NINETEENTH ANNUAL MATHEMATICS CONTEST
Sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

GEOMETRY TEST

1975

Scoring Formula: 4R - W

EDITED BY:

Billy Edwards
and
James G. Ware
University of Tennessee at Chattanooga
Chattanooga, Tennessee

This test was prepared from a list of Geometry questions submitted by Memphis State University.

DIRECTIONS:

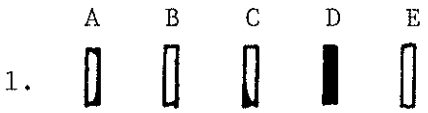
Do not open this booklet until you are told to do so.

This is a test of your competence in high school geometry. For each problem there are listed 5 possible answers; one and only one is correct. You are to work each problem, determine the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. You must use a pencil with soft lead (No. 2 lead or softer). A sample problem follows:

1. If $2x = 3$, then x equals

   (a). 2/3. (b). 3. (c). 6.

   (d). 3/2. (e). none of these



The correct answer for the sample problem is 3/2, which is answer (d); so you would answer this problem by making a heavy black mark under space D as indicated above.

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers so it is advisable not to do much wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet.

The answer sheets will be used for a statewide statistical compilation and will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet to page 1 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

1. The number of degrees in Angle A is

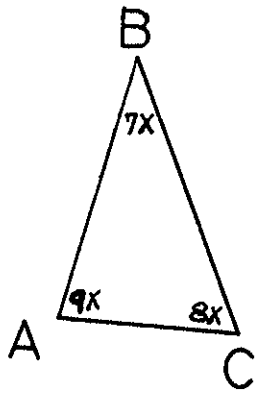
   (a) 60

   (b) $67\frac{1}{2}$

   (c) $52\frac{1}{2}$

   (d) $53\frac{1}{2}$

   (e) $66\frac{1}{2}$



2. In the given figure, $\measuredangle A = 20°$, $\measuredangle E = 45°$, $\measuredangle F = 50°$, $\measuredangle G = 60°$. Hence, $\measuredangle C$ is

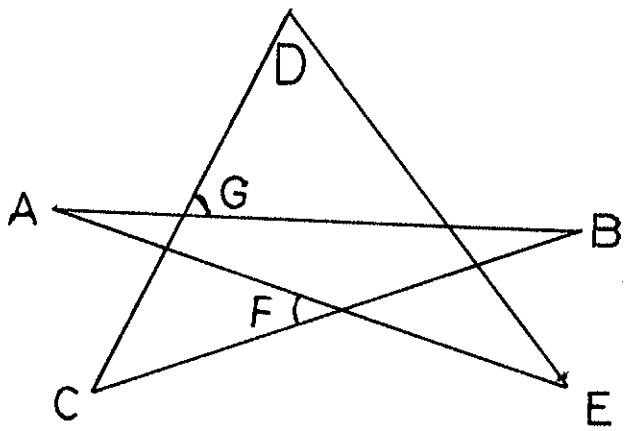
   (a) 55°

   (b) 40°

   (c) 35°

   (d) 30°

   (e) none of the above



3. The axiom of parallelism for Euclidean Geometry (Playfair Axiom) states that, in a plane, the number of distinct lines through a given point not on a given line and parallel to this given line is

   (a) exactly one

   (b) 0

   (c) 2

   (d) infinitely many

   (e) none of these

4. If the hypotenuse of a right triangle is 17 units in length and one leg is 15 units long, then the length of the remaining leg is

   (a) 4 units

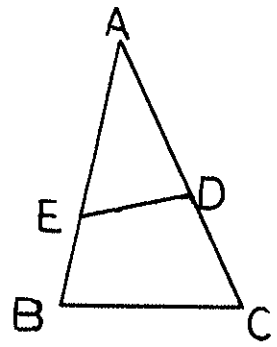
   (b) 8 units

   (c) $\frac{1}{2}$ (15 x 17) units

   (d) $\sqrt{15 \text{ x } 17}$ units

   (e) none of these

5. In the figure, if $\measuredangle ABC = \measuredangle ADE$, then for ratios of lengths of line segments, which of the following ratios is NOT equal to the ratio $\overline{AD} / \overline{AB}$?



(a) $\overline{DE} / \overline{BC}$

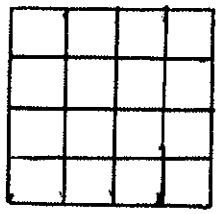
(b) $\overline{AE} / \overline{AC}$

(c) $\overline{AD} / \overline{AC}$

(d) all of the above

(e) none of the above

6. The total number of squares in this 4 x 4 figure is



(a) 16

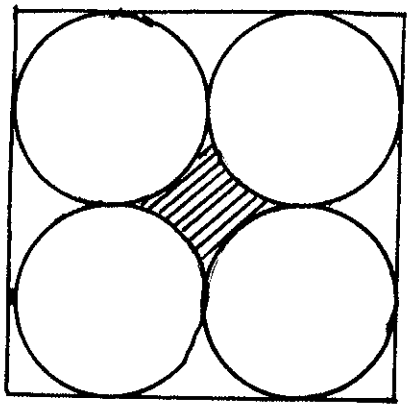
(b) 17

(c) 25

(d) 30

(e) 55

7. Four circles of equal radii are inscribed in a square of side 1 cm as shown. What is the area of the shaded portion in cm^2 ?



(a) $\frac{1}{4} - \frac{\pi}{4}$

(b) $\frac{\pi}{4}$

(c) $1 - \pi$

(d) $\frac{1}{4} - \frac{\pi}{16}$

(e) $1 - \frac{\pi}{16}$

8. In the regular pentagon ABCDE diagonals $\overline{AC}$ and $\overline{BD}$ are drawn, intersecting at F. Then $\measuredangle AFB =$

(a) $60°$

(b) $64°$

(c) $68°$

(d) undetermined

(e) none of the above

9. Given a point 0, the relation defined by "x is farther from 0 than y is from 0"

   (a) is an equivalence relation

   (b) is not symmetric or transitive

   (c) is transitive but not reflexive

   (d) is reflexive or symmetric

   (e) none of the above

10. The perimeter of a triangle is 48 meters. A second triangle is formed by joining the midpoints of the sides of the original triangle. Then this second triangle has a perimeter of

   (a) 6 meters

   (b) 12 meters

   (c) 16 meters

   (d) 24 meters

   (e) 30 meters

11. Rectangle ABCD is divided into five equal parts as shown. If $\overline{AB} = 30$, then $\overline{EF}$ =

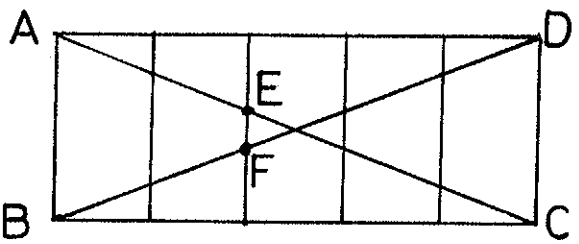
   (a) 6

   (b) 3

   (c) 4

   (d) 5

   (e) 7



12. The center of gravity of a triangle is

   (a) the intersection of perpendicular bisectors

   (b) the intersection of medians

   (c) the intersection of angle bisectors

   (d) the intersection of altitudes

   (e) none of the above

13. The two base angles of a triangle are A and B with $B > A$. If the altitude to the base divides the vertex angle C into two parts C_1 and C_2 with C_2 adjacent to side a, then

(a) $C_1 + C_2 = A + B$

(b) $C_1 - C_2 = A - B$

(c) $C_1 - C_2 = A + B$

(d) $C_1 - C_2 = B - A$

(e) $C_1 + C_2 = B - A$

14. Each side of triangle ABC is 12 cm. $\overline{AD}$ is perpendicular to $\overline{CB}$ and E is the midpoint of $\overline{AD}$. The length of $\overline{BE}$ is

(a) $\sqrt{18}$ cm.

(b) $\sqrt{28}$ cm.

(c) $\sqrt{6}$ cm.

(d) $\sqrt{98}$ cm.

(e) $\sqrt{63}$ cm.

15. If the graphs of $2y + x + 3 = 0$ and $3y + ax + 2 = 0$ are to meet at right angles, then a is

(a) $\pm \frac{2}{3}$

(b) $-\frac{2}{3}$

(c) -6

(d) $-\frac{3}{2}$

(e) 6

16. The negation of p implies q is true when

(a) p is false and q is false

(b) p is false and q is true

(c) p is true and q is false

(d) q implies p

(e) not p implies not q

17. For any two planes in space, which of the following is not a possible intersection ?

(a) $\emptyset$

(b) one line

(c) one point

(d) a plane

(e) none of these

18. An equilateral triangle is inscribed in a circle of radius 1 unit. The area of the triangle in square units is

(a) $3\sqrt{3}$

(b) $\frac{9}{4}$

(c) $\frac{3\sqrt{3}}{2}$

(d) $\frac{3\sqrt{3}}{4}$

(e) none of these

19. A plane is 5 mm from the center of a sphere of radius 13 mm. The area, in square mm , of the circle of intersection is

(a) 144 π

(b) 169 π

(c) 24 π

(d) 432

(e) none of the above

20. A rectangular sheet of paper 8 inches by 10 inches is folded at the upper right hand corner so that the vertex being folded over exactly contacts the opposite side, as indicated in the figure. The area in square inches of the folded triangle BA'E is

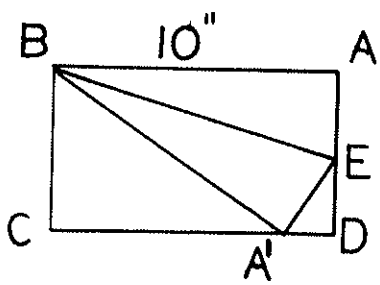
(a) $\frac{5}{2}\sqrt{82}$

(b) 25

(c) 18

(d) 20

(e) none of the above



21. In the equiangular hexagon ABCDEF, $\overline{AB} = \overline{CD} = \overline{EF} = 1$, and $\overline{BC} = \overline{DE} = \overline{FA} = 2$. The area, in square units, of ABCDEF is

(a) 3

(b) $\frac{13\sqrt{3}}{4}$

(c) $\frac{11\sqrt{2}}{2}$

(d) $3\sqrt{3}$

(e) none of the above

22. In a semicircle of radius 2, two semicircles of radius 1 are inscribed. A circle tangent to all three semicircles is inscribed within the larger semicircle. The radius of this circle is

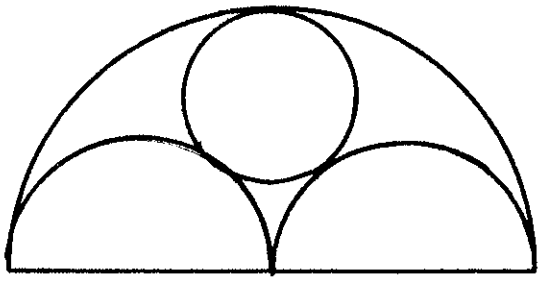
(a) 1

(b) $\frac{4}{5}$

(c) $\frac{3}{4}$

(d) $\frac{2}{3}$

(e) none of the above



23. Let A be the area of a rhombus with diagonals x and y such that $y = 2x$.

The value of x in terms of A is

(a) $\sqrt{A}$

(b) $\frac{\sqrt{2A}}{2}$

(c) $\frac{\sqrt{3A}}{3}$

(d) $\frac{\sqrt{5A}}{5}$

(e) none of these

24. Each of points P and Q lie on a line segment $\overline{AB}$ with the midpoint of $\overline{AB}$ between them. P divides $\overline{AB}$ in the ratio 2:3 and Q divides $\overline{AB}$ in the ratio 4:3. If the length of $\overline{AB}$ is 7, the length of $\overline{PQ}$ is

(a) $\frac{35}{12}$

(b) $\frac{6}{5}$

(c) $\frac{1}{5}$

(d) $\frac{7}{2}$

(e) $\frac{29}{5}$

25. Using only a straight edge and compass, which one of the following constructions is not possible ?

(a) the bisector of an angle

(b) an equilateral triangle

(c) a line segment one fifth as long as a given segment

(d) a regular pentagon

(e) the trisection of an angle

26. The locus of points in a plane, the sum of whose distances from two different fixed points of the plane is constant, is

(a) a line

(b) a circle

(c) an ellipse

(d) a pair of circles

(e) a pair of lines

27. In the figure, a point D is on a semicircle with diameter $\overline{AC}$. $\overline{DB}$ is perpendicular to $\overline{AC}$ at B. If $\overline{AB} = 12$ and $\overline{BC} = 3$, the length of $\overline{DB}$ is

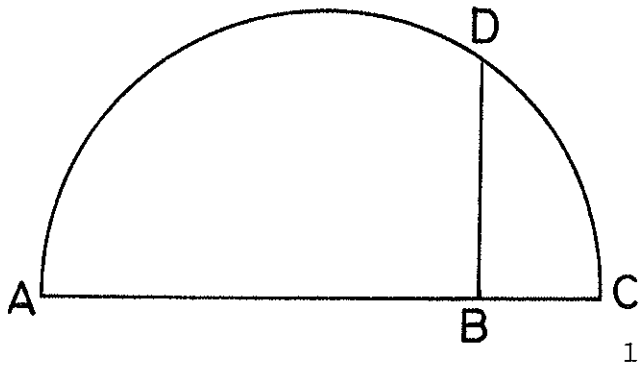
(a) $\sqrt{15}$

(b) 7

(c) 9

(d) 6

(e) none of these



28. As a sphere is submerged in a cylinder of water, the water rises $\frac{1}{4}$ cm. If the radius of the cylinder is 6 cm., then the radius of the sphere is

(a) $\frac{3\sqrt[3]{2}}{2}$ cm.

(b) $\frac{3\sqrt{3}}{2}$ cm.

(c) $\frac{3\pi\sqrt[3]{2}}{2}$ cm.

(d) $\sqrt[3]{9}$ cm.

(e) $3\sqrt[3]{2}$ cm.

29. In the figure, ΔABC is similiar to ΔEDC, $\overline{CE} = 6$, $\overline{BE} = 4$, and $\overline{AC} = (\overline{AD} + 3)$. Thus $\overline{AC}$ is

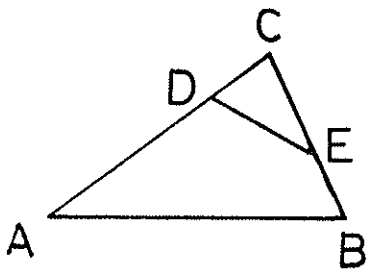
(a) 17

(b) 20

(c) 14

(d) 5

(e) none of the above



30. In triangle ABC, $\measuredangle ACB = 120°$, $\overline{AB} = c$, $\overline{AC} = b$, $\overline{BC} = a$. $a^2 + b^2 - c^2$ is

(a) ab

(b) a + b - c

(c) - ab

(d) - a - b + c

(e) none of the above

31. A rectangular barn is 40 feet long and 16 feet wide. A horse is tied outside the barn at one corner with a rope that is 30 feet long. The number of square feet over which the horse may graze is

(a) 646 π

(b) 724 π

(c) 900 π

(d) 1096 π

(e) none of the above

32. Given: $\overline{AB}$ is a chord of a circle with center 0. $\overline{AB}$ is produced to C so that $\overline{BC} = \overline{OB}$. $\overline{CO}$ is produced to meet the circle in points D and E.

$\frac{\overset{\frown}{EA}}{\overset{\frown}{BD}}$ is

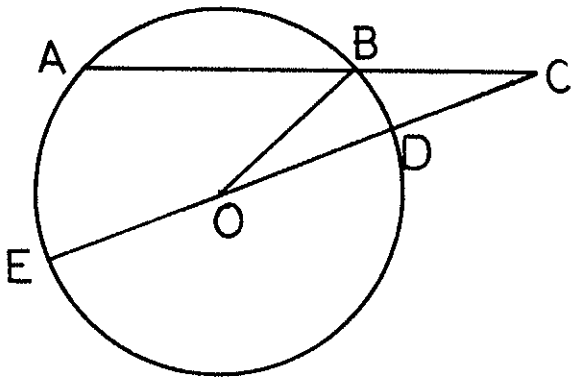
(a) 2.5

(b) 3.0

(c) 3.14

(d) 3.5

(e) 4.0



33. This 6 x 6 board is to be covered with 17 dominoes, each domino covering two adjacent squares, leaving two squares open. The two squares which might be left uncovered are

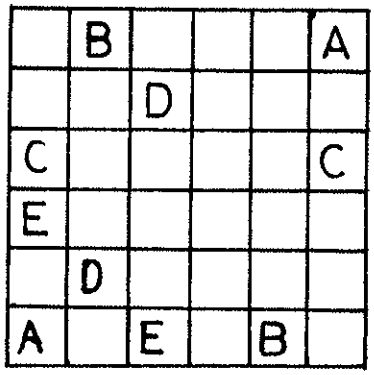
(a) A

(b) B

(c) C

(d) D

(e) E



34. Triangle ABC is isosceles with base $\overline{AC}$. Points P and Q respectively are in $\overline{CB}$ and $\overline{AB}$ such that $\overline{AC} = \overline{AP} = \overline{PQ} = \overline{QB}$. The number of degrees in angle B is

(a) $\frac{180}{7}$

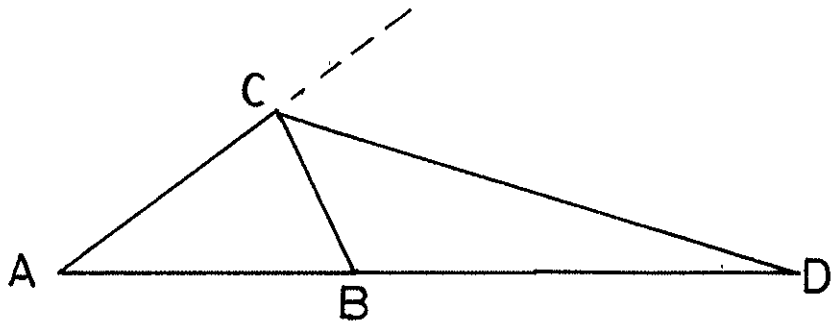
(b) 30

(c) 40

(d) $\frac{100}{3}$

(e) Not determined

35. If triangle ABC has sides $\overline{AC} = 7$ and $\overline{BC} = 4$, with segment $\overline{BD} = 8$, and segment $\overline{CD}$ lying on the exterior bisector of the angle C, the length of $\overline{AB}$ is



(a) 7

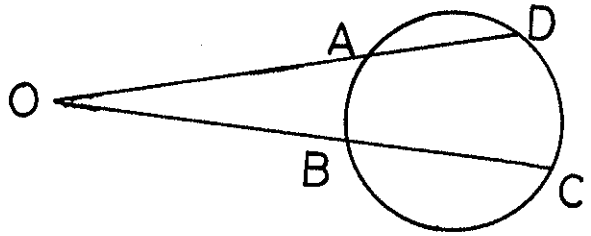
(b) 6

(c) $\frac{27}{4}$

(d) $\frac{13}{2}$

(e) none of these

36. In the figure, arcs are named or described and measured in the counter-clockwise direction. The measure of the angle at O is equal to the measure of



(a) $\overset{\frown}{AB}$

(b) $\overset{\frown}{BC} - \overset{\frown}{DA}$

(c) $\overset{\frown}{CD} + \overset{\frown}{AB}$

(d) $(\overset{\frown}{BC} - \overset{\frown}{DA}) / 2$

(e) none of these

37. A pentagon has angles A = 80°, B = 90°, C = 110°, and D = 120°. What is the degree measure of the fifth angle E ?

(a) 140°

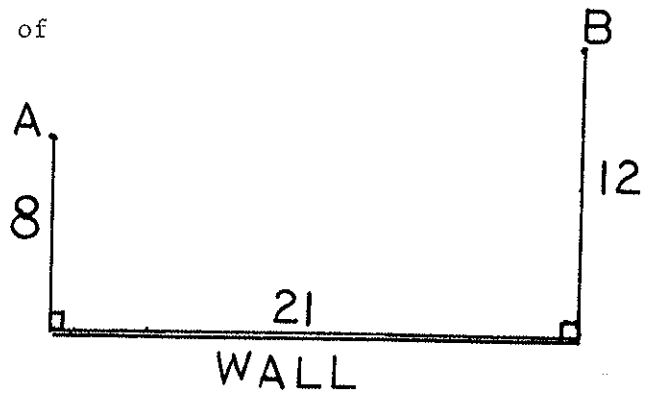
(b) 100°

(c) 108°

(d) 70°

(e) none of these

38. John wishes to run from point A to the brick wall and then back to point B. The shortest path would be a distance of



(a) $(\sqrt{254.25} + \sqrt{174.25})$ units

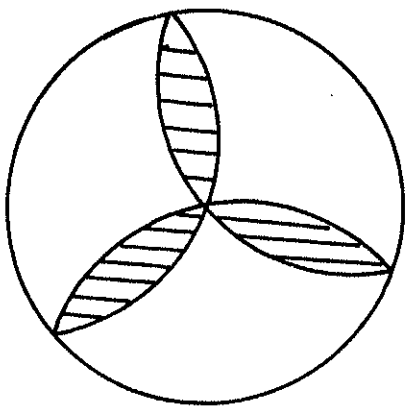
(b) $(2\sqrt{41} + \sqrt{265})$ units

(c) $(2\sqrt{61} + \sqrt{185})$ units

(d) 28 units

(e) $\sqrt{134.56} + \sqrt{302.76}$ units

39. If all of the arcs and the circle have a radius of 2 inches the area of the shaded figure in square inches is



(a) $4\pi - 6\sqrt{3}$

(b) $\frac{2\pi}{3} - \frac{\sqrt{3}}{2}$

(c) $(4 - 3\sqrt{3})\ \pi$

(d) $2\pi - 6\sqrt{3}$

(e) none of the above

40. The point P is interior to the rectangle ABCD. Also $\overline{AP} = 2$, $\overline{BP} = 3$, $\overline{DP} = 1$. $\overline{CP}$ is

(a) 1.5

(b) 4

(c) $\sqrt{5}$

(d) $\sqrt{6}$

(e) none of the above