TWENTIETH ANNUAL MATHEMATICS CONTEST
Sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

GEOMETRY TEST

1976

Scoring Formula: 4R - W

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This test was prepared from a list of Geometry questions submitted by Austin Peay State University.

DIRECTIONS:

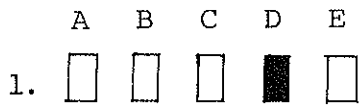
Do not open this booklet until you are told to do so.

This is a test of your competence in high school geometry. For each problem there are listed 5 possible answers; one and only one is correct. You are to work each problem, determine the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. You must use a pencil with soft lead (No. 2 lead or softer). A sample problem follows:

1. If $2x = 3$, then x equals

(a) 2/3. (b) 3. (c) 6.

(d) 3/2. (e) none of these



The correct answer for the sample problem is 3/2, which is answer (d); so you would answer this problem by making a heavy black mark under space D as indicated above.

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers so it is advisable not to do much wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet.

The answer sheets will be used for a statistical compilation and will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet to page 1 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

1. In a 5" x 5" photograph, a man's image $1\frac{1}{4}$" tall is adjacent to a building with an image $2\frac{1}{2}$" tall. If the man's actual height is 5' 8", what is the actual height of the building?

   (a) 14' 2"

   (b) 8'

   (c) 11' 4"

   (d) 12'

   (e) 12' 6"

2. What is the measure of the angle formed by the hands of a clock at 4:30?

   (a) 15°

   (b) 30°

   (c) 45°

   (d) 60°

   (e) 75°

3. Given $\triangle ABC$ with D on $\overline{AC}$, E on $\overline{BC}$, and $\overleftrightarrow{DE} \parallel \overleftrightarrow{AB}$, $m(\angle CDE) = 52^{\circ}$ and $m(\angle DCE) = 48^{\circ}$. Then $m(\angle ABC) =$

   (a) 100°

   (b) 48°

   (c) 52°

   (d) 80°

   (e) 60°

4. The altitude to the hypotenuse of a right triangle divides the hypotenuse into segments of length 2 inches and 8 inches. The area of the largest of the two newly created triangular regions is

   (a) 16 sq. in.

   (b) 20 sq. in.

   (c) 8 sq. in.

   (d) 32 sq. in.

   (e) can not be determined

5. The points A and B are 10 centimeters apart. An ordinary six-sided die is tossed. A circle is constructed with center at A with the length of the radius equal in centimeters to outcome of the die. The die is tossed again and a circle is constructed with the center at B in a similar manner. What is the probability that the two circles are tangent?

(a) $\frac{1}{6}$

(b) $\frac{1}{10}$

(c) $\frac{1}{12}$

(d) $\frac{5}{6}$

(e) 0

6. In the figure below 0 is the center of the circle and the measure of minor arc $\overset{\frown}{BC}$ is 100°. Then $m(\angle ABC) =$

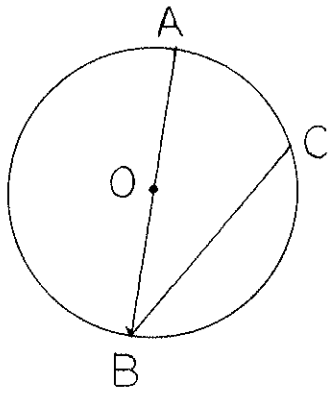
(a) 130°

(b) 100°

(c) 80°

(d) 40°

(e) 30°



7. In the figure below $\overleftrightarrow{AB} \perp \overleftrightarrow{BC}$, $\overleftrightarrow{DE} \parallel \overleftrightarrow{AB}$, $\overline{AC} = \overline{CD} = x$, $\overline{AB} = y$, $\overline{BC} = z$ $\overline{CE} = v$ and $\overline{DE} = w$. Which of the following is a true sentence?

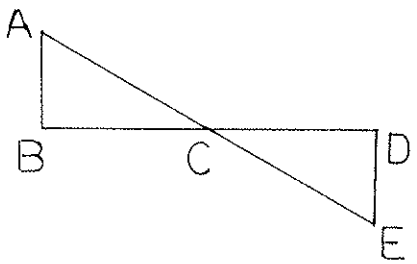
(a) $x^2 = yw$

(b) $x^2 = vz$

(c) $vz = yw$

(d) $xy = vw$

(e) all of the above are false



8. In the figure below, ΔABC is an equilateral triangle, with perimeter k, and $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$ are diameters of the semicircles pictured. An ant walks from A to B on the semicircular path, from B to C on the semicircular path and from C to A on the semicircular path. How far has he walked altogether?

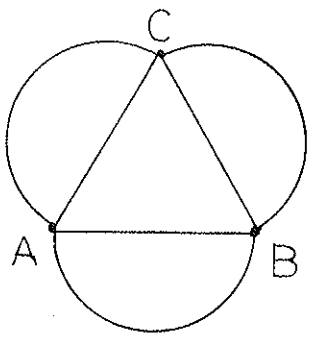
(a) $k\pi$

(b) $\frac{k^2\pi}{2}$

(c) $\frac{3k\pi}{2}$

(d) $\frac{k\pi}{2}$

(e) none of the above



9. The diagonal of a rectangle has length 13 inches and the perimeter is 34 inches. The area of the rectangular region is

(a) 60 sq. in.

(b) 55 1/4 sq. in.

(c) 72 1/4 sq. in.

(d) 110 1/2 sq. in.

(e) 73 2/3 sq. in.

10. In the figure below $\overline{AC} = 8$, $\overline{CB} = 6$, $\overline{AB} = 10$ $\overleftrightarrow{DE} \| \overleftrightarrow{CB}$ and area $(\Delta ADE) = \frac{1}{2}$ area (ΔABC). Then $\overline{DC} =$

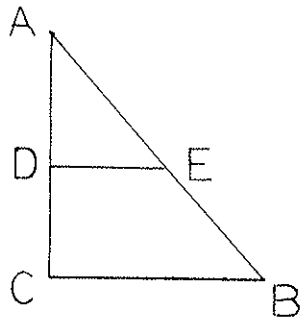
(a) 4

(b) 3

(c) $4\sqrt{2}$

(d) $8 - 4\sqrt{2}$

(e) $8 - 2\sqrt{3}$



11. The sum of the measures of the interior angles of a triangle, convex quadrilateral, convex pentagon, and convex hexagon are respectively 180°, 360°, 540° and 720°. The sum of the measures of the interior angles of a convex polygon of n sides is

(a) $2n\ (180^{\circ})$

(b) $n\ (180^{\circ})$

(c) $(n-1)\ 180^{\circ}$

(d) $(n-2)\ 180^{\circ}$

(e) none of the above

12. In the plane figure below $\overleftrightarrow{AD} \perp \overleftrightarrow{AB}$, $\overleftrightarrow{BC} \perp \overleftrightarrow{AB}$, $\overline{AB} = 3$, $\overline{BE} = 2$, and $\overline{DE} = 3$.

Then $\dfrac{\text{area } (\Delta BCE)}{\text{area } (\Delta DAE)} =$

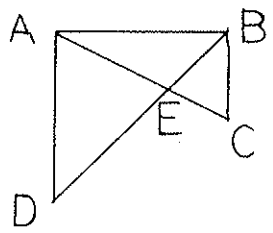
(a) 2/3

(b) 4/9

(c) 2/9

(d) 2/5

(e) none of the above



13. In the figure below $\overline{BC} = 10$ and $\overline{AC} = 15$ and quadrilateral CDEF is a square. What is the area, in square units, of the square?

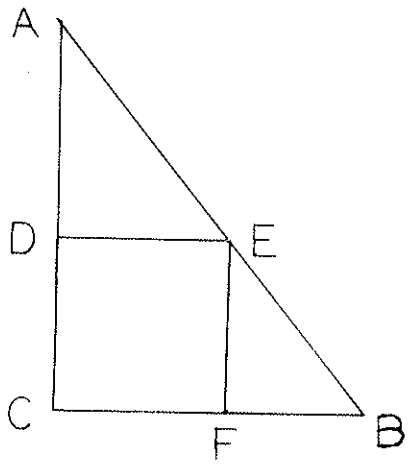
(a) 36

(b) 25

(c) 16

(d) 49

(e) 64



14. In the figure below, quadrilateral ABCE is an isosceles trapezoid, $\overline{AB}$ is a diameter of the circle, 0 is the center of the circle and $m(\angle DAB) = 80^\circ$. Then $m(\angle DBC) =$

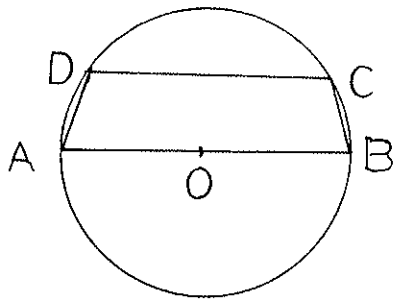
(a) 40°

(b) 50°

(c) 80°

(d) 20°

(e) 70°



15. From the information given, which of the following statements is true?

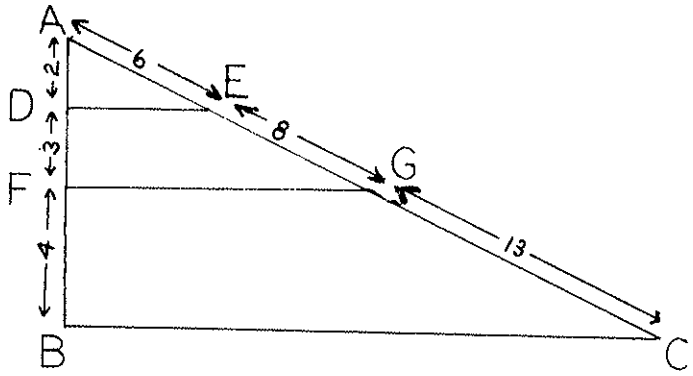
(a) $\overline{DE} \parallel \overline{FG}$

(b) $\overline{FG} \parallel \overline{BC}$

(c) $\triangle ADE \sim \triangle ABC$

(d) $\triangle AFG \sim \triangle ADE$

(e) none of the above



16. Four congruent circles are internally tangent to a square and pairwise tangent to each other as shown below. The length of one side of the square is 1 inch. What is the area of the shaded portion in square inches?

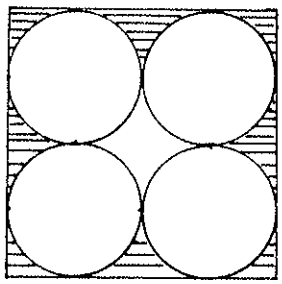
(a) $2 - \frac{\pi}{4}$

(b) $\frac{1}{4} - \frac{\pi}{16}$

(c) $\frac{\pi}{4}$

(d) $\frac{3}{4} - \frac{\pi}{16}$

(e) $\frac{3}{4} - \frac{3\pi}{16}$



17. In the diagram O is the center of the circle, C is on the circle, and the line segment $\overline{AB}$ is tangent to the circle at B. If $\overline{AC} = 2$ and $\overline{AB} = 3$ then the length of a radius of the circle is

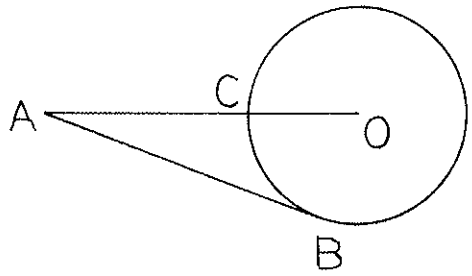
(a) 4/5

(b) 1

(c) 5/4

(d) 3/2

(e) 13/4



18. In the figure below $\overleftrightarrow{AB} \perp \overleftrightarrow{DE}$, $m(\angle BAC) = 70^\circ$, and $m(\angle ACD) = 60^\circ$. Then $m(\angle CDE) =$

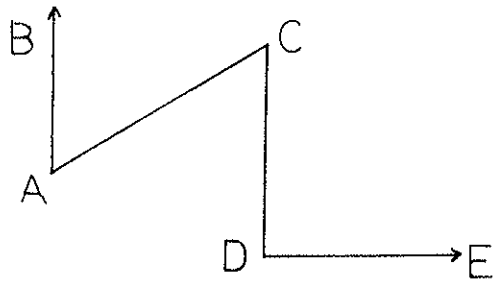
(a) 40°

(b) 50°

(c) 60°

(d) 70°

(e) 80°



19. The polygon ABCD is a square. The boundary of the shaded region is the union of eight congruent segments, AD = y, and GJ = x. What is the area of the shaded region in square units?

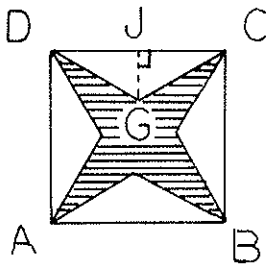
(a) $y^2 - 2x$

(b) $y^2 - 4x$

(c) $y^2 - 4xy$

(d) $4xy - y^2$

(e) $y^2 - 2xy$



20. The center of the inscribed circle of a triangle is

(a) the intersection of the bisectors of the interior angles of the triangle.

(b) the intersection of the altitudes of the triangle.

(c) the intersection of the medians of the triangle.

(d) the intersection of the perpendicular bisectors of the sides of the triangles.

(e) none of the above.

21. Let ℓ_1 and ℓ_2 be lines. Then ℓ_1 is parallel to ℓ_2 if and only if $\ell_1 \cap \ell_2 = \emptyset$ or $\ell_1 = \ell_2$. Using the definition of "is parallel to" given above, the relation "is not parallel to" is

(a) an equivalence relation.

(b) reflexive, symmetric and not transitive.

(c) transitive, symmetric and not reflexive.

(d) not reflexive, not symmetric and not transitive.

(e) symmetric, not reflexive and not transitive.

22. Let k be a line with equation $x - 3y = 6$, let ℓ be a line with equation $y = x$, and let m be the line which is the reflection of k in ℓ. Then an equation for m is

(a) $x - 3y = -6$

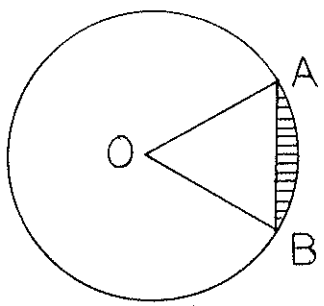
(b) $3x + y = -6$

(c) $3x - y = 6$

(d) $3x + y = 6$

(e) $3x - y = -6$

23. In the figure below 0 is the center of the circle, $\overline{0A} = r$ and $m(\angle A0B) = 60^\circ$. Then the area of the shaded region in square units is



(a) $\frac{r^2}{\sqrt{3}}$

(b) $\frac{\pi - \sqrt{3}}{4} \cdot r^2$

(c) $\frac{r^2}{9}$

(d) $\frac{2\pi - 3\sqrt{3}}{12} \cdot r^2$

(e) none of the above

24. The altitudes of a triangle are concurrent at a point called the

(a) incenter

(b) centroid

(c) circumcenter

(d) orthocenter

(e) alticenter

25. Find the area of the region of a rhombus if one diagonal has length 10 and one side has length 13.

(a) 169 square units

(b) 120 square units

(c) 130 square units

(d) 100 square units

(e) 84.5 square units

26. In an isosceles triangle an altitude is drawn from the vertex (formed by the congruent sides) to the noncongruent side. The ratio of the perimeter of the triangle to the length of the altitude of that triangle is 4 to 1. What is the ratio of the length of one of the congruent sides to the length of the noncongruent side?

(a) 4 to 1

(b) 6 to 5

(c) 3 to 5

(d) 5 to 3

(e) 5 to 6

27. The length of the radius of the circle with equation $x^2 + y^2 + 4x - 6y = 6$ is

(a) 6

(b) $\sqrt{6}$

(c) 19

(d) $\sqrt{19}$

(e) none of the above

28. The midpoints of the sides of a quadrilateral are the vertices of a

(a) parallelogram.

(b) square in every case.

(c) trapezoid with only two parallel sides.

(d) quadrilateral with no pair of parallel sides.

(e) rhombus in every case.

29. The distance between the centers of two tangent circles with radii r_1 and r_2 is always

(a) $r_1 + r_2$

(b) $r_1 - r_2$

(c) $|r_1 - r_2|$

(d) $|r_1| - |r_2|$

(e) none of the above

30. Write the equation of the line which is the perpendicular bisector of the segment whose end points are (-6,2) and (4,8).

(a) $5x - 3y + 4 = 0$

(b) $3x + 5y - 22 = 0$

(c) $5x - 3y - 10 = 0$

(d) $5x + 3y - 10 = 0$

(e) none of these

31. Which statement is correct concerning the Pythagorean Theorem?

(a) The converse and inverse of the theorem are true.

(b) The contrapositive is true but not the inverse or converse.

(c) The contrapositive is not true but the inverse is true.

(d) The inverse is not true but the converse is true.

(e) Neither the inverse, the converse or contrapositive are true.

32. In the figure below, the two circles are internally tangent at A, and $\overline{DC} = \overline{DA}$. If the measure of minor arc $\overset{\frown}{AB}$ is 140° then m(<DAC) =

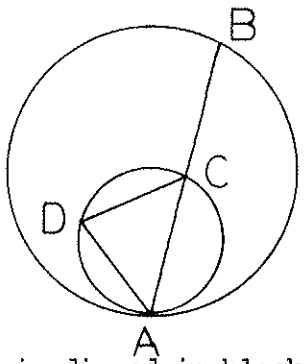
(a) 55°

(b) 140°

(c) 110°

(d) 60°

(e) none of the above



33. A cube measuring 10 inches on each edge is dipped in black paint, allowed to dry, and then sliced into 1000 smaller cubes measuring 1 inch on each edge. Of the 1000 smaller cubes, how many have 2 or more black faces?

(a) 104

(b) 120

(c) 96

(d) 100

(e) 324

34. The circumcenter, the orthocenter and centroid

(a) are collinear.

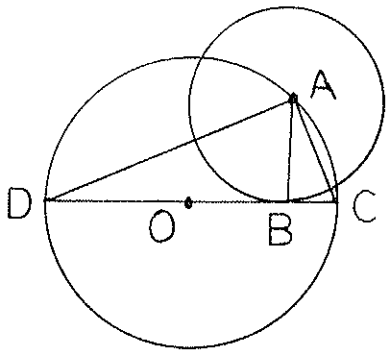
(b) lie on a circle with center at incenter.

(c) are vertices of a right triangle.

(d) are vertices of an equilateral triangle.

(e) none of the above

35. In the figure below 0 is the center of a circle with radius r_1, and A is the center of a circle with radius r_2. Diameter $\overline{DC}$ is tangent to circle A at point B. $\overline{AD} \cdot \overline{AC} = ?$



(a) $3r_1^2$

(b) $2r_1r_2$

(c) $r_1 + r_2$

(d) $4r_1r_2$

(e) none of the above

36. Point P has coordinates (1,3) and point Q has coordinates (7,-5). Then the point R which is on $\overline{PQ}$ and is 1/4 of the way from P to Q has coordinates

(a) (5/2, 1)

(b) (2, -1/2)

(c) (3/2, -2)

(d) (-3/2, 2)

(e) (7/2, 11/2)

37. Let A and B be distinct points in a plane. The set of all points in the plane which are twice as far from A as from B is

(a) a set of two points.

(b) a line.

(c) two parallel lines.

(d) a circle.

(e) a hyperbola.

38. Let a, b and c be positive real numbers. What is the volume of the solid bounded by the surface with equation

$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ and the coordinate planes?

(a) $|1/2|$ abc cubic units

(b) $|1/3|$ abc cubic units

(c) $|1/4|$ abc cubic units

(d) $|1/5|$ abc cubic units

(e) $|1/6|$ abc cubic units

39. Let the coordinates of A be (-6,4) and the coordinates of C be (6,8). Find the coordinates of point B which is on the x-axis, such that the sum of the distances from A to B and from B to C is a minimum.

(a) $(-\sqrt{2}, 0)$

(b) (0, 0)

(c) (1, 0)

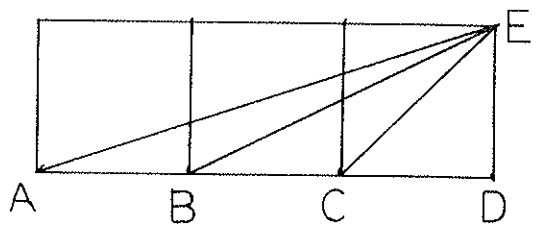
(d) (-2, 0)

(e) (-1, 0)

40. Given three unit squares placed side by side to form a 3 x 1 rectangle (see the figure), consider the following two statements.

(i) ΔEAC is similar to ΔBEC.
(ii) $m(\angle EAB) + m(\angle EBC) = m(\angle ECD)$.

Which of the following is correct?



(a) (i) is true, (ii) is false.

(b) (i) is false, (ii) is true.

(c) neither is true.

(d) both are true.

(e) impossible to determine which is true.