

EIGHTEENTH ANNUAL MATHEMATICS CONTEST  
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THE TENNESSEE MATHEMATICS TEACHER'S ASSOCIATION

GEOMETRY TEST

1974

Scoring Formula:  $4R - W$

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This test was prepared from a list of Geometry questions submitted by Austin Peay State University.

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DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school geometry. For each problem there are listed 5 possible answers; one and only one is correct. You are to work each problem, determine the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. You must use a pencil with soft lead (No. 2 or softer). A sample problem follows:

1. If  $2x = 3$ , then  $x$  equals

- (a).  $2/3$ .      (b). 3.      (c). 6.  
(d).  $3/2$ .      (e). none of these.

1.      A      B      C      D      E  
                    

The correct answer for the sample problem is  $3/2$ , which is answer (D); so you would answer this problem by making a heavy black mark under space D as indicated above.

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers so it is advisable not to do much wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet.

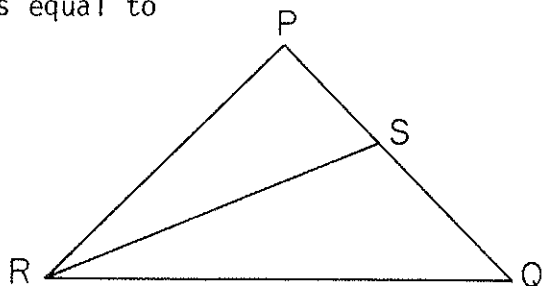
The answer sheets will be used for a statewide statistical compilation and will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

Figures given on this test are not necessarily drawn to scale.

1. In triangle PQR, it is given that  $PR = PQ$ ,  $m(\angle Q) = 40^\circ$ , and RS bisects angle PRQ. The measure of angle SRQ is equal to

- (a).  $10^\circ$ .
- (b).  $20^\circ$ .
- (c).  $25^\circ$ .
- (d).  $40^\circ$ .
- (e).  $50^\circ$ .

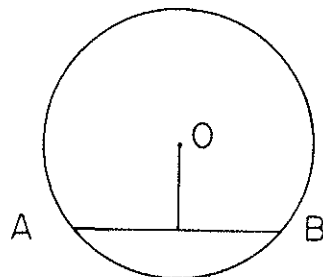


2. If two angles of a quadrilateral are supplementary, the other two angles are

- (a). acute.
- (b). obtuse.
- (c). complementary.
- (d). supplementary.
- (e). equal and supplementary.

3. In the circle, chord AB is 12 inches long and 8 inches from center O. What is the length, in inches, of the radius of the circle?

- (a).  $\sqrt{80}$  inches
- (b). 10 inches
- (c).  $\sqrt{208}$  inches
- (d). 16 inches.
- (e). 20 inches



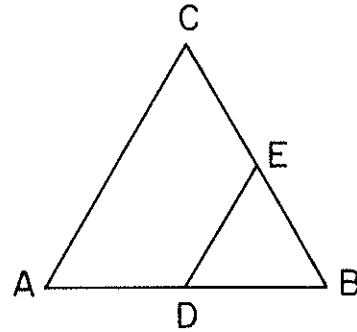
4. Which of the following statements concerning the diagonals of a square is (are) true?

- I. The diagonals are equal.
- II. The diagonals are perpendicular.
- III. The diagonals bisect each other.

- (a). II only
- (b). I and II only
- (c). I and III only
- (d). II and III only
- (e). I, II, and III

5. In the figure, if  $CA = CB$  and  $ED = EB$ , then which of the following can be concluded?

- (a).  $CA$  must be parallel to  $ED$
- (b).  $CA$  cannot be parallel to  $ED$
- (c).  $\triangle ABC$  is equilateral
- (d).  $\triangle BDE$  is equilateral
- (e).  $AD = CE$



6. Which of the following should be proved equal in order to show that two parallelograms are congruent?

- (a). One pair of corresponding angles.
- (b). One pair of corresponding sides.
- (c). Two pairs of adjacent sides and the included angles.
- (d). A pair of diagonals.
- (e). Two pairs of diagonals.

7. If a straight line is drawn from one vertex of a hexagon (six-sided polygon) to another vertex, which of the following pairs of polygons could be produced?

- (a). Two quadrilaterals
- (b). Two triangles
- (c). Two pentagons
- (d). A quadrilateral and a pentagon
- (e). A triangle and a quadrilateral

8. If the hypotenuse of a right triangle is 10 inches long and one acute angle measures  $60^\circ$ , then one leg must have a length of

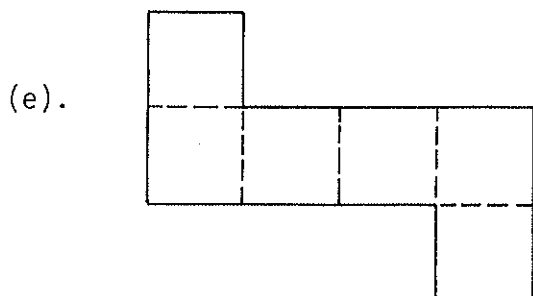
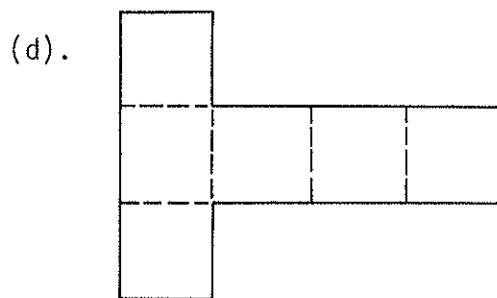
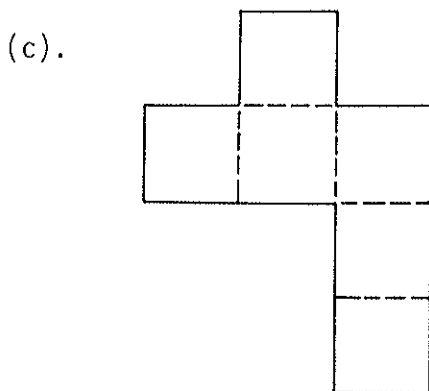
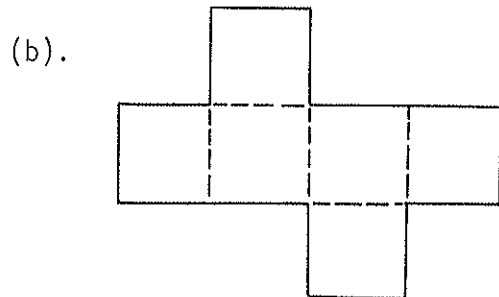
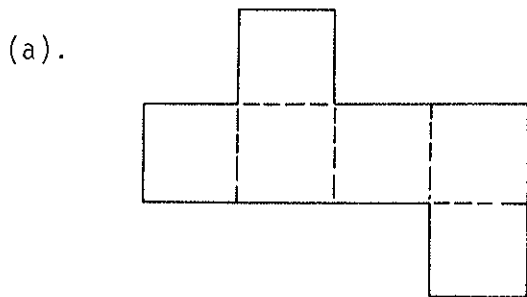
- (a).  $3 \frac{1}{3}$  inches.
- (b). 5 inches.
- (c).  $5\sqrt{2}$  inches.
- (d). 6 inches.
- (e). 9 inches.

9. The statement "p implies q and q implies p" means exactly the same as all of the following except
- (a). "if p then q and conversely".
  - (b). "p if and only if q".
  - (c). "p and q are equivalent".
  - (d). "p and q are unrelated".
  - (e). "p is necessary and sufficient for q".
10. If triangle ABC is inscribed in a circle of diameter 10 and angle A is acute, then what can be concluded about the length of BC?
- (a).  $BC < 5$
  - (b).  $BC = 5$
  - (c).  $BC < 10$
  - (d).  $BC = 10$
  - (e).  $BC > 10$
11. A point P is 12 inches from a plane m. The set of points in m which are 13 inches from P is
- (a). an ellipse.
  - (b). a point.
  - (c). two distinct points.
  - (d). a segment.
  - (e). a circle.
12. If  $\overleftrightarrow{AB}$  is oblique to plane m the number of planes that contain  $\overleftrightarrow{AB}$  and are perpendicular to m is
- (a). exactly 0.
  - (b). exactly 1.
  - (c). exactly 2.
  - (d). an infinite number.
  - (e). either 0 or an infinite number depending upon  $\overleftrightarrow{AB}$  and plane m.

13. Which of the following is not equivalent to the statement "Quadrilateral ABCD is a rectangle"?

- (a). Quadrilateral ABCD is a parallelogram and angle ABC is a right angle.
- (b). Quadrilateral ABCD has the property that  $\overline{AC}$  and  $\overline{BD}$  bisect each other.
- (c). Quadrilateral ABCD is a parallelogram and  $\overline{AC} \cong \overline{BD}$ .
- (d). Quadrilateral ABCD has the property that  $\angle ABC \cong \angle BCD \cong \angle CDA \cong \angle DAB$ .
- (e). Quadrilateral ABCD is a parallelogram and the area of its region is  $(AB)(BC)$ .

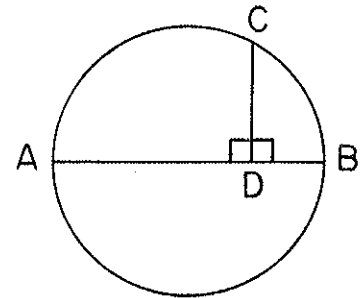
14. Which of the following two dimensional figures could not be folded along the dotted segments so as to form a cube?



15. The radii of two coplanar circles are 3 and 8 and the distance between their centers is 13. The length of a common external tangent segment is
- (a). 11.
  - (b). 7.
  - (c). 16.
  - (d). 12.
  - (e). 21.

16. In the figure,  $\overline{AB}$  is a diameter of the circle and  $\overline{CD}$  is perpendicular to  $\overline{AB}$ . If  $AD = a$  and  $DB = c$  then  $CD$  is equal to

- (a).  $ac$ .
- (b).  $\sqrt{ac}$ .
- (c).  $(1)/(ac)$ .
- (d).  $a^2c^2$ .
- (e).  $a\sqrt{c}$ .



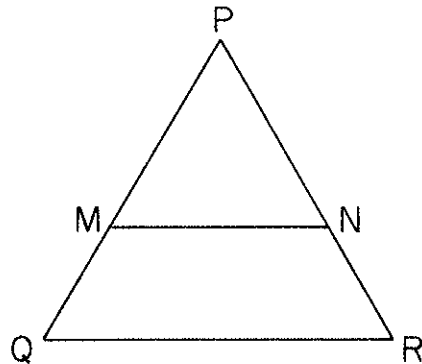
17. Given two circles with a common tangent at a point A such that the second circle passes through the center of the first. Which of the following is true?
- (a). Every chord of the first circle that passes through A is bisected by the second circle.
  - (b). Every chord of the first circle that passes through A is trisected by the second circle.
  - (c). Every chord of the first circle that passes through A is tangent to the second circle.
  - (d). Every chord of the second circle that passes through A is perpendicular to the line that passes through the center of the first circle and point A.
  - (e). None of the above are true.

18. Let angle  $\angle ABC$  be a right angle in  $\triangle ABC$  and let  $D$  be the midpoint of  $\overline{AC}$ . If  $E$  is the point of intersection of  $\overline{BC}$  and the line through  $D$  parallel to  $\overline{AB}$ , then  $CE/CB$  is equal to
- (a).  $2/1$ .
  - (b).  $1/2$ .
  - (c).  $1/3$ .
  - (d).  $1/1$ .
  - (e).  $2/3$ .

19. Suppose  $\overline{AB}$  and  $\overline{BC}$  are nonparallel sides of a parallelogram and  $m(\angle A) = 30^\circ$ . Find the area of the region of the parallelogram if  $AB = 10$  and  $BC = 8$ .
- (a). 48
  - (b).  $(8 + 5\sqrt{3})5$
  - (c).  $25\sqrt{3}$
  - (d). 60
  - (e). 40

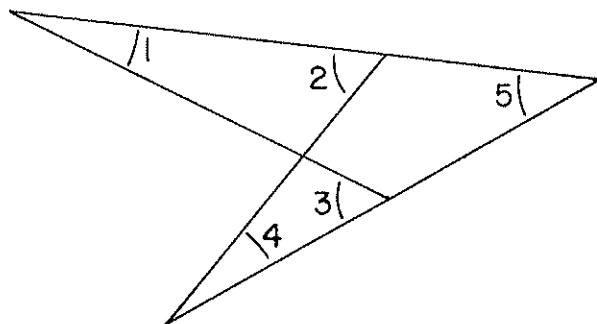
20. In the figure, it is given that  $\frac{PM}{MQ} = \frac{PN}{NR}$ . Which one of the following must always be true?

- (a).  $\overline{MN}$  is parallel to  $\overline{QR}$ .
- (b).  $MN = (QR)/2$ .
- (c).  $m(\angle PQR) = m(\angle PRQ)$ .
- (d).  $PM = MQ, PN = NR$ .
- (e). Triangle  $PQR$  is equilateral.



21. Find  $m(\angle 5)$  given that  $m(\angle 2) = 60^\circ$ ,  $m(\angle 3) = 80^\circ$  and the sum of the measures of angles 1 and 4 is  $80^\circ$ .

- (a).  $40^\circ$
- (b).  $30^\circ$
- (c).  $20^\circ$
- (d).  $70^\circ$
- (e). none of these

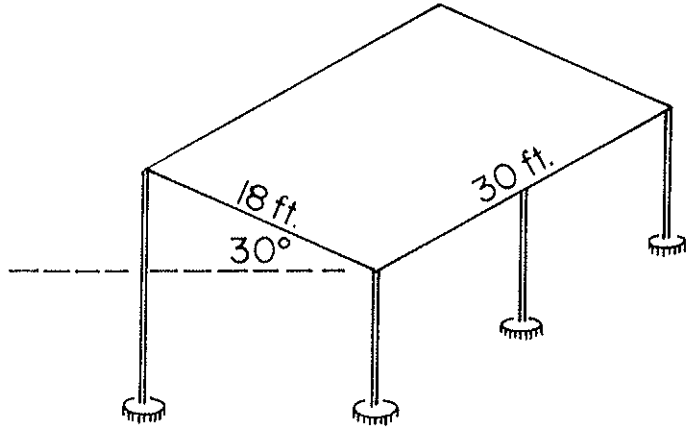


22. Which of the following is false?
- (a). The area of the region of an isosceles right triangle is equal to one-fourth the square of the length of the hypotenuse.
  - (b). If the diagonals of a convex quadrilateral are perpendicular, then the area of the region of the quadrilateral is equal to  $1/2$  the product of the lengths of the diagonals.
  - (c). Let  $D$  be the midpoint of  $\overline{AC}$  where  $\overline{AC}$  is a side of  $\triangle ABC$ . Then the area of the region for  $\triangle ABD$  is equal to the area for the region of  $\triangle CBD$ .
  - (d). If  $\triangle ABC \sim \triangle DEF$  and  $(AB/DE) = 2$  and if the perimeter of  $\triangle ABC$  is 10, then the perimeter of  $\triangle DEF$  is 5.
  - (e). If  $\triangle ABC \sim \triangle DEF$  and  $AB/DE = 2$  and if the area of the region for  $\triangle ABC$  is 64, then the area of the region of  $\triangle DEF$  is 32.
23. Let  $x$  and  $y$  be positive real numbers. Then the set of all points at a distance  $x$  from one plane, and at a distance  $y$  from a second plane which intersects the first plane obliquely is
- (a). four parallel lines.
  - (b). four points in one plane.
  - (c). four planes.
  - (d). a line parallel to both planes.
  - (e). two parallel lines.
24. Points  $A$ ,  $B$ , and  $C$  are located on a circle. Segment  $\overline{AB}$  is congruent to segment  $\overline{AC}$ . If segment  $AB = 15$  inches and segment  $BC = 24$  inches, what is the diameter of the circle?
- (a).  $19 \frac{1}{2}$  inches
  - (b). 30 inches
  - (c). 25 inches
  - (d).  $15\sqrt{2}$  inches
  - (e). 12.5 inches



25. A farmer has built a flat-roof shelter for his livestock. The roof measures 30 feet long and 18 feet wide and is set at an angle of inclination measuring  $30^\circ$  from the horizontal. Find the area of the projection of the roof on the ground. (Assume that the ground is horizontal.)

- (a).  $270\sqrt{3}$  square feet
- (b). 270 square feet
- (c). 540 square feet
- (d). 405 square feet
- (e). 135 square feet



26. Let  $A$  be the area of the region bounded by an isosceles triangle. What is the measure of each of the congruent sides if the length of the third side is  $L$ ?

- (a).  $2A/L$
- (b).  $(\sqrt{16A^2 + L^4})/2L$
- (c).  $\sqrt{(A^2 + L^2)}/2$
- (d).  $(L + A)\sqrt{3/2}$
- (e).  $(AL)/2$

27. The exterior angle theorem states: The measure of any exterior angle of a triangle is greater than the measure of each of its remote interior angles. Which of the following is an immediate consequence of this theorem?

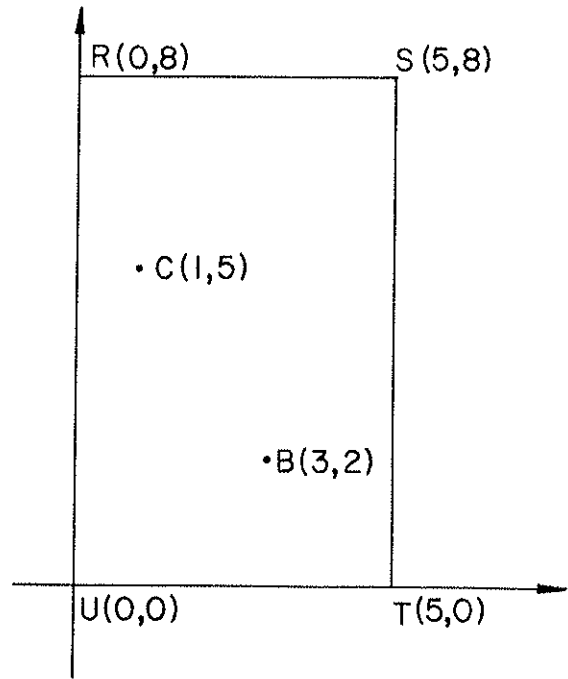
- (a). The shortest segment joining a point to a line is the perpendicular segment.
- (b). Any angle congruent to a right angle is a right angle.
- (c). Every angle has exactly one bisector.
- (d). If two angles form a vertical pair, then they are congruent.
- (e). In any triangle  $ABC$  we have  $m(\sphericalangle A) + m(\sphericalangle B) + m(\sphericalangle C) = 180^\circ$ .

28. The length of any diagonal that is not a face diagonal of a rectangular parallelepiped with edges of length  $a$ ,  $2a$ , and  $3a$  where  $a$  is a positive real number is
- (a).  $6a^3$ .
  - (b).  $a\sqrt{14}$ .
  - (c).  $6a$ .
  - (d).  $a^3\sqrt{14}$ .
  - (e). none of the above.
29. The sum of the measures of the interior angles of a convex  $n$ -sided polygon is
- (a).  $(360^\circ)/(n - 2)$ .
  - (b).  $180^\circ - 2n$ .
  - (c).  $90^\circ (n - 2)$ .
  - (d).  $n(180^\circ) - 360^\circ$ .
  - (e).  $[(n - 2)/n](180^\circ)$ .
30. The radii of two coplanar circles are 3 and 6 and the distance between their centers is 12. The length of a common internal tangent is
- (a).  $3\sqrt{7}$ .
  - (b).  $2\sqrt{35}$ .
  - (c). 8.
  - (d).  $\sqrt{135}$ .
  - (e). 3.
31. A line segment  $\overline{AB}$  assumes all positions in a plane such that point  $A$  is on line  $L_1$  and point  $B$  is on line  $L_2$ , which is perpendicular to  $L_1$ . If  $C$  is the midpoint of  $\overline{AB}$ , the set of points determined by all positions assumed by  $C$  forms
- (a). a square.
  - (b). a line segment.
  - (c). a circle.
  - (d). an ellipse.
  - (e). a triangle.

32. The Playfair form of the parallel postulate states: Given a line and an external point, there is one and only one line which passes through the given point and is parallel to the given line. The parallel postulate is necessary to the proof of which of the following theorems?
- (a). Given two lines and a transversal. If the lines are parallel, then each pair of alternate interior angles are congruent.
  - (b). If two lines be in the same plane and are perpendicular to the same line, then they are parallel.
  - (c). Given a line and a point not on the line, there is always at least one line which passes through the given point and is parallel to the given line.
  - (d). Given two lines and a transversal. If a pair of alternate interior angles are congruent, then the lines are parallel.
  - (e). Given two lines and a transversal. If a pair of corresponding angles are congruent, then a pair of alternate interior angles are congruent.
33. Consider the following definition:  
For each line  $L$ , for each line  $M$ ,  $L$  is parallel to  $M$  if and only if: (1)  $L$  and  $M$  are subsets of the same plane and (2)  $L \cap M = \emptyset$ .  
With respect to this definition, "is parallel to" is a relation on the set of all lines which is
- (a). an equivalence relation.
  - (b). symmetric but not reflexive and not transitive.
  - (c). symmetric and transitive but not reflexive.
  - (d). reflexive and transitive but not symmetric.
  - (e). not reflexive, not symmetric and not transitive.
34. Select the false statement concerning the reflection about a given line  $L$ .
- (a). The reflection about line  $L$  preserves collinearity.
  - (b). If the image of point  $A$  in the reflection about line  $L$  is point  $B$  then the image of point  $B$  in the reflection about  $L$  is  $A$ .
  - (c). The reflection about line  $L$  preserves orientation of points.
  - (d). The reflection about line  $L$  is an isometry.
  - (e). The reflection about line  $L$  preserves angle measure.

35. Use the billiards table diagrammed below, with sides  $\overline{RS}$ ,  $\overline{ST}$ ,  $\overline{TU}$ , and  $\overline{RU}$ . At what point on side  $\overline{RS}$  will the cue ball strike if the player strikes the cue ball located at  $(1,5)$  so that it strikes sides  $\overline{RU}$ ,  $\overline{RS}$ , and  $\overline{ST}$  in that order before hitting the ball located at  $(3,2)$ ?

- (a).  $(1, 8)$
- (b).  $(4/3, 8)$
- (c).  $(5/3, 8)$
- (d).  $(2, 8)$
- (e).  $(3/2, 8)$



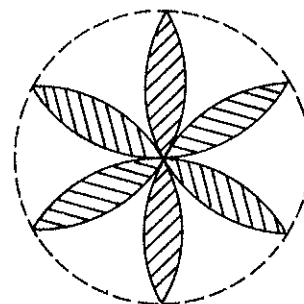
36. Given: If A is not guilty, then B and C are both guilty.  
Either A is not guilty or B is guilty.  
Either B is not guilty or C is not guilty.  
Where does the guilt lie?

- (a). A is the only guilty party.
- (b). B is the only guilty party.
- (c). C is the only guilty party.
- (d). A and B are guilty.
- (e). A and C are guilty.

37. Suppose Gary starts at Nashville, Tennessee, and travels 500 miles due north to point A. From point A he travels 500 miles due east to point B. From point B he travels 500 miles due south to point C. Which of the following best describes Gary's position at point C on the earth's surface relative to Nashville?
- (a). Gary is 500 miles east of Nashville.
  - (b). Gary is more than 500 miles east of Nashville.
  - (c). Gary is less than 500 miles east of Nashville.
  - (d). Gary is approximately 500 miles north and east of Nashville.
  - (e). Gary is approximately 500 miles south and east of Nashville.

38. The radius of each circular arc which makes up this six-petal design is the same as the radius of the circle which contains the outer tips of all the petals. If the radius is 1 what is the area of the shaded region?

- (a). 2
- (b). 3
- (c).  $4\pi - 6$
- (d).  $2\pi - 3\sqrt{3}$
- (e).  $\pi + \sqrt{2}$



39. An equilateral triangle whose sides are of length  $s$  is revolved about an altitude. What is the volume of the solid figure generated?
- (a).  $(\pi s^3)/2\sqrt{3}$
  - (b).  $(\pi s^3)/12$
  - (c).  $(\sqrt{3}/24)\pi s^3$
  - (d).  $(\sqrt{3}/4)\pi s^2$
  - (e).  $(\sqrt{3}/4)s^2$

40. Using only straight edge and compass it is possible to perform all of the following constructions except one. Which one is not possible?

- (a). Trisect a  $90^\circ$  angle.
- (b). Bisect a  $72^\circ$  angle.
- (c). Trisect a  $60^\circ$  angle.
- (d). Construct a regular hexagon.
- (e). Construct a regular octagon.



