THIRTEENTH ANNUAL MATHEMATICS CONTEST

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GEOMETRY TEST

1969

Scoring Formula: 4R - W

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DIRECTIONS:

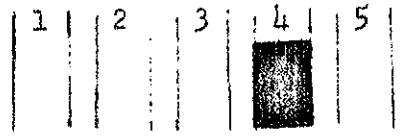
Do not open this booklet until you are told to do so.

This is a test of your competence in Geometry. For each of the 40 problems there are listed 5 possible answers. You are to work each problem and determine which is the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. A sample follows:

1. The sum of the measures of the angles of a triangle is:

(1) 360° (2) 45° (3) 90°

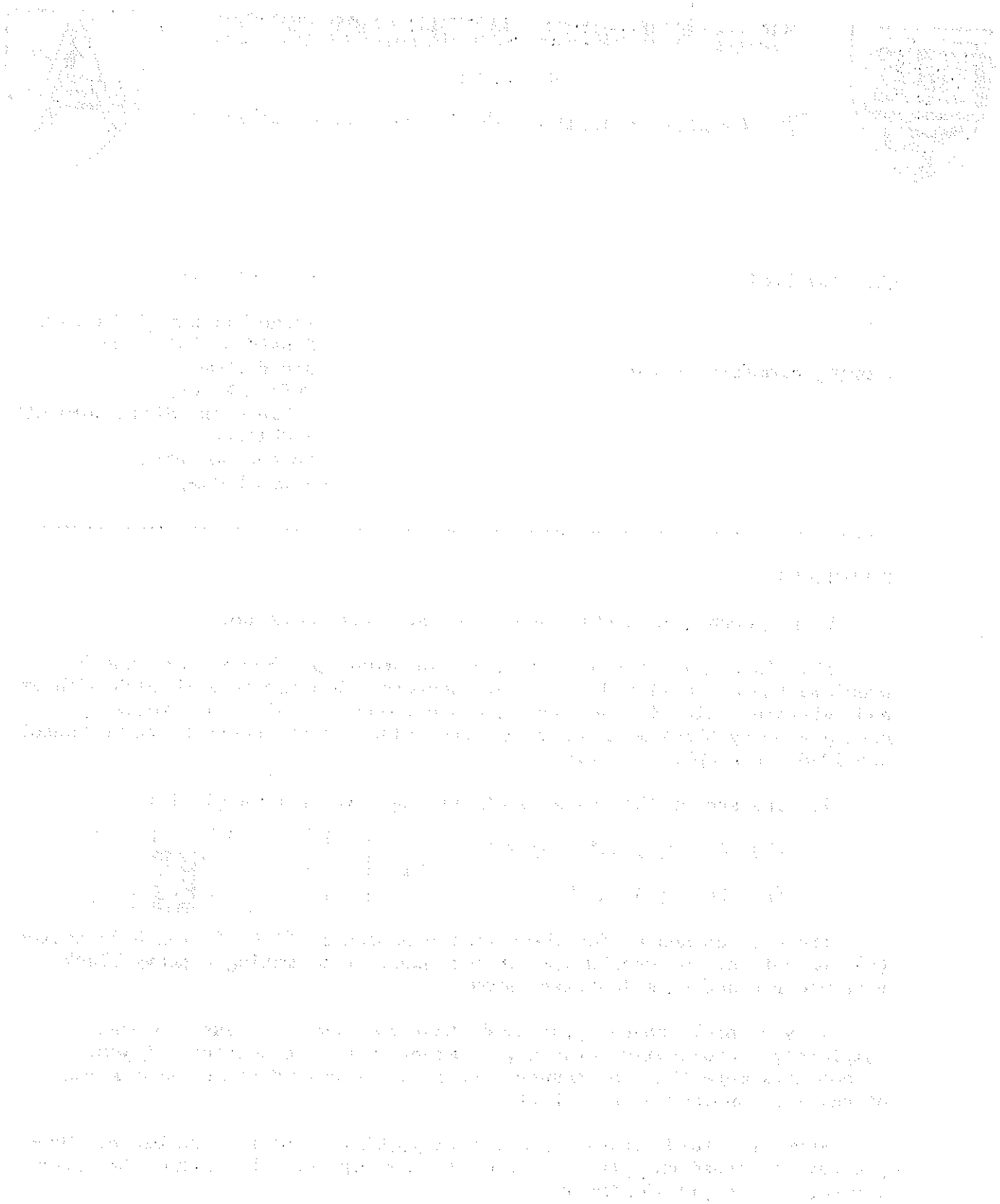
(4) 180° (5) 270°



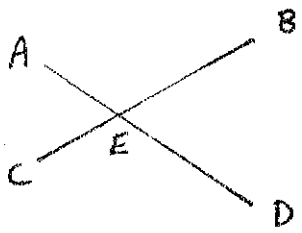
The correct answer for the sample question is "180", which is answer (4); therefore, you should answer this question by making a heavy black mark under space 4 as indicated above.

If you should change your mind about an answer, be sure to erase completely. Avoid wild guessing, as wrong answers count against you. Do not mark more than one answer for any question. Make no stray marks of any kind on your answer sheet.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next page. The working time for the entire test is 80 minutes.



5. In the figure, the measures of $\overset{\frown}{AC}$ and $\overset{\frown}{BD}$ are respectively 30° and 90°. $\angle BEA$ is measured by



(1) 90°
(2) 60°
(3) 120°
(4) 75°
(5) none of these

6. A rectangular parallelopiped has 3 edges whose measures are represented by a,b,and c respectively. Any diagonal (not containing as many as two points of the same face) has length of

(1) $\sqrt{a^2 + b^2}$

(2) $\sqrt{a^2 + b^2 + c^2}$

(3) $\sqrt{2a + 2b + 2c}$

(4) $\frac{ab + bc + ac}{3}$

(5) none of these

7. Consider any plane triangle denoted by the symbol, ΔABC; which of the following statements is <u>correct</u> for the given ΔABC under consideration?

(1) it consists of three non-collinear points such that $\overline{AB} \cap \overline{BC} \cap \overline{AC}$ is ΔABC.

(2) ΔABC contains none of its angles.

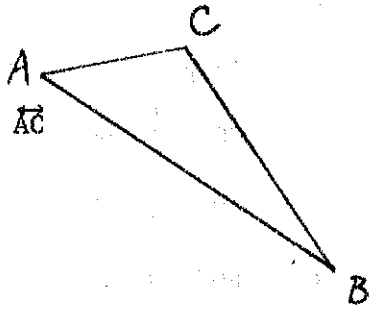
(3) the sides of its angles are segments.

(4) the sides of the triangle are rays

(5) none of the above

8. The interior of ΔABC is

(1) the union of the half-planes bounded above by $\overleftrightarrow{AC}$ and $\overleftrightarrow{BC}$ and below by $\overleftrightarrow{AB}$.



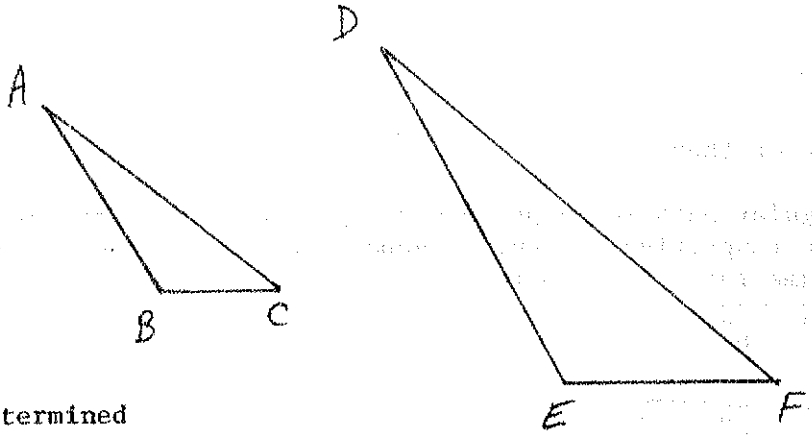
(2) the intersection of the half planes bounded above by $\overleftrightarrow{AC}$ and $\overleftrightarrow{BC}$ and below by $\overleftrightarrow{AB}$

(3) $\{\angle CAB\} \cup \{\angle CBA\}$

(4) $\{$the interior of $\angle CAB\} \cup \{$the interior of $\angle CBA\}$

(5) $\overline{AC} \cup \overline{CB} \cup \overline{BA}$

1. $\triangle ABC$ is similar to $\triangle DEF$ in figure. If AB = 3 and DE = 5, what is the ratio of the respective areas?



(1) $\frac{9}{25}$

(2) $\frac{3}{5}$

(3) $\frac{1}{4}$

(4) $\frac{5}{8}$

(5) cannot be determined by given data.

2. The measure of the altitude from the vertex angle of an isosceles triangle is 15 inches. What is the measure of the line segment from the vertex of the isosceles triangle to the point of intersection of the angle bisectors of the triangle?

(1) $7\frac{1}{2}$ "

(2) 12"

(3) 10"

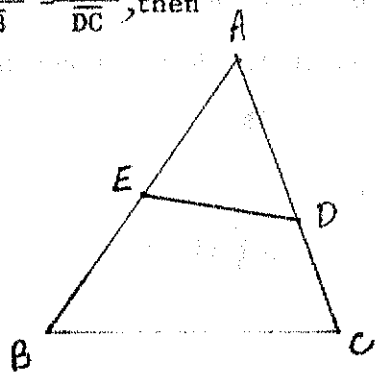
(4) 6"

(5) cannot be determined from given data.

3. For which of the following types of triangles does the incenter and circumcenter coincide?

(1) right
(2) acute
(3) equilateral
(4) isosceles
(5) all of these

4. In triangle ABC of the figure; if $\frac{\overline{AE}}{\overline{EB}} = \frac{\overline{AD}}{\overline{DC}}$, then



(1) $\overline{ED} = \frac{1}{2} \cdot \overline{BC}$

(2) $\overline{ED} = \frac{2}{3} \cdot \overline{BC}$

(3) $\overline{ED} \parallel \overline{BC}$

(4) $\overline{AB} = \overline{AC}$

(5) $\overline{AE} = \frac{1}{2} \cdot \overline{AB}$

9. In $\triangle ABC$, $\{\angle CAB\} \cap \{\angle CBA\}$ =

(1) $\overline{AB} \cup \{C\}$

(2) $\overrightarrow{AB} \cup \overrightarrow{AC}$

(3) $\{A\} \cup \{B\} \cup \{C\}$

(4) $\overrightarrow{AB} \cup \{C\}$

(5) $\triangle ABC$

10. The number of different planes that can be determined by 15 distinct points, no four of which are co-planar, is

(1) 15
(2) 210
(3) 225
(4) 455
(5) 2730

11. The altitudes of a triangle are concurrent at a point called the

(1) incenter
(2) centroid
(3) circumcenter
(4) orthocenter
(5) alticenter

12. The line AB cuts three parallel planes in the points A, E and B, respectively. The line CD cuts the same planes in the points C, F and D, respectively. If $\overline{AE}$ has length 3'', $\overline{EB}$ has length 4'' and $\overline{CD}$ has length 6'', the length of $\overline{CF}$ is

(1) $\frac{9}{2}$

(2) $\frac{18}{7}$

(3) $\frac{24}{7}$

(4) $\frac{7}{4}$

(5) $\frac{7}{2}$

13. Two lines that have no point in common and do not lie in the same plane are said to be:

(1) oblique lines
(2) parallel lines
(3) perpendicular lines
(4) skew lines
(5) slant lines

14. SAS, ASA, and SSS refer to statements of sufficient conditions for congruency of triangles. Consider a structurally axiomatic treatment of geometry in which the axiom set contains elements that are consistent and independent of one another. If this axiom set had for one of its elements SAS, we would be obliged to consider the ASA and SSS statements as

(1) theorems
(2) postulates
(3) axioms
(4) definitions
(5) undefined statements

15. Consider the theorem: If two lines intersect, then their union lies in exactly one plane. If L and L* represent the two lines, which of the following could not be a step in the direct proof of the theorem?

(1) $L \cap L^*$ is a point P*.

(2) L* contains a point P such that $P \neq P^*$.

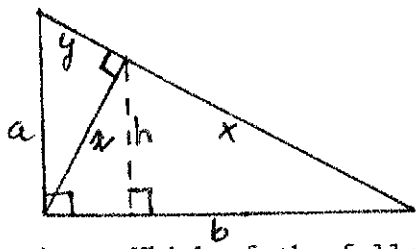
(3) There exists a plane K such that $L \cup \{P\} \subset K$

(4) $L \cup L^* \not\subset K$.

(5) No other plane different from K contains $L \cup L^*$.

16. In the figure, $a \cdot b = (x + y) \cdot$?

(1) h+z
(2) y
(3) a-h
(4) z
(5) h



17. ABCD is a rectangle and E is an interior point. Which of the following is always true? a,b,c,d refer to the measures of the lengths of $\overline{AE}$, $\overline{BE}$, $\overline{CE}$ and $\overline{DE}$ respectively.

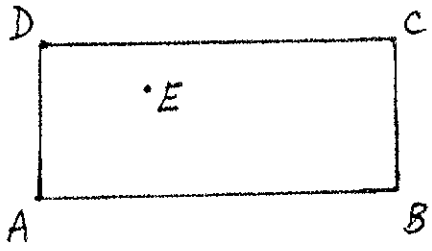
(1) $a + c = b + d$

(2) $a^2 + c^2 = b^2 + d^2$

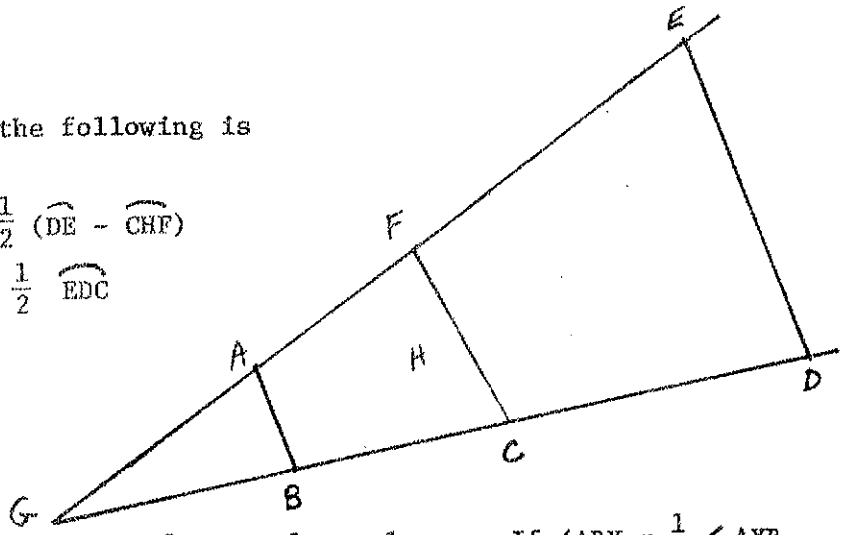
(3) $\frac{a}{c} = \frac{d}{b}$

(4) $a \cdot c = d \cdot b$

(5) none of these



18. In the figure which of the following is not necessarily true?



(1) $\angle G$ is measured by $\frac{1}{2}(\overset{\frown}{DE} - \overset{\frown}{CHF})$

(2) $\angle ABC$ is measured by $\frac{1}{2}\ \overset{\frown}{EDC}$

(3) $ED \parallel AB$

(4) $m\angle DEF = m\angle FCB$

(5) none of these

19. AX and BX are 2 adjacent sides of a regular polygon. If $\angle ABX = \frac{1}{3}\angle AXB$, how many sides has the polygon?

(1) 3
(2) 4
(3) 5
(4) 6
(5) 7

20. The diagonals of a _____ do not necessarily bisect each other.

(1) rhombus
(2) parallelogram
(3) parallelopiped
(4) trapezoid
(5) none of these

21. The sides of a triangle are 5", 9", and 12". What is the length of the altitude to the 12" side?

(1) $\frac{55}{12}$

(2) $\sqrt{\frac{55}{12}}$

(3) $\frac{104}{9}$

(4) $\sqrt{\frac{104}{9}}$

(5) none of these

22. A,B,C, and D are 4 distinct points in order on the circumference of a circle. $\overset{\frown}{AB}$, $\overset{\frown}{BC}$, $\overset{\frown}{CD}$ and $\overset{\frown}{DA}$ are bisected respectively by points E,F,G and H. Then

(1) $EG \perp FH$
(2) $EG \parallel FH$
(3) $EG = FH$
(4) EG and FH do not meet
(5) cannot be determined from given data

23. If a circle is inscribed in a 3-4-5 right triangle, then the diameter of the circle is

(1) $1\frac{1}{2}$"

(2) $1\frac{3}{4}$"

(3) 2"

(4) $2\frac{1}{2}$"

(5) cannot be determined from given data.

24. A piece of paper 24" x 18" is folded once in such a way that 2 diagonally opposite corners are made to coincide. What is the length of the crease?

(1) $\sqrt{800}$"

(2) $\sqrt{45}$"

(3) $22\frac{1}{2}$"

(4) 45"

(5) none of these

25. A set of points S is said to be convex, if for $x,y \in S$, $x \neq y$ always implies $\overline{xy} \subset S$ (i.e. the line segment joining x and y is contained in S) According to this definition which of the following sets is not always convex?

(1) line
(2) triangle
(3) half plane
(4) half space
(5) quadrilateral

26. Euclidean geometry is the study of invariants under certain transformations. Which of the following is not one of these transformations?

(1) projections
(2) rotations
(3) reflections
(4) parallel shifts (or translations)
(5) expansions

27. An "If-then" statement is called a "conditional" or "implication". The _____ of a false conditional $\overset{\text{is}}{\text{are}}$ always true.

(1) contrapositive and converse
(2) converse and inverse
(3) inverse and contrapositive
(4) contrapositive
(5) none of these

28. $r = -3/\cos\theta$ is the polar equation of

(1) a line
(2) a circle
(3) an ellipse
(4) an hyperbola
(5) a parabola

29. A point is 5 inches from a line. The locus of points 4 inches from the given point and 3 inches from the given line is

(1) a circle
(2) 2 points
(3) empty
(4) a plane
(5) none of these

30. Which of the following is always true?

(1) A line perpendicular to a line in a plane is perpendicular to the plane.

(2) A plane parallel to a line in a plane is parallel to the plane.

(3) The set of points equidistant from two fixed points is a line which is the perpendicular bisector of the line segment joining the two points.

(4) A plane perpendicular to a line in a plane is perpendicular to the plane.

(5) none of these

31. $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ are altitudes of ΔABC. Their intersection P is also the intersection of the _____ of ΔDEF. ΔABC is an acute triangle.

(1) altitudes
(2) medians
(3) angle bisectors
(4) perpendicular besectors of the sides
(5) none of these

32. In a plane, ΔABC has vertices A (-2,3), B (4,5) and C (2,-3). Find the point of intersection of the perpendicular bisectors of the sides of the triangle.

(1) $(2, 1\frac{1}{3})$

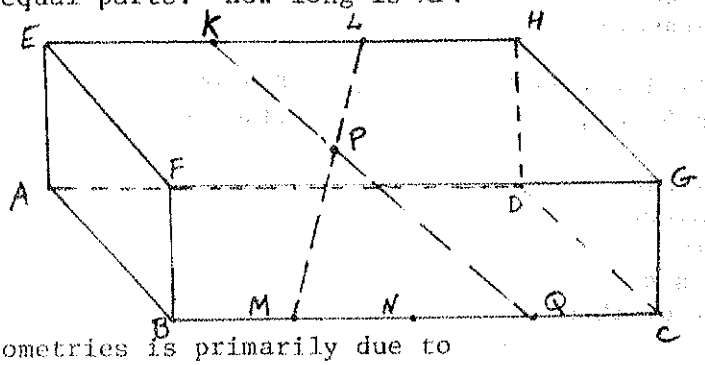
(2) $(2\frac{1}{10}, 1\frac{2}{5})$

(3) $(2\frac{1}{9}, 1\frac{4}{9})$

(4) $(1\frac{1}{12}, 1\frac{5}{12})$

(5) $(1\frac{10}{11}, 1\frac{3}{11})$

33. In the rectangular solid, BC = 12; GC = $2\sqrt{2}$; DC = 4; K and L trisect $\overline{EH}$; M, N and Q divide BC into 4 equal parts. How long is KP?



(1) 2.4
(2) 2.6
(3) 2.8
(4) 3.0
(5) 3.2

34. The rise of non-Euclidean geometries is primarily due to

(1) Einstein.
(2) The refutation of the uniqueness of the Euclidean parallel postulate.
(3) spherical geometry
(4) a "proof" of the Euclidean parallel postulate.
(5) an internal failure in the Euclidean model of geometry.

35. In a plane the set of points at a distance of 5 units from the fixed point (2,-3) is defined by the following equation.

(1) $x^2 - 4x + y^2 + 6y = 12$

(2) $x^2 - 4x + y^2 + 6y = 25$

(3) $x^2 + 4x + y^2 - 6y - 25 = 0$

(4) $(x+2)^2 + (y-3)^2 = 25$

(5) none of these

36. If the diagonals of an isosceles trapezoid are drawn, how many pairs of congruent triangles are formed?

(1) 0 (2) 1 (3) 2 (4) 3 (5) 4

37. If triangle ABC is a right triangle with the right angle at C and with $\overline{BC} = 1$, $\overline{CA} = \sqrt{3}$ and $\overline{AB} = 2$, then $m\angle B =$

(1) 45°

(2) 30°

(3) $2(m\angle A)$

(4) $\frac{1}{2}(m\angle A)$

(5) none of these

38. The sphere $\{(x,y,z): x^2 + y^2 + z^2 = 6x + 2y + 15\}$ has as its center the point

(1) $(4,0,\sqrt{23})$

(2) $(-3,1,0)$

(3) $(3,1,5)$

(4) the origin

(5) none of these

39. A right circular cone is circumscribed by a sphere and the base of the cone contains the center of the sphere. The ratio of the total surface area of the cone to that of the sphere is closer to

(1) $\frac{1}{2}$

(2) $\frac{2}{3}$

(3) $\frac{3}{5}$

(4) $\frac{3}{4}$

(5) $\frac{5}{8}$

40. How many regular polyhedra are there?

(1) 0 (2) 3 (3) 5 (4) 7 (5) 9