ELEVENTH ANNUAL MATHEMATICS CONTEST

Sponsored by

THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

GEOMETRY TEST

1967

Scoring Formula: 4R - W.

Prepared by:

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DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in Geometry. For each of the 40 problems there are listed 5 possible answers. You are to work each problem and determine which is the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. A sample follows:

1. The sum of the angles of a triangle is:

(1) 360° (2) 45° (3) 90°

(4) 180° (5) 270°

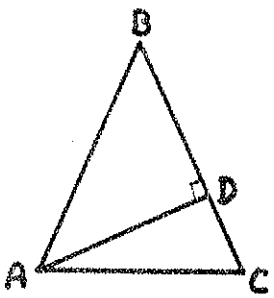
1. 

The correct answer for the sample question is "180°", which is answer (4); therefore, you should answer this question by making a heavy black mark under space 4 as indicated above.

If you should change your mind about an answer, be sure to erase completely. Avoid wild guessing, as wrong answers count against you. Do not mark more than one answer for any question. Make no stray marks of any kind on your answer sheet.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next page. The working time for the entire test is 80 minutes.

1. Given isosceles triangle ABC, with $\overline{AB} \cong \overline{BC}$; $\overline{AD}$, the altitude on $\overline{BC}$. Which of the following statements is always true?



(1) $\angle B$ is twice as large as $\angle A$.

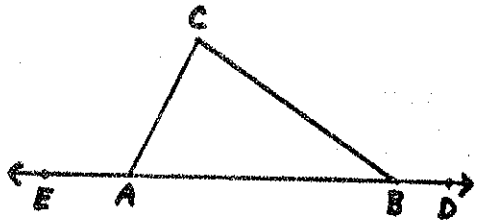
(2) $\angle DAC$ is one-half the size of $\angle A$.

(3) $\angle DAC$ is one-third the size of $\angle B$.

(4) $\angle B$ is twice as large as $\angle DAC$.

(5) none of these.

2. Given $\triangle ABC$, with exterior angles CAE and CBD at A and B, respectively. If the degree is taken as the unit of measurement, which of the following statements is always true? ($m\angle EAC$ means the measure of angle EAC)



(1) $m\angle EAC + m\angle CBD = 180$.

(2) $m\angle EAC + m\angle CBD - m\angle ACB = 180$.

(3) $m\angle CBD = m\angle CAB + m\angle CBA$.

(4) $m\angle CBD - m\angle EAC = 90$.

(5) none of these.

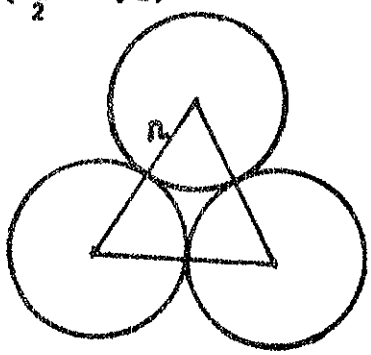
3. One angle of an equiangular polygon is seven-fourths the size of a right angle. If n is the number of sides of this polygon, then n =

(1) 17 (2) 16 (3) 12 (4) 18 (5) 20.

4. Three congruent circles of radius r are tangent externally to each other. Which of the following expressions represents the area included between the circles?

(1) $r(\sqrt{3} - 3\pi)$ (2) $r(\sqrt{2} - 3r^2\pi)$ (3) $r^2(\frac{\pi}{2} - \sqrt{2})$

(4) $r^2(\sqrt{3} - \frac{\pi}{2})$ (5) none of these.



5. The measure of the side of a square is 18 inches. What are the measures in inches of the circumferences of the inscribed and circumscribed circles, respectively?

(1) 18π, $18\pi\sqrt{2}$ (2) $6\pi r^2$, $9\pi\sqrt{2}$ (3) 9π, 18π

(4) 6π, 9π (5) 9π, 12π.

6. Which of the following sets of angles could be the face angles of a convex polyhedral angle? The sets are listed as to sizes in degrees.

(1) 45, 45, 90 (2) 60, 160, 60 (3) 105, 118, 130

(4) 128, 114, 135, 111 (5) 130, 116, 110, 112.

7. The sum of the areas of two circles is 20 sq. yd. and the difference of their areas is 15 sq. yd. Find the lengths of their radii in yards.

(1) $\sqrt{20}$, $\sqrt{15}$ (2) 20, 15 (3) $\sqrt{\frac{20}{2\pi}}$, $\sqrt{\frac{15}{2\pi}}$ (4) $\sqrt{70\pi}$, $\sqrt{40\pi}$ (5) $\sqrt{\frac{35}{2\pi}}$, $\sqrt{\frac{5}{2\pi}}$.

8. Given: O, any circle; $\overline{AM}$, a diameter; F, a point, on $\overline{AM}$ other than A, M, and O; $\overline{BF} \perp \overline{AM}$. If m $\overline{BF} = \sqrt{7}$, (means the measure of $\overline{BF}$ is $\sqrt{7}$), which of the following statements is always true?

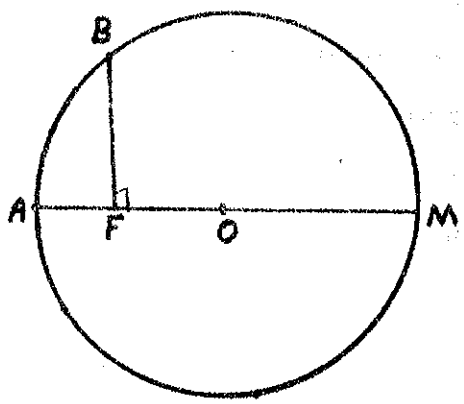
(1) m $\overline{AF} = 1$

(2) m $\overline{FM} = 7$

(3) m $\overline{AF} \times$ m $\overline{FM} = 7$

(4) m $\overline{AF} = 2$, and m $\overline{FM} = 3.5$

(5) m $\overline{AF}$ X m $\overline{FM} = 2\sqrt{7}$.



Which of the following statements is not necessarily true?

(1) All angles inscribed in a semicircle are right angles.

(2) If the opposite angles of a quadrilateral are supplementary, the four vertices of the quadrilateral lie on a circle.

(3) The segment joining the midpoints of two sides of a triangle is one-half the length of the third side.

(4) In a plane, if line l is parallel to line m, and line m is parallel to line r, then line l is parallel to line r.

(5) In space of 3 dimensions, if plane Q is parallel to line l, and plane R is parallel to line l, then plane Q is parallel to plane R.

10. With the use of a ruler (unmarked straight edge) and compass alone, it is theoretically possible to construct angles of the following sizes: 36°, 90°, and 60°. On the basis of this, and considering the fact that we can always bisect an angle (disregarding the angle of 0°), which of the angles listed below cannot be constructed with the ruler and compass only?

(1) 3° (2) 6° (3) $25\frac{1}{2}^\circ$ (4) 51° (5) 10°.

11. How far from a circle with radius 6 must a point lie in order for the secant from that point through the center to be twice as large as the tangent from that point?

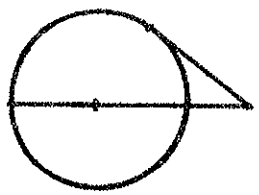
(1) 8

(2) 4

(3) 12

(4) 5

(5) 3.



12. Given right triangle ABC, with right angle at B; $\overline{BF} \perp \overline{AC}$ at F.

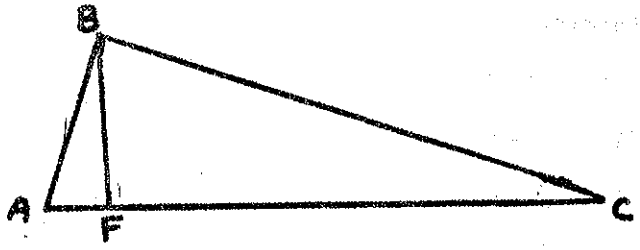
If $\overline{AF} = 4$, $\overline{FC} = 9$, which of the following statments is false?

(1) $\overline{BF} = 6$

(2) $\overline{AB} = 2\sqrt{13}$

(3) $\overline{BC} = 3\sqrt{13}$

(4) $\triangle ABF \sim \triangle BFC$



(5) one of the above statements is false.

13. If P is any interior point on the common chord $\overline{QX}$ of intersecting circles O and O′, and if $\overline{KA}$, a chord in O′, and $\overline{FM}$, a chord in O, intersect in P, then which statement below is always true?

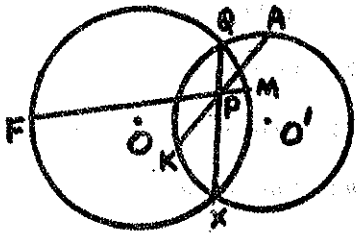
(1) F, M, K, and A are cyclic (a circle exists which contains them).

(2) $\overline{PA}$ bisects $\angle QPM$.

(3) $\overline{FM}$ is necessarily a diameter of O.

(4) Radius of O is greater than that of O′.

(5) none of these.



14. For one of the following true statements the converse is false. Identify that statement.

(1) If a quadrilateral is a parallelogram, its diagonals bisect each other.

(2) If a quadrilateral is a parallelogram, its opposite angles are congruent.

(3) If a quadrilateral is a parallelogram, its opposite sides are congruent.

(4) If two sides of a quadrilateral are parallel and congruent the figure is a parallelogram.

(5) If two angles form a vertical pair they are congruent to each other.

15. Given: circle O; $\overline{PA}$, a secant intersecting the circle at B and A; the length of $\overline{BP}$ the same as that of the radius; and $\overline{PC}$, a secant through O, terminating at C in the circumference.

Which of the following statements is always true?

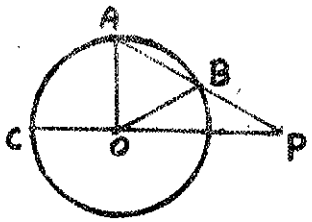
(1) $\angle AOC$ is a right angle.

(2) $\angle AOB$ contains 60°.

(3) $\angle AOB$ contains 45°.

(4) $m\angle AOC = 3\, m\angle P$.

(5) none of these.



16. Which of the following statements concerning geometry is false?

(1) Geometry treats of the following concepts: point, line, surface, and solid, taken individually or in combination.

(2) Geometry is based fundamentally on undefined terms and unproved statements.

(3) Geometry is space science.

(4) The terms "postulate" and "axiom" are often used synonymously.

(5) In geometry a postulate is often deductively derived from a theorem.

17. In proving that a geometric figure is composed of the set of points that satisfy a given condition, it is:

(1) Sufficient to prove that every point of the figure is a point that satisfies the condition.

(2) Sufficient to prove that every point that satisfies the condition is a point of the figure.

(3) Sufficient to prove that every point not on the figure is not a point that satisfies the condition.

(4) Sufficient to establish steps (1) and (2), or steps (1) and (3).

(5) Always necessary to use the indirect method.

18. Which of the following statements below concerning similar triangles is incorrect?

(1) Corresponding altitudes of two similar triangles have the same ratio as any two corresponding sides.

(2) The corresponding medians of two similar triangles have the same ratio as any two corresponding sides.

(3) The areas of two similar triangles have the same ratio as any two corresponding sides.

(4) The perimeters of two similar triangles have the same ratio as any two corresponding sides.

(5) If two right triangles have an acute angle of one congruent to an acute angle of the other, the triangles are similar.

19. A pipe two inches in diameter is to be replaced by one that will deliver twice as much water perminute. Assuming the carrying power of a pipe under the same water pressure is proportional to the cross section area, the measure of the diameter in inches must be which of the following numbers?

(1) 2.5 (2) 4 (3) $2\sqrt{2}$ (4) 2π (5) 3.

20. A cylindrical tank is lying on its side in a horizontal position. If the tank is 48 inches long and 16 inches in diameter, how many cubic inches of liquid does it contain when the depth of the liquid is 4 inches?

(1) $12\pi - 16\sqrt{3}$

(2) $8\pi + 16\sqrt{3}$

(3) $12\pi(8\sqrt{3})$

(4) $16\pi + 12\sqrt{3}$

(5) $256(4\pi - 3\sqrt{3})$.

21. The length of a rectangular parallelepiped is three times the height, and the width is twice the height. If a diagonal is $3\sqrt{14}$, find the volume.

(1) 54 (2) 162 (3) 72 (4) 64 (5) 40.

22. The diagonal of a cube is $8\sqrt{3}$. The area of the cube is:

(1) 512 (2) 64 (3) 384 (4) 438 (5) 164.

23. If A and B are distinct points, which of the following does not represent one of $\overline{AB}$, $\overrightarrow{AB}^{\circ}$, $\overrightarrow{AB}$, or $\overleftrightarrow{AB}$?

(1) $\{A\} \cup \{x \mid A - x - B\}$

(2) $\{A,B\} \cup \{x \mid A - x - B\} \cup \{x \mid A - B - x\}$

(3) $\{A,B\} \cup \{x \mid A - B - x\}$

(4) $\overrightarrow{AB} \cup \overrightarrow{BA}$

(5) $\{A,B\} \cup \{x \mid A - x - B\}$.

24. One of the following statements concerning three distinct collinear points A, B and C, is false. Identify this statement. If B is between A and C, then:

(1) $\overline{AB} \cap \overline{BC} = \{B\}$

(2) $\overrightarrow{AC} \cup \overleftarrow{BC} = \overrightarrow{AC}$

(3) $\overrightarrow{CA} \cap \overrightarrow{AC} = \overline{AC}$

(4) $\overrightarrow{AC} \cap \overleftarrow{BC} = \overrightarrow{BC}$

(5) $\overline{AB} \cup \overline{BC} = \overline{AC}$.

25. On a cartesian plane, three points and their coordinates are as follows: A(-4,3), B(-1, -1), and C(3,2). Which of the following statements about these points is true?

(1) A, B, anc C are the vertices of an isosceles triangle.

(2) A, B, and C are the vertices of an equilateral triangle.

(3) A, B, and C are the vertices of a right triangle.

(4) A, B, and C lie on a straight line.

(5) none of these.

26. On a cartesian plane, three points and their coordinates are as follows: A(-5,6), B(-1,3), and C(7,-3). Which of the following statements about these points is true?

(1) A, B, and C are the vertices of an equilateral triangle.

(2) A, B, and C are the vertices of an isosceles triangle.

(3) A, B, and C are the vertices of a right triangle.

(4) A, B, and C are collinear.

(5) A, B, and C are the vertices of a scalene triangle.

27. The base of an isosceles triangle is the segment joining the points whose coordinates are (6,1) and (-1,2). The abscissa of the vertex is 3. The ordinate of the vertex is:

(1) 5 (2) 10 (3) 4 (4) 6 (5) 8.

28. In the figure, if $\overline{AB}$ is parallel to $\overline{CD}$, then which of the following must be true?

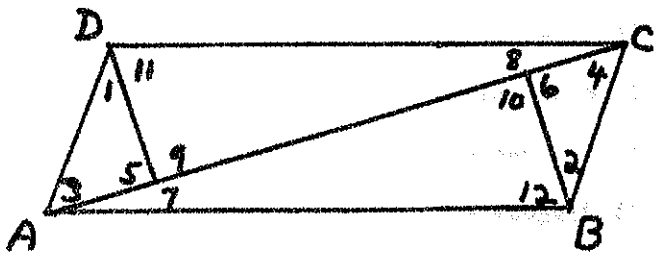
(1) $\angle 1 = \angle 2$

(2) $\angle 7 = \angle 8$

(3) $\angle 3 = \angle 4$

(4) $\angle 5 = \angle 6$

(5) $\angle 11 = \angle 12$.



29. For proving that a convex quadrilateral is a parallelogram, which of the following procedures is not acceptable?

(1) Showing that opposite sides are parallel.

(2) Showing that opposite sides are congruent.

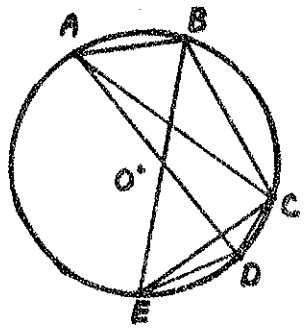
(3) Showing that opposite angles are congruent.

(4) Showing that consecutive angles are complementary.

(5) Showing that consecutive angles are supplementary.

30. Given: Circle O.

If the measures are in degrees, and m $\overset{\frown}{AB}$ = 40, m $\overset{\frown}{BC}$ = 70, m $\overset{\frown}{CD}$ = 30, and m $\overset{\frown}{EA}$ = 130, which of the angles measures 55?



(1) $\angle$ ABC

(2) $\angle$ BCE

(3) $\angle$ CDA

(4) $\angle$ DEB

(5) $\angle$ ACE.

31. In one of the following situations in three dimensional space, perpendiculars do not exist, or are not unique. Identify this instance.

(1) A line $\perp$ to a given line through a point not on the given line.

(2) A line $\perp$ to a given plane through a point not on the given plane.

(3) A line $\perp$ to a given plane through a point on the given plane.

(4) A plane $\perp$ to a given line through a point on the given line.

(5) A line $\perp$ to a given line through a point on the given line.

32. A polygon of 10 sides contains at most:

(1) 25 diagonals

(2) 30 diagonals

(3) 35 diagonals

(4) 15 diagonals

(5) 20 diagonals.

33. The length of a diagonal of a square is a + b. The area of the square is:

(1) $(a + b)^2$

(2) $\frac{1}{2}(a + b)^2$

(3) $a^2 + b^2$

(4) $\frac{1}{2}(a^2 + b^2)$

(5) $\frac{1}{4}(a^2 + b^2)$.

34. The length of a diagonal of a square is a + b. The perimeter of the square is:

(1) $\frac{a + b}{2}$

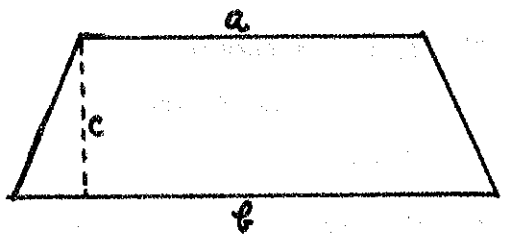
(2) $2\sqrt{2}(a + b)$

(3) $\frac{1}{2}(a + b)^{\frac{1}{2}}$

(4) $\frac{1}{2}(2a + b)^{\frac{1}{2}}$

(5) $\frac{1}{2}(2a + b)^{\frac{1}{2}}$.

35. The bases of an isosceles trapezoid are represented by a and b, where $b > a$. The altitude is represented by c. Which of the following expressions stands for the perimeter of the trapezoid?



(1) $\sqrt{4c^2 + (b - a)^2}$

(2) $\frac{1}{2} b^2 (a + c)$

(3) $(a + b) + \sqrt{4c^2 - (b - a)^2}$

(4) $(a + b)\sqrt{4c^2 - (b - a)^2}$

(5) $(a + b) + \sqrt{4c^2 + (b - a)^2}$.

36. If the area of the interior region of an equilateral triangle is $4\sqrt{3}$ sq. units, what is the length of a side?

(1) $2\sqrt{3}$ units

(2) $\frac{4}{3}\sqrt{3}$ units

(3) $\sqrt{3}$ units

(4) 4 units

(5) none of these.

37. The length of the diagonal of a square is b + c units. The perimeter of a second square which has four times the area of the first square is:

(1) $4\sqrt{2}\,(b + c)$ units.

(2) $4\,(b + c)$ units

(3) $\sqrt{2}\,(b + c)$ units

(4) $\frac{\sqrt{2}\,(b + c)^2}{4}$ units.

(5) none of these.

38. The lengths of the sides of a triangle are as follows: 6 inches; 8 inches; and 10 inches. The area of the triangle is:

(1) 24 sq. in.

(2) 12 sq. in.

(3) 18 sq. in.

(4) 16 sq. in.

(5) none of these.

39. Given: $\overline{AB}$, a chord, other than a diameter, in circle O; P, any point on minor arc AB different from A and B; C, a point on major arc AB such that $m\,\overset{\frown}{AC} = m\,\overset{\frown}{BC}$. Then:

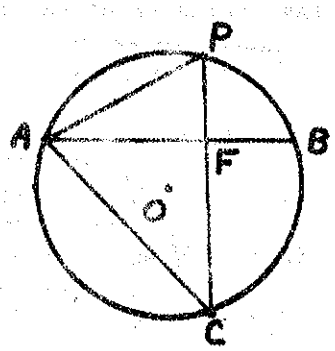
(1) $\triangle$ AFC is isosceles.

(2) $\triangle$ AFC is necessarily acute.

(3) $\triangle$ APC $\sim$ $\triangle$ AFC.

(4) $\triangle$ APF $\sim$ $\triangle$ AFC.

(5) None of these is correct.



40. Given: any triangle ABC, and $\overline{DE}$, any segment with D in $\overline{AB}$ and E in $\overline{AC}$, such that $\overline{DE}$ is parallel to $\overline{BC}$. Then the set of all points F, consisting of the midpoints of $\overline{DE}$ is the same set of points as:

(1) The median of the triangle from A.

(2) The perpendicular bisector of $\overline{BC}$.

(3) The bisector of $\angle A$.

(4) The altitude to $\overline{BC}$ from A.

(5) None of these.