SIXTEENTH ANNUAL MATHEMATICS CONTEST

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GEOMETRY TEST

1972

Scoring Formula: 4R-W

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This test was prepared from a list of Geometry questions submitted by eleven colleges and univeristies across Tennessee.

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school geometry. For each problem there are listed 5 possible answers; one and only one is correct. You are to work each problem, determine the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. You must use a pencil with soft lead (No. 2 lead or softer). A sample problem follows:

1. If $2x = 3$, then x equals

   (1). 2/3 (2). 3 (3). 6

   (4). 3/2 (5). none of these

	1	2	3	4	5
1.					

The correct answer for the sample problem is 3/2, which is answer (4); so you would answer this problem by making a heavy black mark under space 4 as indicated above.

If you should change your mind about an answer, be sure to erase completely. Avoid wild guessing, as wrong answers count against you. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

1. Given the statements:

   (p) If a man lives in Chicago, he lives in Illinois.
   (q) If a man does not live in Chicago, he does not live in Illinois,

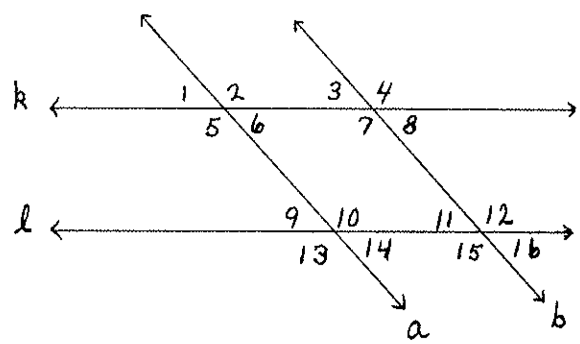
   1. (p) and (q) have the same conclusions.
   2. (p) and (q) have the same hypothesis.
   3. (q) is the inverse of (p).
   4. (q) is the converse of (p).
   5. (q) is the contrapositive of (p).

2. If X is the midpoint of $\overline{AB}$, then

   1. A is on ray $\overrightarrow{XB}$.
   2. $\overrightarrow{XB}$ and $\overrightarrow{BX}$ represent the same ray.
   3. $\overline{AX} + \overline{XB} = \overleftrightarrow{AB}$.
   4. $AX = XB$ or $m(\overline{AX}) = m(\overline{XB})$.
   5. $\overline{AX} = \overline{XB}$.

3. In the figure it is given that a || b, k || l, and a is not perpendicular to k; which of the following is <u>not</u> true?

   1. $m(\measuredangle 1) = m(\measuredangle 3)$
   2. $m(\measuredangle 1) = m(\measuredangle 8)$
   3. $m(\measuredangle 1) = m(\measuredangle 16)$
   4. $m(\measuredangle 1) \neq m(\measuredangle 10)$
   5. $m(\measuredangle 2) \neq m(\measuredangle 12)$



4. How many planes are determined by four parallel lines, no three of which lie in the same plane?

   1. 2
   2. 4
   3. 6
   4. 8
   5. none of the above.

5. If the hypotenuse of a right triangle is 10 inches and one of the acute angles is 60°, then the length of one leg must be:

   1. $5\sqrt{2}$ inches

   2. 5 inches

   3. 8 inches

   4. 6 inches

   5. none of these

6. Find the area of a square inscribed in a circle of radius 2.

   1. $2\sqrt{2}$ sq. units

   2. $\sqrt{2}$ sq. units

   3. 2 sq. units

   4. 8 sq. units

   5. 16 sq. units

7. If FQD is an equilateral triangle in which $\overline{QM}$ is perpendicular to $\overline{FD}$, then which one of the following assertions is true?

   1. $(QF)^2 + (QD)^2 = (QM)^2$

   2. $(FM)^2 + (MD)^2 = (QD)^2$

   3. The area of $\Delta QFD = (FD)\ (QM)$

   4. $(QM)^2 = (3/4)\ (QF)^2$

   5. none of these

8. At a point on a line in space, how many lines can be drawn perpendicular to the line?

   1. 0

   2. 1

   3. 2

   4. infinite number

   5. none of these

9. The midpoint of the line segment whose endpoints are (-7, 6) and (3, -4) has the coordinates:

   1. (6, 3)
   2. (2, -1)
   3. (-2, 1)
   4. (7, -4)
   5. none of these

10. Given in $\triangle$ABC, D and F the midpoints of AB and AC, respectively. Which one of the following assertions is true?

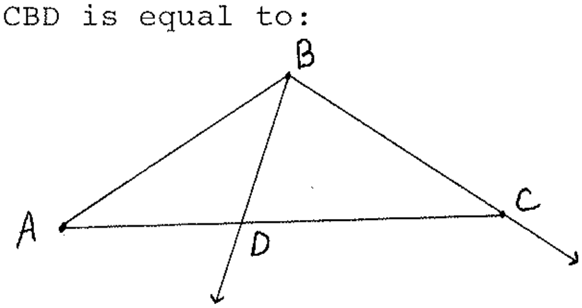
   1. The area of $\triangle$ADF is one eighth the area of $\triangle$ABC.
   2. $\triangle$ADF is isosceles.
   3. $m(\overline{DF}) = (1/3)\ m\ (\overline{BC})$.
   4. $\overline{DF} \parallel \overline{BC}$, and $2m(\overline{DF}) = m(\overline{BC})$.
   5. none of these

11. Given right $\triangle$MQP with right angle at Q. K is the foot of the altitude from Q to $\overline{MP}$. All but one of the statements below are true. Which statement is false?

   1. $\dfrac{MK}{QK} = \dfrac{QK}{KP}$
   2. $\dfrac{MP}{MQ} = \dfrac{MQ}{MK}$
   3. $\dfrac{QP}{MP} = \dfrac{KP}{QP}$
   4. $(QP)^2 = (QK)^2 + (KP)^2$
   5. none of the above is true

12. If the geometric concepts are sets of points, then, in the given figure, $(\triangle ABC \cap \overrightarrow{BD}) \cap \measuredangle CBD$ is equal to:

   1. $\measuredangle$BDC
   2. $\overline{BD} \cup \overline{BC}$
   3. {B, D}
   4. $\overline{BC}$
   5. $\overline{DC}$



13. Consider the circle 0 with chords $\overline{AB}$ and $\overline{CD}$ intersecting at S. If AS = 2 in., SB = 3 in., SD = 4 in., then SC is

1. 1 inch
2. 1 1/2 inches
3. 2 inches
4. 2 1/2 inches
5. none of these

14. A point moves so that the sum of the squares of its distances from two given fixed lines, which are perpendicular to each other, is always the same positive number. What is the locus of the moving point?

1. 4 points
2. a square
3. two straight lines
4. a straight line
5. a circle

15. The area of a triangle having sides $\sqrt{3}$, $2\sqrt{2}$, $\sqrt{11}$ is:

1. $\frac{\sqrt{33}}{2}$
2. $\sqrt{66}$
3. $\sqrt{22}$
4. $\sqrt{6}$
5. impossible to determine

16. A circle is inscribed in an equilateral triangle. A second equilateral triangle is inscribed in the circle. The ratio of the areas of the two triangles is:

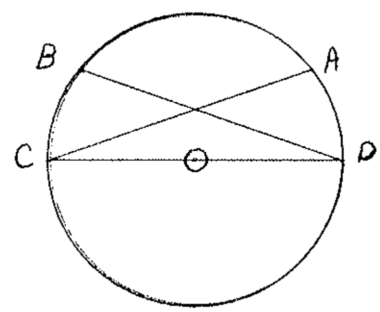
1. $1 : \sqrt{16}$
2. $1 : \sqrt{3}$
3. $2 : 3$
4. $5 : 7$
5. $\sqrt{2} : \sqrt{8}$

17. If a triangle is drawn on the surface of a sphere, the sum of its three internal angles is:

    1. equal to 180°
    2. less than 180°
    3. between 180° and 270°
    4. less than 540°
    5. less than or equal to 270°

18. In the given figure, circle 0 has diameter CD = 4 inches, AC = 3 inches, and $\measuredangle$ ACD $\cong$ $\measuredangle$ BDC. What is the measure of BC?

    1. 5 inches
    2. $\sqrt{2}$ inches
    3. $\sqrt{7}$ inches
    4. 3 1/2 inches
    5. 2 inches



19. Two circles have radii 3 inches and 6 inches respectively. If the line of centers is 18 inches, the length of the common internal tangent is:

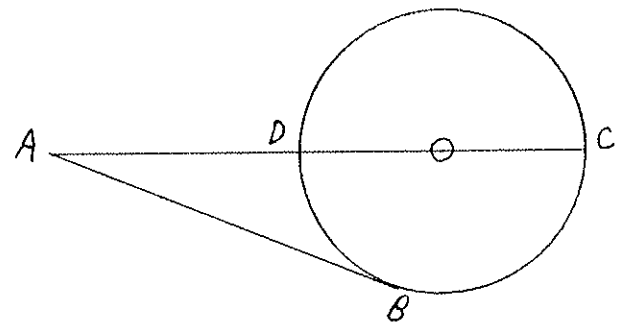
    1. 9 inches
    2. $9\sqrt{3}$ inches
    3. $6\sqrt{3}$ inches
    4. cannot be determined from the information given
    5. none of these

20. In a circle 0, diameter $\overline{CD}$ bisects $\measuredangle$ ACB where A and B are on 0. If m($\measuredangle$ACD) = 10°, then m($\widehat{ACB}$) is equal to:

    1. 20°
    2. 340°
    3. 40°
    4. 320°
    5. 80°

21. In the figure, $\overline{AB}$ is a tangent to the circle and $\overline{AC}$ is a secant through the center of the circle. If AB is 2 and the external segment AD of the secant is 1, what is the radius of the circle?

1. 2/3
2. 4/3
3. 3/2
4. 5/3
5. 1/2



22. The following statements refer to the concept of convexity as applied to planar regions. All but one of the statements below are false. Identify the true statement.

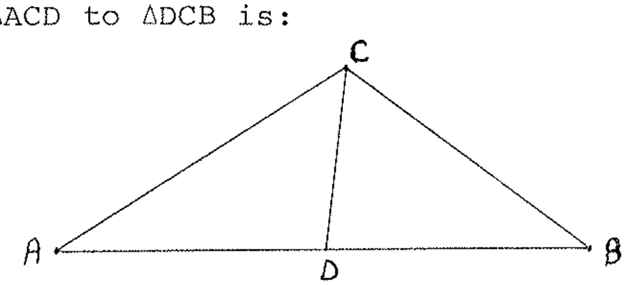
1. Every subset of a convex set is convex.
2. All subsets of non-convex sets are non-convex.
3. The intersection of two convex sets is convex.
4. The interior of an angle is not a convex set.
5. Since the interior of a circle is a convex set, the area of a circle is also a convex set in the plane.

23. If the circle $(x - a)^2 + y^2 = R^2$ is wholly contained within the circle $x^2 + y^2 = 1$, then

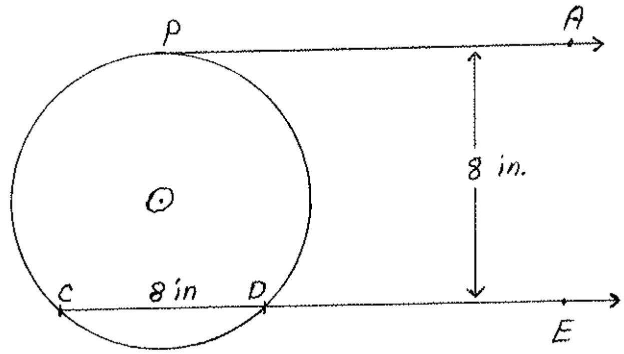
1. $a - R > 1$
2. $a + R < 1$
3. $a = R$
4. $a + R = 2$
5. none of the above

24. In ΔABC, as given in the figure, $AC = b$, $CB = a$, $\angle ACD \cong \angle DCB$, then the ratio of the area of ΔACD to ΔDCB is:

1. $b^2 : a^2$
2. $b : a/2$
3. $b/2 : a$
4. $b/2 : a/2$
5. none of the above



25. $\overrightarrow{PA}$ is tangent to circle 0. Chord $\overline{CD}$ is extended to E and $\overrightarrow{PA} \parallel \overrightarrow{CE}$. Based on the measures indicated in the figure, the length of the radius of the circle is:

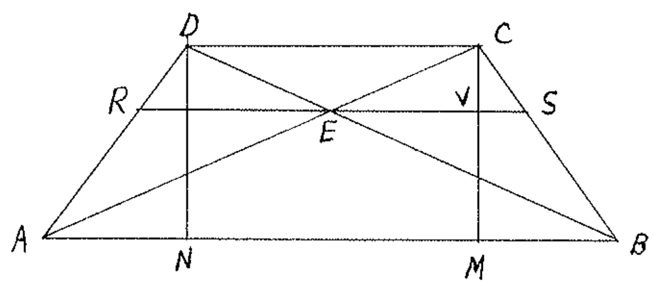


1. 6 in.
2. $6\sqrt{3}$ in.
3. 8 in.
4. 5 in.
5. $3\sqrt{2}$ in.

26. The locus of a point in the plane equidistant from two intersecting lines is

1. a line
2. a circle
3. two parallel lines
4. two intersecting lines
5. none of these

27. In the figure, ABCD is a trapezoid with AB = 15, DC = 9, and CB = AD = 5. $\overline{CM}$ and $\overline{DN}$ are perpendicular to $\overline{AB}$ and $\overline{RS}$ is parallel to $\overline{DC}$. Then MV equals:



1. 4
2. 2 1/2
3. 2
4. 3
5. none of these

28. Which one of the following is false?

1. If two parallel planes are cut by a third plane, their lines of intersection are parallel.

2. If each of two intersecting planes is parallel to a given line, the intersection of the planes is parallel to that line.

3. Two lines perpendicular to intersecting lines are parallel.

4. If three or more straight lines in a plane intercept congruent parts on one transversal, they intercept congruent parts on any transversal.

5. If two sides of a quadrilateral are equal and parallel, then the other two sides are also equal and parallel.

29. A circle of unit radius is inscribed within a rhombus. A diagonal of the rhombus divides the rhombus into two equilateral triangles. Find the length of a side of the rhombus.

1. $\frac{4\sqrt{3}}{3}$

2. $\frac{2\sqrt{3}}{3}$

3. $4/3$

4. $\frac{\sqrt{3}}{3}$

5. $4\sqrt{3}$

30. The largest possible area of a right triangle whose hypotenuse is 6 units in length is:

1. 9 sq. units

2. $6\sqrt{3}$ sq. units

3. $9\sqrt{2}$ sq. units

4. $6\sqrt{2}$ sq. units

5. none of these

31. In the circle shown, with center at 0, m(∡ AEB) = 15° and m(∡ COD) = 8°. The m(∡ CFD) is:

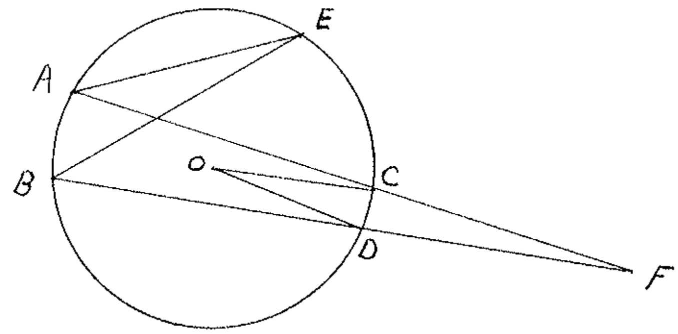
1. 15°

2. 18.5°

3. 8°

4. 11°

5. 22°



32. ABC is an equilateral triangle inscribed in the circle with center 0. If AB is 5, what is the area of the shaded region?

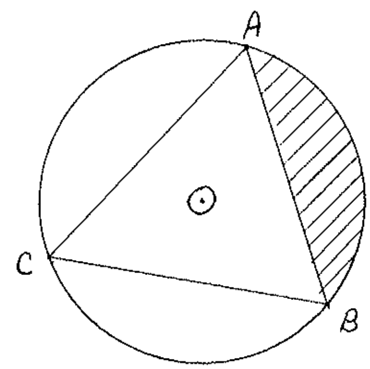
1. $25 \dfrac{(2\pi - 3\sqrt{3})}{18}$

2. $25 \dfrac{(4\pi - 3\sqrt{3})}{36}$

3. $25 \dfrac{(2\pi - 3\sqrt{3})}{9}$

4. $25 \dfrac{(4\pi - 3\sqrt{3})}{12}$

5. $25 \dfrac{(2\pi - 3\sqrt{3})}{6}$



33. ABCD is a trapezoid, with $\overline{AB}$ parallel to $\overline{CD}$. E and F are taken on $\overline{AD}$ and $\overline{BC}$ respectively, so that $\dfrac{AE}{ED} = \dfrac{BF}{FC} = \dfrac{2}{1}$.

If AB = 7, and DC = 10, find the length of EF.

1. 7 1/2

2. 8

3. 8 1/2

4. 9

5. none of these

34. In the accompanying figure $\overline{AP}$ and $\overline{BP}$ are tangent to the circle, from P the shortest distance to the circle is a; if the radius of the circle is r, what is the measure for $\overline{CD}$? Assume a and r are measured in the same units.

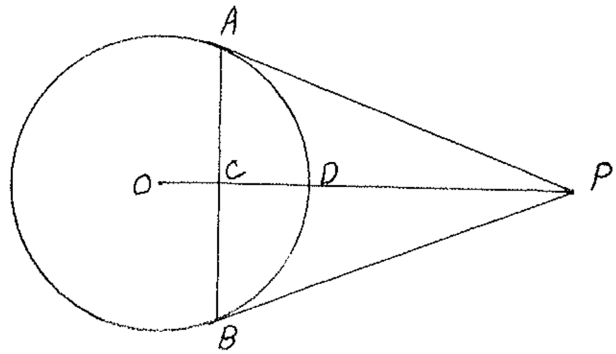
1. $\frac{a - \sqrt{r^2 - 4a}}{2}$

2. $\frac{ar}{a - r}$

3. $\frac{a + r}{ar}$

4. $\frac{ar}{a + r}$

5. $\frac{a - r}{ar}$



35. A cylinder with the diameter of its base equal to its height is inscribed in a sphere of radius a. The ratio of the volume of the cylinder to the volume of the sphere is:

1. 1/3

2. 4/3

3. 4a/3

4. $(3\sqrt{2})/8$

5. $3/(4\sqrt{2})$

36. In the drawing, the large circle has radius 5 and the small circle has radius 2. Find the area of the shaded portion.

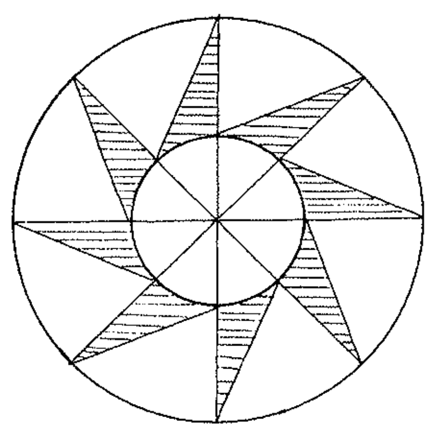
1. 8

2. $\frac{20 - 4\pi}{3}$

3. $8\sqrt{2} - 4\pi$

4. $20\sqrt{2} - 4\pi$

5. $\frac{2\pi}{3}$



37. If P is any point inside the equilateral triangle ABC, and AB = 6 in., the sum of the perpendicular distances from P to each of the 3 sides is:

1. $3\sqrt{2}$ in.
2. $3\sqrt{3}$ in.
3. $6\sqrt{3}$ in.
4. dependent on the position of P.
5. $\frac{3\sqrt{6}}{2}$ in.

38. A circle is inscribed in a triangle having sides of 6, 8, 10. The area of the inscribed circle is:

1. π
2. 4π
3. 2π
4. 3π
5. $4/3\pi$

39. The area of the intersection of $x^2 + y^2 \leq 1$ and $(x - 1)^2 + y^2 \leq 1$ is:

1. $\pi/2$
2. $\pi/3$
3. $2\pi/3$
4. $3\pi/4$
5. none of the above

40. In ΔABC, $AB = c$, $BC = a$, and $AC = b$. If D is a point on $\overline{AB}$ such that $\overline{CD}$ bisects $\measuredangle ACB$, then the length of $\overline{CD}$ is:

1. $\sqrt{ab\left[1 - \left(\frac{c}{a + b}\right)^2\right]}$
2. $\sqrt{ac\left[1 - \left(\frac{b}{a + b}\right)^2\right]}$
3. $\sqrt{bc\left[1 - \left(\frac{a}{a + b}\right)^2\right]}$
4. $\sqrt{ab\left[1 - \left(\frac{ab}{a + b}\right)^2\right]}$
5. none of these