TWELFTH ANNUAL MATHEMATICS CONTEST

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THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

GEOMETRY TEST

1968

Scoring Formula: 4R - W.

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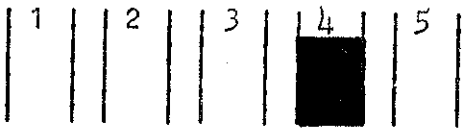
DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in Geometry. For each of the 40 problems there are listed 5 possible answers. You are to work each problem and determine which is the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. A sample follows:

1. The sum of the measures of the angles of a triangle is:

(1) 360° (2) 45° (3) 90°
(4) 180° (5) 270°

1. 

The correct answer for the sample question is "180", which is answer (4); therefore, you should answer this question by making a heavy black mark under space 4 as indicated above.

If you should change your mind about an answer, be sure to erase completely. Avoid wild guessing, as wrong answers count against you. Do not mark more than one answer for any question. Make no stray marks of any kind on your answer sheet.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next page. The working time for the entire test is 80 minutes.

1. In the figure, if AB is parallel to CD, the length of line segment x in terms of a and b is

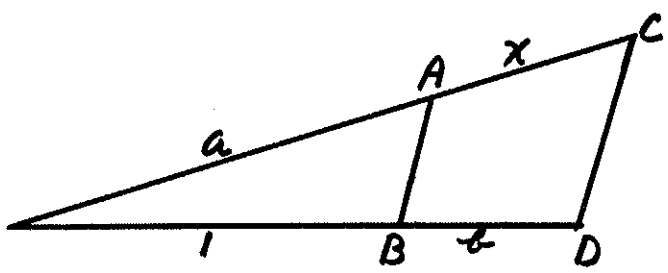
(1) $\frac{a}{b}$.

(2) $\frac{b}{a}$.

(3) ab.

(4) $\sqrt{a}$.

(5) $\sqrt{b}$.



2. For a certain right circular cylinder, the radius is doubled and the height halved. The ratio of the volume of the new cylinder to that of the old is

(1) 1:2. (2) 2:1. (3) 1:4. (4) 4:1. (5) none of these.

3. In the circle, chords AB and CD intersect at P. If the lengths of the line segments are AP = 2, PB = 3, CD = $\frac{11}{2}$, and PD $<$ 2, then PD equals

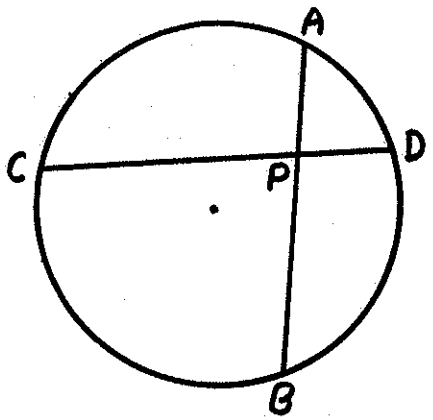
(1) $\frac{3}{2}$.

(2) $\frac{4}{3}$.

(3) $\frac{5}{4}$.

(4) $\frac{6}{5}$.

(5) $\frac{7}{6}$.



4. In the figure, PQ and PR are tangent to a circle with center at O. If $\angle$RPQ measures 70°, $\angle$QOP measures

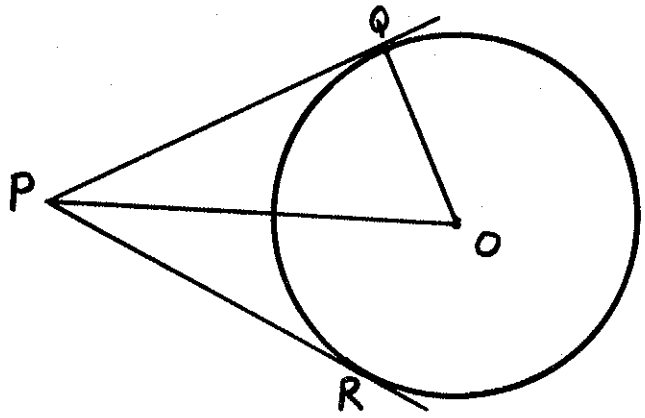
(1) 20°.

(2) 30°.

(3) 35°.

(4) 55°.

(5) 70°.



5. The locus of the vertex of a right triangle with a given hypotenuse as base is

   (1) an ellipse.

   (2) a hyperbola.

   (3) a circle.

   (4) a straight line.

   (5) a triangle.

6. Triangles ABC and A'B'C' are similar, with corresponding sides BC = 5 and B'C' = 2. The ratio of the area of triangle ABC to the area of triangle A'B'C' is

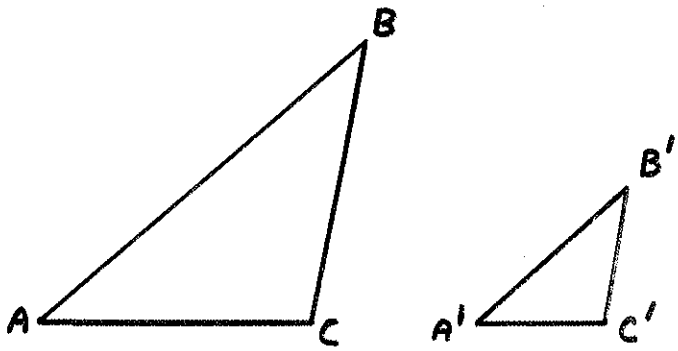
   (1) 1.25

   (2) 2.50

   (3) 5.00

   (4) 6.25

   (5) 10.00.



7. If the diagonals of a quadrilateral bisect each other, the quadrilateral is always a

   (1) square.

   (2) rhombus.

   (3) rectangle.

   (4) trapezoid.

   (5) parallelogram.

8. The sum of the measures of the exterior angles of a convex polygon, formed by extending each side in succession is

   (1) 1 right angle.

   (2) 1 straight angle.

   (3) 2 right angles.

   (4) 2 straight angles.

   (5) 540°

9. Two sides of a triangle are 3 and 5. The third side can not be

   (1) 3.

   (2) 5.

   (3) 6.

   (4) 7.

   (5) 9.

10. Which pair of the following angles are not necessarily equal?

   (1) vertical angles

   (2) complements of the same angles

   (3) supplements of equal angles

   (4) adjacent angles

   (5) base angles of an isoceles triangle

11. Which of the following does not constitute a proof of congruence of two triangles?

   (1) Three sides of one triangle equal respectively three sides of the other triangle.

   (2) Two sides and an opposite angle of one triangle are equal to two sides and an opposite angle of the other triangle.

   (3) Two sides and the included angle of one triangle equal respectively two sides and the included angle of the other triangle.

   (4) Two angles and a side of one triangle equal respectively two angles and a side of the other triangle.

   (5) None of the above.

12. Two angles whose sides are respectively perpendicular are always

   (1) equal.

   (2) supplementary.

   (3) either equal or supplementary.

   (4) both equal and supplementary.

   (5) either equal or complementary.

13. The radii of two pulley wheels are 5 inches and 1 inch, respectively, and the line of centers is 8 inches. The length of a belt (in inches) passed around the wheels is

(1) $\frac{19}{3} + 2\sqrt{3}$

(2) $\frac{19}{3} + 8\sqrt{3}$

(3) $\frac{22}{3} + 8\sqrt{3}$

(4) $\frac{22}{3} + 2\sqrt{3}$

(5) none of the above.

14. Three circles are drawn so that each is externally tangent to the other two. The lines of centers are 10", 14", and 18" respectively. The radius of the largest circle is

(1) 3" (2) 7" (3) 11" (4) 14". (5) It cannot be determined.

15. Find the length of the diameter of the circle denoted by d.

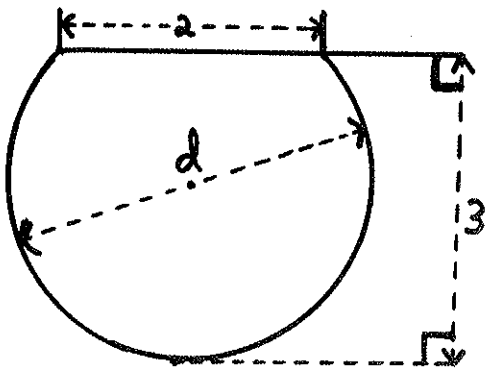
(1) $\frac{10}{3}$

(2) $\frac{16}{5}$

(3) $\frac{19}{6}$

(4) $\frac{7}{2}$

(5) $\frac{28}{9}$.



16. If two spherical surfaces of unequal radii intersect in at least two points, the intersection is

(1) indeterminate.

(2) a shape like a football.

(3) an ellipse.

(4) a circle.

(5) another spherical surface.

17. A right parallelepiped has a rhombus as a base. The length of each edge equals e. One of the angles of the base measures 60°. The length of the longest diagonal is

(1) $2e$ (2) $e\sqrt{2}$ (3) $e\sqrt{3}$ (4) $\frac{3}{2}e$. (5) Sufficient information is not given.

18. In the figure, line segments AC, DC and DB are equal in length. If angle ACD measures 20°, angle ACB measures

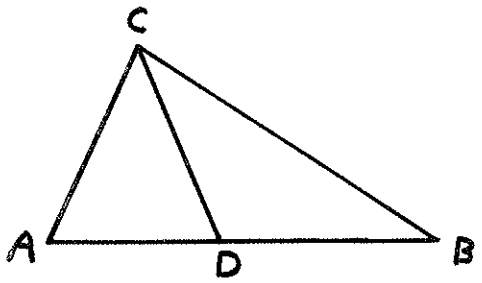
(1) 40°

(2) 50°

(3) 60°

(4) 70°

(5) 80°.



19. ABC is a right triangle with right angle at C. D is a point of AB such that CD is perpendicular to AB. Then

(1) $\frac{CD}{DB} = \frac{AC}{AD}$

(2) $\frac{CD}{DB} = \frac{CD}{AD}$

(3) $\frac{AB}{AC} = \frac{BC}{CD}$

(4) $\frac{AB}{BC} = \frac{BC}{CD}$

(5) $\frac{AC}{AB} = \frac{BC}{AC}$.

20. The point equidistant from the vertices of a triangle is the

(1) intersection of the medians.

(2) intersection of the internal angle bisectors.

(3) intersection of the altitudes.

(4) intersection of the external angle bisectors.

(5) intersection of the perpendicular bisectors of the sides.

21. Let S_1 and S_2 be two spheres with radii r_1 and r_2, respectively. If S_2 is to hold 5 times as much liquid as S_1, then

(1) $r_1 = \sqrt[3]{5}\, r_2$

(2) $r_2 = \sqrt[3]{5}\, r_1$

(3) $r_1 = \sqrt{5}\, r_2$

(4) $r_2 = \sqrt{5}\, r_1$

(5) $r_1 = 5r_2$

22. In a simple (convex) polyhedron having 7 vertices and 12 edges, the number of faces is

(1) 5 (2) 7 (3) 9 (4) 17 (5) 19.

23. In $\triangle ABC$, BD is the median to side AC. If the length of BD is $\frac{1}{2}$ the length of AC, then triangle ABC is

(1) equilateral.

(2) right.

(3) acute.

(4) obtuse.

(5) none of the above.

24. Triangle ABC is similar to triangle DEF with $\angle A$ congruent to $\angle D$ and $\angle B$ congruent to $\angle E$. If $\cong$ means "is congruent to", then

(1) $\triangle$ ACB is always congruent to $\triangle$ DEF.

(2) $\triangle$ ABC is congruent to $\triangle$ DEF if $AC \cong DE$.

(3) $\triangle$ ABC is congruent to $\triangle$ DEF if $BC \cong EF$.

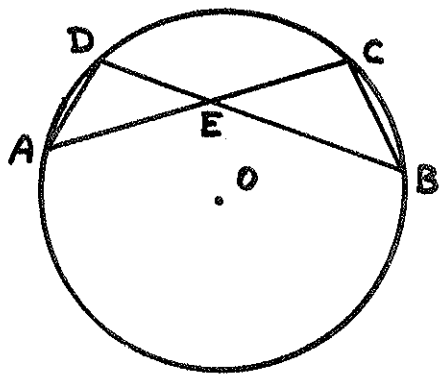
(4) $\triangle$ ABC is congruent to $\triangle$ DEF if $\angle C \cong \angle F$.

(5) $\triangle$ ABC is never congruent to $\triangle$ DEF.

25. The maximum number of distinct planes that can be determined by 10 distinct points is

(1) 120 (2) 10 (3) 45 (4) 1 (5) indeterminate.

26. In the figure, chords AC and DB of circle O intersect at E. Then



(1) $\triangle ADE \sim \triangle BCE$.

(2) $\frac{ED}{EC} = \frac{EA}{EB}$.

(3) $\frac{EB}{EC} = \frac{ED}{EA}$.

(4) $\overset{\frown}{AD}$ is congruent to $\overset{\frown}{BC}$.

(5) AC is congruent to BD.

27. If a rhombus may have at least one right angle, and if in quadrilateral AEBF, AB is the perpendicular bisector of EF and EF is the perpendicular bisector of AB, then the quadrilateral AEBF is always a

(1) square.

(2) rhombus.

(3) rhombus but not a square.

(4) parallelogram but not a rhombus.

(5) rhomboid but not a rhombus.

28. Let the coordinates of points A, B, C, and D be (2,2), (3,7), (-9,-1), and (-4,-2), respectively. Then quadrilateral ABCD is a

(1) trapezoid with opposite angles complementary.

(2) trapezoid with $AB \parallel CD$.

(3) quadrilateral which is not a trapezoid.

(4) trapezoid with AB congruent to CD.

(5) trapezoid with BC congruent to AD.

29. The contrapositive of a theorem

(1) denies the stated conclusion.

(2) affirms the hypothesis.

(3) is equivalent to the theorem.

(4) is not logically connected with the theorem.

(5) none of the above.

30. Given a circle with diameter AB and chord CD which intersects AB. If CD is not a diameter then

(1) ADBC forms a rectangle.

(2) $\angle$ ADC is congruent to $\angle$ BCD.

(3) AC $\parallel$ DB.

(4) $\angle$ BCA is congruent to $\angle$ ADB.

(5) figure ACBD is a trapezoid.

31. If a quadrilateral is inscribed in a circle, then

(1) consecutive angles are equal.

(2) consecutive angles are complementary.

(3) opposite angles are supplementary.

(4) the figure is a parallelogram.

(5) consecutive angles are supplementary.

32. The intersection of the three altitudes of a triangle

(1) is always inside the triangle.

(2) lies $\frac{2}{3}$ of the way from any vertex to the opposite side.

(3) is the center of the inscribed circle.

(4) is not always unique.

(5) may be outside the triangle.

33. Given parallel lines M, M' containing points, A, B, C, D, and straight line segments AB, AC, BD, CD, then

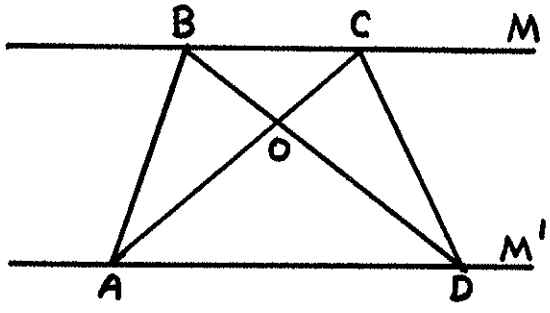
(1) $\triangle$ ABD is congruent to $\triangle$ ACD.

(2) $\triangle$ AOB is similar to $\triangle$ DOC.

(3) BC = $\frac{AD}{2}$.

(4) $\triangle$ AOD is isosceles.

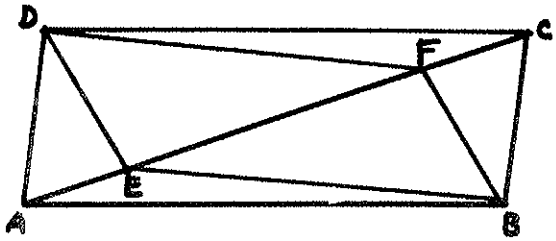
(5) $\triangle$ DBA has the same area as $\triangle$ ACD.



34. The number of faces of a regular polyhedron cannot be

(1) 4 (2) 6 (3) 8 (4) 10 (5) 20.

35. Given parallelogram ABCD with diagonal AC. BF bisects $\angle ABC$ and DE bisects $\angle ADC$. DF and BE are straight lines. Then



(1) $\triangle$ BFC is isosceles.

(2) $AF = \frac{2AC}{3}$.

(3) $\angle$ BAD and $\angle$ DCB are complementary.

(4) $\angle$ EDF and $\angle$ BED are supplementary.

(5) none of the above.

36. With the use of straight edge, pencil and compass it is not possible to construct

(1) a square equal in area to a given circle.

(2) a square equal in area to a given rectangle.

(3) a triangle equal in area to a given polygon.

(4) a square equal in area to a given triangle.

(5) a triangle equal in area to a given square.

37. The bisector of an internal angle of any triangle

(1) divides the triangle into two triangles of equal areas.

(2) divides the triangle into two triangles of equal perimeters.

(3) bisects the opposite side.

(4) is the mean proportional between the two sides adjacent to the angle.

(5) none of the above.

38. In the figure, CD is the diameter of the semi-circle CBED with center at O and AB = OD in length. If $\angle$ DOE measures 60°, then angle CAB measures

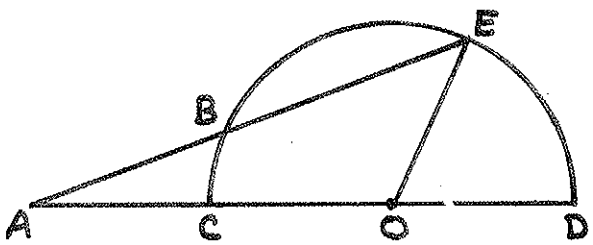
(1) 15°

(2) 20°

(3) 24°

(4) 30°

(5) none of the above.



39. A circular segment is cut from a regular pentagon, with one vertex as center and a side of the pentagon as radius of the circle. If the area of the segment is 30π, the length of each side of the pentagon is

(1) 10

(2) $\frac{30}{\pi}$

(3) $\sqrt{150}$

(4) 3π

(5) 12.

40. Let P be any point inside an equilateral triangle of side S units long. The sum of the perpendicular distances from P to each of the three sides

(1) is S.

(2) is $\frac{S}{2}\sqrt{3}$.

(3) is $\frac{S\sqrt{3}}{3}$.

(4) is $S\sqrt{3}$.

(5) varies with the position of the point.