistant from (6,3) and (2a,1) is

(a) $y = (a - 3)x - 2a^2 + 6a + 1$

(b) $y = (a - 3)x + 11 - a^2$

(c) $y = (3 - a)x + a^2 - 7$

(d) $y = (3 - a)x - a^2 - 7 + 6a$

(e) $y = (a - 3)x + a^2 - 6a + 11$

31. The polygon formed by $y = 2x + 2$, $y = -2x + 2$ and $y = -2$ is

(a) an equilateral triangle

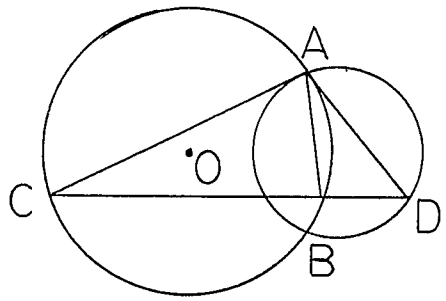
(b) an isosceles triangle

(c) a right triangle

(d) a scalene triangle

(e) none of these

32. $\overline{AC}$ and $\overline{AD}$ are tangents to the larger and smaller circles respectively and $\overline{CB}$ is a secant with $m\overline{CB} = 9$. If $m\overline{BD} = 4$ then $m\overline{AB}$ is



(a) 5

(b) $4\sqrt{3}$

(c) 6

(d) $4\sqrt{2}$

(e) cannot be determined

33. A square is inscribed in a circle of radius a. The perimeter of the square is

(a) $4\sqrt{a}$

(b) $4\pi a$

(c) $2\sqrt{2}\,a$

(d) $4\sqrt{2}\,a$

(e) $\frac{3\pi a^2}{4}$

34. Suppose two circles of radii 4 inches and 9 inches have a common external tangent $\overline{AB}$ such that $m\overline{AB}$ is 15 inches. How apart are their centers?

(a) 15 inches

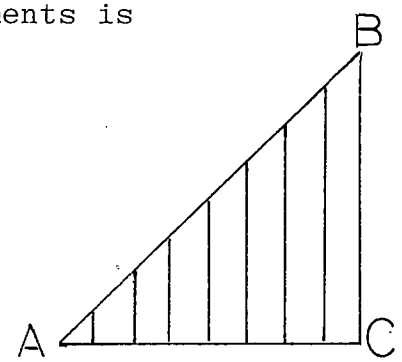
(b) $5\sqrt{10}$ inches

(c) $\frac{135}{4}$ inches

(d) 17 inches

(e) none of these

35. $\overline{AC}$ in right triangle ABC is divided into eight congruent segments. Seven line segments parallel to $\overline{BC}$ are drawn to $\overline{AB}$ from the division points. If $m\overline{BC} = 10$, the sum of the measures of the seven segments is



(a) 33

(b) 35

(c) 45

(d) 34

(e) none of these

36. If a triangle of base 8 is equal in area to a circle of radius 8, what is the height of the triangle?

(a) 9π

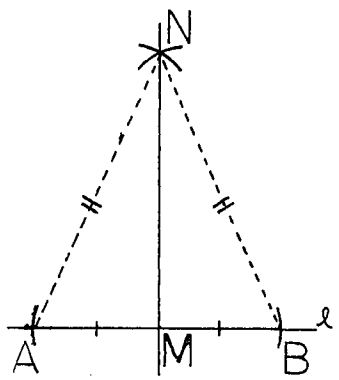
(b) 10π

(c) 12π

(d) 16π

(e) cannot be determined from the information given

37. The diagram shows the construction of a perpendicular to a line ℓ at a point M on ℓ. Which of the following is used in the proof of this construction to show that $\Delta AMN \cong \Delta BMN$?



(a) Two right triangles are congruent if the hypotenuse and leg of one are congruent to the corresponding parts of the other.

(b) Two triangles are congruent if two angles and the included side on one are congruent to the corresponding parts of the other.

(c) Two triangles are congruent if the three sides of one are congruent to the three sides of the other.

(d) Two right triangles are congruent if the hypotenuse and an acute angle of one are congruent to the corresponding parts of the other.

(e) none of the above

38. The altitude of one equilateral triangle equals the length of a side of another equilateral triangle. The ratio of their areas is

(a) $\sqrt{3} : 1$

(b) $2 : 1$

(c) $4 : 3$

(d) $6 : 8$

(e) $1 : 1$

39. Inside square RSTU with side of length x, quarter circle arcs with radii x and centers at R and S are drawn. These arcs intersect at a point P inside the square. The distance from P to side $\overline{TU}$ is

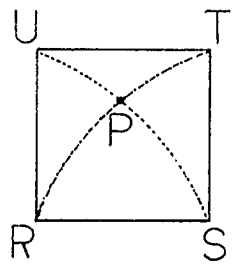
(a) $\frac{x}{2}(\sqrt{3} + 4)$

(b) $\frac{x}{2}\sqrt{3}$

(c) $\frac{x}{2}(1 + \sqrt{3})$

(d) $\frac{x}{2}(\sqrt{3} - 1)$

(e) $\frac{x}{2}(2 - \sqrt{3})$



40. In the accompanying figure, the two circles are internally tangent at point P. If $\overline{PA} \cong \overline{PB}$ and $m\overset{\frown}{PC} = 80°$, then $m\angle APB$ is

(a) 80°

(b) 90°

(c) 100°

(d) 110°

(e) 120°