FOURTEENTH ANNUAL MATHEMATICS CONTEST

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THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

GEOMETRY TEST

1970

Scoring Formula: 4R-W.

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DIRECTIONS:

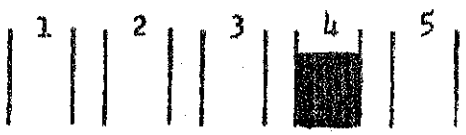
Do not open this booklet until you are told to do so.

This is a test of your competence in Geometry. For each of the 40 problems there are listed 5 possible answers. You are to work each problem and determine which is the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. A sample follows:

1. The sum of the measures of the angles of a triangle is:

(1) 360° (2) 45° (3) 90°

(4) 180° (5) 270°



The correct answer for the sample question is "180", which is answer (4); therefore, you should answer this question by making a heavy black mark under space 4 as indicated above.

If you should change your mind about an answer, be sure to erase completely. Avoid wild guessing, as wrong answers count against you. Do not mark more than one answer for any question. Make no stray marks of any kind on your answer sheet.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next page. The working time for the entire test is 80 minutes.

1. Any two equilateral triangles are:

   (1) congruent.

   (2) obtuse.

   (3) similar.

   (4) equal in area.

   (5) perspective from a point.

2. If two parallel lines are cut by a transversal, the alternate interior angles formed are:

   (1) complementary.

   (2) supplementary.

   (3) congruent.

   (4) acute.

   (5) obtuse.

3. The volume of a sphere of radius r is given by:

   (1) $V = \frac{1}{3}\pi r^3$

   (2) $V = 4\pi r^2$

   (3) $V = \frac{4}{3}\pi r^2$

   (4) $V = 4\pi r^3$

   (5) $V = \frac{4}{3}\pi r^3$

4. If AB=9, the intersection of a circle with center A, radius 5, and a circle with center B, radius 4, will contain the following number of points:

   (1) 0

   (2) 1

   (3) 2

   (4) 3

   (5) 4

5. If $\overline{AB} \perp \overline{BC}$, then

   (1) $AB < AC$

   (2) $AB < BC$

   (3) $AC < BC$

   (4) $AC < AB$

   (5) None of the above.

6. If the sides of one angle in a plane are perpendicular respectively to the sides of another angle in that plane, then the angles are:

   (1) congruent or supplementary.

   (2) supplementary or complementary.

   (3) complementary or congruent.

   (4) acute.

   (5) None of the above.

7. If a triangle is isosceles, then

   (1) the altitude is one third as long as the base.

   (2) the bisectors of the base angles are congruent.

   (3) the base angles are complementary.

   (4) all three angles are congruent.

   (5) all three angles are acute.

8. The sum of the measures of the interior angles of a regular pentagon is:

   (1) 180°

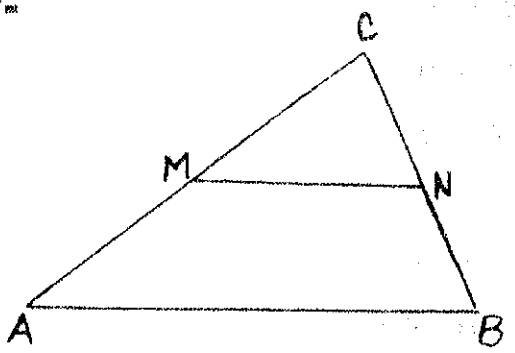
   (2) 360°

   (3) 540°

   (4) 1080°

   (5) none of the above.

9. If AM=MC and BN=NC, then MN=



(1) MC

(2) NB

(3) AB

(4) 1/2 (AC)

(5) 1/2 (AB)

10. The set of vertices of all isosceles triangles in a plane having a given segment $\overline{AB}$ as base is contained in:

(1) a circle.

(2) a line.

(3) an ellipse.

(4) a ray.

(5) a single point.

11. If the diagonals of a quadrilateral bisect each other, then the quadrilateral is generally known as a:

(1) square.

(2) rhombus.

(3) trapezoid.

(4) parallelogram.

(5) rectangle.

12. If the line L intersects a circle of radius r in points A and B, then the measure of the segment $\overline{AB}$ is:

(1) less than 2r.

(2) less than or equal to 2r.

(3) equal to 2r.

(4) greater than or equal to 2r.

(5) none of the above.

13. The locus of all points which lie inside or on a circle C and are equidistant from two given points on C is:

(1) a cord of C which does not necessarily contain the center of C.

(2) a radius of C.

(3) a circle concentric with C.

(4) a diameter of C.

(5) none of the above.

14. A square is symmetric with respect to the following number of lines:

(1) 1

(2) 2

(3) 3

(4) 4

(5) 5

15. The sum of the measures of the exterior angles of a regular polygon having 12 sides made by producing each of the sides in succession is:

(1) 360°

(2) 120°

(3) 1200°

(4) 180°

(5) none of the above.

16. If $\triangle ABC$ has a right angle at vertex C, then the altitude from vertex B

(1) separates $\triangle ABC$ into two similar triangles.

(2) separates $\triangle ABC$ into two congruent triangles.

(3) bisects side $\overline{AC}$.

(4) is perpendicular to side $\overline{AB}$.

(5) none of the above.

17. If the circumference of a circle is 4 inches, then the area of the circle is

    (1) 16π sq. in.

    (2) 4π sq. in.

    (3) $\pi/4$ sq. in.

    (4) $4/\pi$ sq. in.

    (5) $2/\pi$ sq. in.

18. A parallelopiped is a special kind of

    (1) polygon.

    (2) polynominal.

    (3) rectangle.

    (4) prism.

    (5) pyramid.

19. The point (in the plane) which is equidistant from all three vertices of a triangle is the

    (1) intersection of the medians.

    (2) intersection of the altitudes.

    (3) intersection of the internal angle bisectors.

    (4) intersection of the perpendicular bisectors of the sides.

    (5) center of the inscribed circle.

20. Two lines in space are parallel if

    (1) they are both perpendicular to the same line.

    (2) they are both parallel to the same line.

    (3) they are both parallel to the same plane.

    (4) they do not intersect.

    (5) any one of the above.

21. The contrapositive of the statement "If A, then B" is the statement:

(1) If (not A), then (not B).

(2) If B, then A.

(3) If (not B), then (not A).

(4) If (not B), then A.

(5) If (not A), then B.

22. A tetrahedron is a special kind of

(1) triangle.

(2) rectangle.

(3) prism.

(4) pyramid.

(5) polygon.

23. Two planes, perpendicular to the same plane, are

(1) parallel.

(2) perpendicular.

(3) identical.

(4) orthogonal.

(5) none of the above.

24. If points A, B, and D lie on a circle with center C so that $\angle ACB$ and $\angle ADB$ intercept the same arc, then the ratio $(m\angle ACB) : (m\angle ADB)$

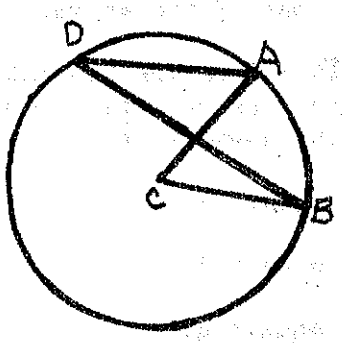
(1) equals 1:2

(2) equals 1:1

(3) equals 4:1

(4) equals 2:1

(5) varies with the position of D



25. One angle of a rhombus has measure 120°. If the shorter diagonal has measure 12 inches, then the measure of a side of the rhombus is

(1) 12 in.

(2) $4\sqrt{3}$ in.

(3) 13 in.

(4) $12\sqrt{3}$ in.

(5) $12\sqrt{2}$ in.

26. If ABCD is a trapezoid with AB=16, AD=6, DE=5, and DC=10, then the area of ABCD (in square units) is:

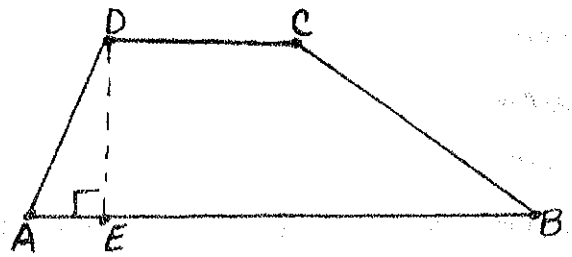
(1) 200

(2) 65

(3) 39

(4) 80

(5) none of the above.



27. A straight line can be described as the set of all points in space which are equidistant from

(1) two intersecting lines.

(2) two intersecting planes.

(3) two parallel lines.

(4) two parallel planes.

(5) none of the above.

28. If $\overline{AB}$ and $\overline{BC}$ are tangent to the circle with center O at points A and C respectively, and $m\angle ABC=120°$ and OB=12, then the length of the segment $\overline{AB}$

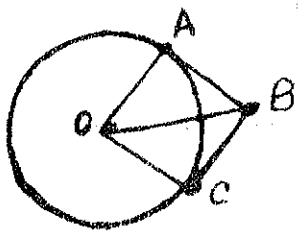
(1) equals 2.

(2) equals 3.

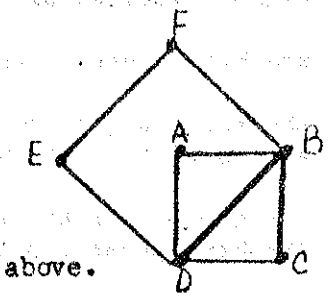
(3) equals 4.

(4) equals 6.

(5) cannot be determined from this information.



29. If ABCD and BDEF are squares, and the area of ABCD is 3 sq. in., then the area of BDEF



(1) equals $2\sqrt{3}$ sq. in.

(2) equals 6 sq. in.

(3) equals 4 sq. in.

(4) can be computed but equals none of the above.

(5) cannot be computed from the given information.

30. If a sphere and a circle have the same center, then their intersection <u>may</u> contain

(1) exactly one point.

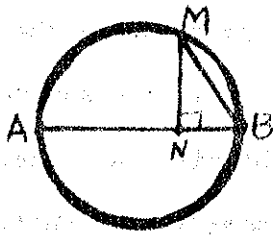
(2) exactly two points.

(3) exactly four points.

(4) an infinite number of points.

(5) any one of the above.

31. If diameter $\overline{AB}$ has measure 18, $\overline{MN} \perp \overline{AB}$, and NB=8, then MB=



(1) 8

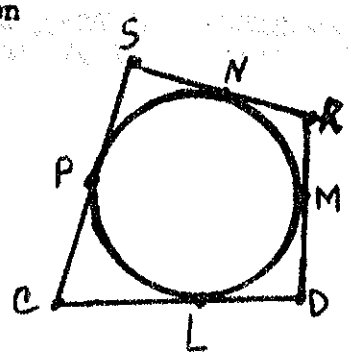
(2) 10

(3) 12

(4) 14

(5) 16

32. If the sides of quadrilateral CDRS are tangent to a circle at L, M, N, and P as in the figure, then



(1) SR+CD < SC+RD

(2) SR+CD > SC+RD

(3) SR+CD = SC+RD

(4) SR+CD = SC+SR

(5) none of the above.

33. Two right triangles are congruent if

(1) the hypotenuse of one is congruent to the hypotenuse of the other.

(2) the acute angles of one are congruent respectively to the acute angles of the other.

(3) the hypotenuse and a leg of one are congruent respectively to the hypotenuse and a leg of the other.

(4) their areas are equal.

(5) none of the above.

34. The intersection of the three altitudes of a triangle

(1) lies 2/3 of the way from any vertex to the opposite side.

(2) lies 1/3 of the way from any vertex to the opposite side.

(3) is the center of the inscribed circle.

(4) is the center of the circumscribed circle.

(5) none of the above.

35. The volume of a circular cone is the product of the altitude and the area of the base if

(1) the radius of the base equals the altitude.

(2) the area of the base is one square unit.

(3) the cone is a right circular cone.

(4) the altitude has measure 1/3 unit.

(5) none of the above.

36. If $\triangle ABC \sim \triangle DEF$, $AB/DE=3$, and the area of $\triangle DEF$ is 27 square units, then the area of $\triangle ABC$ (in square units) is

(1) 81

(2) 243

(3) 9

(4) 3

(5) 162

37. If two circles have radii r_1 and r_2 respectively (with $r_1 > r_2$) and are tangent, then the distance between their centers is

(1) $(r_1)(r_2)$

(2) $r_1 - r_2$

(3) $|r_1 - r_2|$

(4) $r_1 + r_2$

(5) a root of the equation $x^2 - 2r_1x + (r_1^2 - r_2^2) = 0$

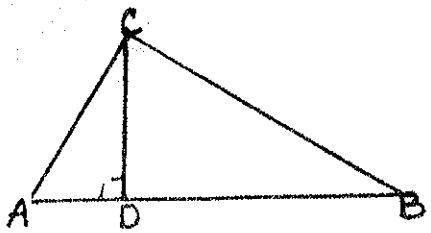
38. In $\triangle ABC$, $AB = 14$, $BC = 15$, $AC = 13$; find the length of the altitude $\overline{CD}$.

(1) 9

(2) 10

(3) 11

(4) 12



(5) none of the above.

39. In a plane, $\overline{AB}$ and $\overline{AC}$ are tangents to a circle with center O. $m\angle BAC = 60°$. If the radius $OB = r$, and $AB = 4$, then

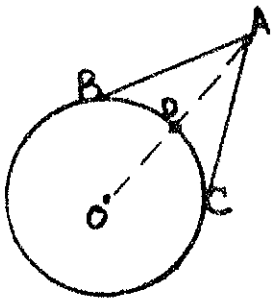
(1) $AD = 2r$

(2) $AD < r$

(3) $AD = r\sqrt{3}$

(4) $AD = r$

(5) none of the above.



40. In the figure, the sphere, with radius r, is inscribed in the right circular cone. The center of the sphere is C. The measures of the angles between the altitude of the cone and the radii of the sphere to the points of tangency are 60°. Find the volume of the cone in terms of r.

(1) πr^3

(2) $2\pi r^3$

(3) $3\pi r^3$

(4) $4\pi r^3$

(5) $6\pi r^3$